



Learn Physics in Right Way

CSIR NET-JRF Physical Sciences July-2025 Complete Solution

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CSIR NET-JRF Physical Sciences Paper July-2025 Solution-Mathematical Physics

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PART B

Q21. The solutions of the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of

- (a) ellipses with different eccentricities (b) circles with different centers
(c) circles with different radii (d) ellipses with different foci

Ans.: (c)

Solution.: $\frac{dy}{dx} = -\frac{x}{y+1} \Rightarrow (y+1)dy = -xdx \Rightarrow \frac{(y+1)^2}{2} = -\frac{x^2}{2} + C \Rightarrow x^2 + (y+1)^2 = 2C = R^2$.

It's a circles with different radii.

Q22. For the function $f(z) = \exp\left[z-1 + \frac{1}{z-1}\right]$

- (a) $z = 1$ is a pole of order one (b) $z = 1$ is an essential singularity
(c) $z = 1$ is a pole of order two (d) $z = 1$ is a removable singular point

Ans.: (b)

Solution.: $f(z) = \exp\left[z-1 + \frac{1}{z-1}\right] = \exp[z-1] \exp\left[\frac{1}{z-1}\right]$
 $\Rightarrow f(z) = \left[1 + (z-1) + \frac{(z-1)^2}{2!} + \dots\right] \left[1 + \frac{1}{(z-1)} + \frac{1}{2!} \frac{1}{(z-1)^2} + \dots\right]$

Since there are infinite terms negative powers of $(z-1)$, $z = 1$ is an essential singularity.

Q23. For the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, which of the following is true?

- (a) $A^3 = 5A^2 - 4A - 2$ (b) $A^3 = 4A^2 - 6A + 3$
(c) $A^3 = 5A^2 - 5A - 1$ (d) $A^3 = 8A^2 + 3A - 4$

Ans.: (a)

Solution.: $|A - \lambda I| = \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 3-\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)[- \lambda(3-\lambda) - 1] + 1(\lambda - 0) = 0$

$$\Rightarrow (2-\lambda)[-3\lambda + \lambda^2 - 1] + \lambda = 0 \Rightarrow 2\lambda^2 - 6\lambda - 2 - \lambda^3 + 3\lambda^2 + \lambda + \lambda = 0$$

$$\Rightarrow \lambda^3 - 5\lambda^2 + 4\lambda + 2 = 0 \Rightarrow \lambda^3 = 5\lambda^2 - 4\lambda - 2$$

From Cayley-Hamilton Theorem $A^3 = 5A^2 - 4A - 2I$.

Q24. The value of the integral $\int_1^e dy \int_0^5 dx \delta(x^2 - y^2) \ln(xy)$ is

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{e}$

(d) $\frac{e}{5}$

Ans.: (a)

Solution.: $I = \int_{y=1}^e dy \int_{x=0}^5 dx \delta(x^2 - y^2) \ln(xy)$

Let $f(x) = x^2 - y^2 = 0 \Rightarrow x = y$ and $x = -y$ (but $x \geq 0$ in integration range, so only $x = y$ if $y \in [0, 5]$; here $y \in [1, e]$, so $x = y$ is inside $[0, 5]$)

$$\delta(x^2 - y^2) = \frac{\delta(x - y)}{|2x|} = \frac{\delta(x - y)}{2x} \quad \because x > 0$$

$$I = \int_{y=1}^e \int_{x=0}^5 \frac{\delta(x - y)}{2x} \ln(xy) dx dy = \int_{y=1}^e \frac{1}{2y} \ln(y \cdot y) dy = \int_{y=1}^e \frac{\ln y}{y} dy$$

Let $\ln y = t \Rightarrow \frac{1}{y} dy = dt \Rightarrow I = \int_{t=0}^1 t dt = \frac{1}{2}$.

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PART C

Q46. Let $P_n(x)$ be a polynomial of degree n with real coefficients, where $n = 0, 1, 2, 3, \dots$. If

$$\int_2^4 P_n(x) P_m(x) dx = \delta_{mn}, \text{ then}$$

(a) $P_1(x) = \pm \sqrt{\frac{3}{2}}(3-x)$

(b) $P_1(x) = \pm \sqrt{\frac{3}{2}}(2-x)$

(c) $P_1(x) = \pm \sqrt{\frac{3}{2}}(1-x)$

(d) $P_1(x) = \pm \sqrt{3}(3+x)$

Ans.: (a)

Solution.: Let $P_0(x) = a_0 \Rightarrow \int_2^4 [P_0(x)]^2 dx = 1 \Rightarrow a_0^2(4-2) = 1 \Rightarrow a_0 = \pm \frac{1}{\sqrt{2}}$

