(a) Maxwell-Boltzmann Distribution

In statistical mechanics, the **Maxwell - Boltzmann distribution** describes particle speeds in gases, where the particles move freely without interacting with one another, except for very brief elastic collision in which they may exchange momentum and kinetic energy, but do not change their respective states of intermolecular excitation, as a function of the temperature of the system, the mass of the particle, and speed of the particle. Particle in this context refers to the gaseous atoms or molecules - no difference is made between the two in its development and result.

Maxwell - Boltzmann system constituent identical particles that are **distinguishable** in nature which means we can distinguish them by name, color, put any number or any level on particle.

For example, if we want to identify two distinguishable particles, we can say that first particle is A and second particle is B. In another way, we can also identify the colour of particles as red for first particle and black for second particle. There is **no any restriction** on number of particles which can occupy any energy level.

Quantum mechanically, the wave function of particle will not overlap to each other because mean separation of particles is more than the thermal wavelength, which is identified by λ. (where $\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$ is defined as the thermal wavelength).

Number of ways (W) that n_i number of distinguishable particle which can be adjusted in g_i number of quantum is $W = g_i^{n_i}$.

Suppose there are l states with energies, $E_1, E_2, E_3 ....E_l$ and degeneracy of each state $g_1, g_2, g_3 ....g_l$ respectively the i^{th} level can be shown schematically

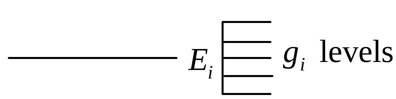

Figure 1

If there is N numbers of distinguishable particles out of these $n_1, n_2, n_3.......n_l$ particles is adjusted in energy level $E_1, E_2, E_3.......E_l$ respectively.

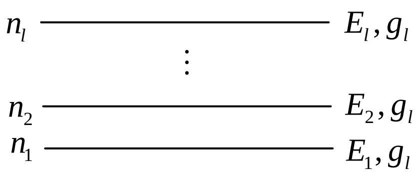

Figure 2

Total number of particle is constant $n_1 + n_2 + \cdots + n_l = N$, $\sum_{l=1}^{l} n_i = N$

$$\sum_{l=1}^{l} dn_i = dN = 0$$

Total energy of configuration is constant $n_1E_1 + n_2E_2 + \cdots$ $\sum_{l=1}^{l} n_iE_i = U$

$$\sum_{l=1}^{l} E_i dn_i = dU = 0$$

Now, number of ways selecting n_1 out of N particles then distribute it in energy state E_1 which degeneracy is g_1 then $W_1 = \frac{N!}{n_1!(N-n_1)!}.g_1^{n_1}$

Next we have to choose n_2 number of particles from remaining $N - n_1$ number of particles and have to adjusted to energy level E_2 which degeneracy is g_2

$$W_2 = \frac{(N-n_1)!}{n_2!(N-n_1-n_2)!}.g_2^{n_2}$$

The total number W of distinct ways of obtaining the distribution of $n_1, n_2, ..., n_l$ particles among the energy states $E_1, E_2, \cdots, E_l$

$$W = W_1 \cdot W_2 \cdot W_3 \cdots\cdots W_l$$

$$W = \frac{N!}{n_1!(N-n_1)!}.g_1^{n_1} \times \frac{(N-n_1)}{n_2!(N-n_1-n_2)!}.g_2^{n_2} \times \cdots \frac{(N-n_1-n_2\cdots n_{l-1})}{n_l!(N-n_1-n_2\cdots n_l)}$$

$$W = \frac{N!}{n_1!n_2!...n_l!} g_1^{n_1}.g_2^{n_2}.....g_l^{n_l} = N!\prod_{i=1}^{l}\left[\frac{g_i^{n_i}}{n_i!}\right]$$