(d) Maxwell-Boltzmann Velocity Distribution

Maxwell-Boltzmann distribution law is applicable for Ideal gas, where molecules have no vibrational or rotational energies.

In the equilibrium state of the molecules, molecules have completed their random motion and probability that a molecule has a given velocity component is independent of other two components.

In the given figure 1, dv is volume element in velocity space for a molecule at velocity $\vec{v} \equiv (v_x, v_y, v_z)$, where $v^2 = v_x^2 + v_y^2 + v_z^2$.

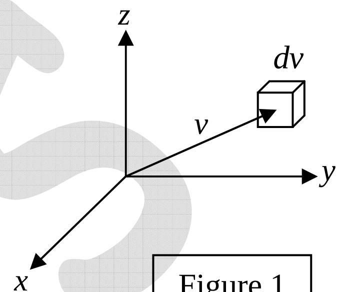

Figure 1

We need to calculate number of molecules simultaneously having component in the range v_x to $v_x + dv_x, v_y$ to $v_y + dv_y$ and v_z to $v_z + dv_z$ $v^2 = v_x^2 + v_y^2 + v_z^2$, which is equation of sphere and $dv_x dv_y dv_z$ can be replaced by $4\pi v^2 dv$. There is an assumption in Maxwell-Boltzmann distribution law that probability that a molecule selected at random has velocities in a given range is a purely function of the magnitude of velocity and the width of the interval.

The Distribution in Terms of Magnitude

$$\phi(E)\,dE = \frac{2\pi}{(\pi k_B T)^{3/2}} E^{1/2} \exp\left(-\frac{E}{k_B T}\right) dE$$

$$E = \frac{1}{2}mv^2 \Rightarrow dE = mvdv$$

$$f(v)\,dv = \frac{dN}{N} = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) 4\pi v^2 dv \quad 0 < v < \infty$$

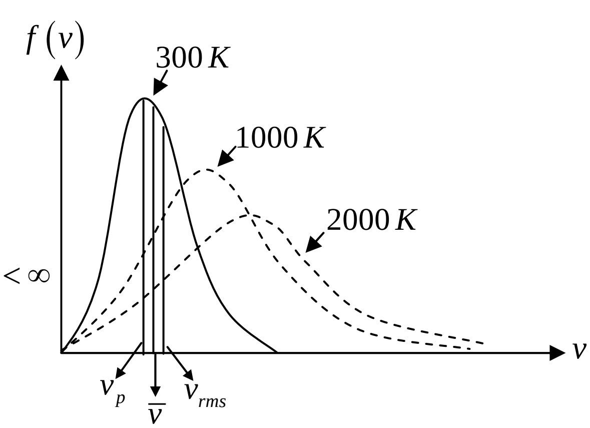

Figure 2: probability distribution $f(v)$ at equilibrium temperature (T)

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$$f\left(v_x, v_y, v_z\right) dv_x dv_y dv_z = \frac{dN}{N} = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m\left(v_x^2 + v_y^2 + v_z^2\right)}{2k_B T}\right) dv_x dv_y dv_z ,$$

where $-\infty < v_x < \infty, -\infty < v_y < \infty, -\infty < v_z < \infty$