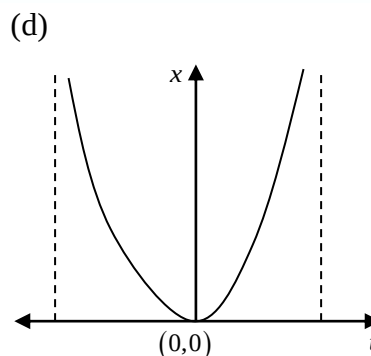
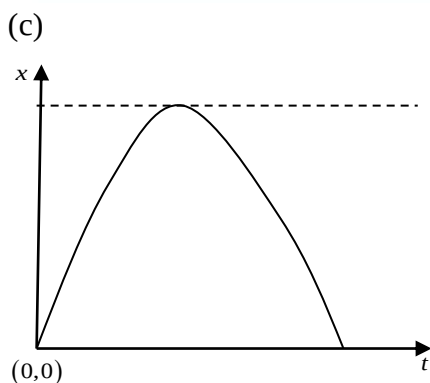
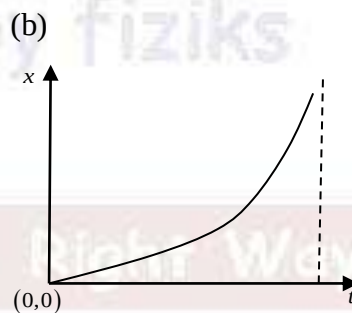
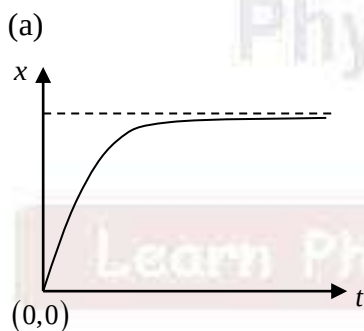
Let $P_1(x) = Ax + B \Rightarrow \int_2^4 P_1(x) P_0(x) dx = 0 \Rightarrow \pm \frac{1}{\sqrt{2}} \int_2^4 (Ax + B) dx = 0 \Rightarrow \int_2^4 (Ax + B) dx = 0$

$$\left[\frac{Ax^2}{2} + Bx \right]_2^4 = 0 \Rightarrow B = -3A \Rightarrow P_1(x) = A(x-3)$$

$$\Rightarrow \int_2^4 [P_1(x)]^2 dx = 1 \Rightarrow A^2 \int_2^4 (x-3)^2 dx = 1 \Rightarrow A^2 \frac{2}{3} = 1 \Rightarrow A = \pm \sqrt{\frac{3}{2}}. \text{ Thus } P_1(x) = \pm \sqrt{\frac{3}{2}}(3-x).$$

Q49. Which one of the following curves best represents the solution of the differential equation

$$\frac{dx}{dt} + x = 1, \text{ with the initial condition } x(0) = 0?$$



Ans.: (a)

Solution.: $\frac{dx}{dt} + x = 1$, with the initial condition $x(0) = 0$?

$$\Rightarrow \frac{dx}{dt} = 1 - x \Rightarrow \frac{dx}{1-x} = dt \Rightarrow -\ln(1-x) = t + c_1 \Rightarrow 1-x = ce^{-t}$$

$$\because x(0) = 0 \Rightarrow 1-0 = ce^{-0} \Rightarrow c = 1 \Rightarrow x(t) = 1 - e^{-t}$$

Q47. The value of the integral $\int_0^\infty \frac{\cos \alpha x}{1+x^2} dx$, where α is a positive real number, is

- (a) $\frac{\pi}{2} e^{-\alpha}$ (b) $\pi e^{-\alpha}$ (c) $\frac{\pi}{2} e^{-(\alpha/2)}$ (d) $\pi e^{-(\alpha/2)}$

Ans.: (a)

Q50. From a straight-line segment of unit length, three points are chosen at random, one after another. The probability that they are in increasing order is

- (a) $\frac{1}{3}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{6}$

Ans.: (d)

Solution.: We have straight line segment of unit length. Three points A, B, C are chosen at random, one after another along the segment. We want the probability that they are in increasing order along the segment, i.e. $A < B < C$ in terms of positions.

“One after another” means we choose A first, then B , then C , independently and uniformly from $[0, 1]$.

The three points A, B, C are distinct with probability 1. Any ordering of the three distinct numbers is equally likely if they were chosen without regard to order but here they are labeled by the order of selection.

The possible strict inequalities between A, B, C are ${}^3P_3 = 3! = 6$.

$A < B < C, A < C < B, B < A < C, B < C < A, C < A < B, C < B < A$.

$$\text{So, } P(A < B < C) = \frac{1}{6}.$$



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Solution-Classical Mechanics

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PART B

Q25. A sphere of radius R is held fixed on the horizontal ground. A point particle of mass m slides without friction from the top under the action of earth's gravity, as shown in the figure. The speed of the particle when it leaves the surface of the sphere is

(a) $\sqrt{\frac{2}{3}gR}$

(b) $\sqrt{\frac{3}{4}gR}$

(c) $\sqrt{2gR}$

(d) $\sqrt{gR}$

Ans.: (a)

Solution:

Let the particle be at polar angle θ from the top when it loses contact. Radial force balance

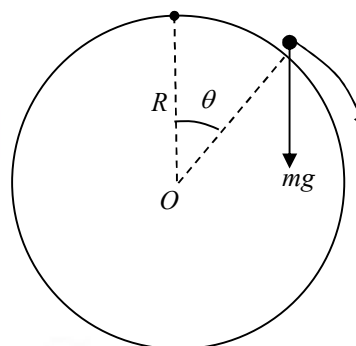
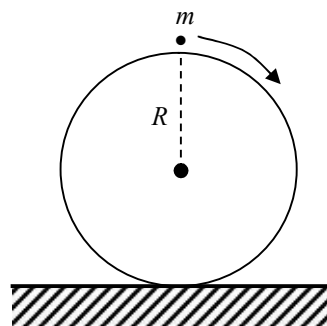
$$\frac{mv^2}{R} = mg \cos \theta \Rightarrow v^2 = gR \cos \theta$$

Here normal reaction $N = 0$

Apply conservation of energy: $mgR(1 - \cos \theta) = \frac{1}{2}mv^2$

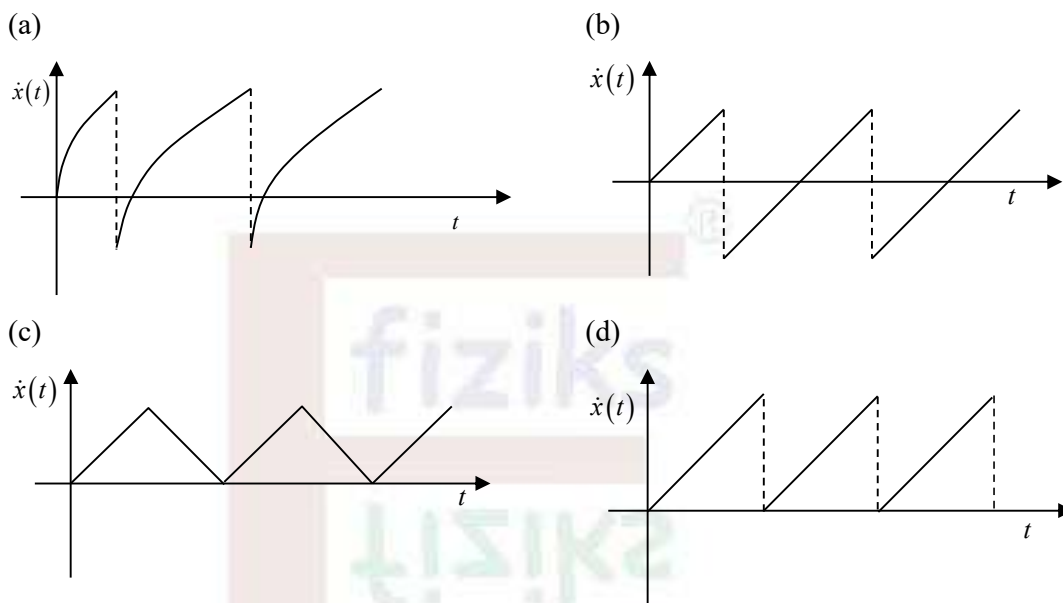
$$\cancel{m} \cancel{g} R (1 - \cos \theta) = \frac{1}{2} \cancel{m} \cancel{g} R \cos \theta \Rightarrow 1 = \frac{3}{2} \cos \theta \Rightarrow \cos \theta = \frac{2}{3}$$

$$\text{Thus } v^2 = gR \cos \theta = \frac{2}{3}gR \Rightarrow v = \sqrt{\frac{2}{3}gR}$$



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Q26. A particle of mass m is subjected to a potential $V(x) = V_0 \Theta(x) - kx$, where V_0 and k are positive constants and V_0 is much larger than the energy of the particle. The function $\Theta(x) = 1$ for $x \geq 0$ and equals 0 otherwise. The particle starts from rest at $t = 0$ and $x = -5$. In the limit $V_0 \rightarrow \infty$, the graph for $\dot{x}(t)$ is best represented by



Ans.: (b)

Solution.:

(i) Particle cannot exist in $x > 0$ because $V(x)$ becomes infinity there

$$[V(x) = V_0 - kx \text{ and } V_0 \rightarrow \infty]$$

(ii) For $x < 0$: $V(x) = -kx$, $F = -\frac{dV}{dx} = k \Rightarrow a = \frac{k}{m} = \text{constant}$

$$\frac{dv}{dt} = a \Rightarrow v = at + u. \text{ At } t = 0, u = 0 \Rightarrow v = at$$

$$\frac{dx}{dt} = at \Rightarrow x = \frac{1}{2}at^2 + x_0. \text{ At } t = 0, x = -5 \Rightarrow x_0 = -5. \text{ Thus } x = \frac{1}{2}at^2 - 5$$

$$\text{As } t \text{ increases, } x \text{ increases and it becomes zero at } t = t_1; 0 = \frac{1}{2}at_1^2 - 5 \Rightarrow t_1 = \sqrt{\frac{10}{a}}$$

At $t = t_1$ collision with rigid boundary at $x = 0$ takes place velocity changes from $v_{\max}$ to $-v_{\max}$.

(iii) Now after the collision the same constant acceleration a acts, so $v(t)$ again rises linearly from $-v_{\max}$ through 0 to $+v_{\max}$. This cycle repeats with constant period.

So option (b) is correct option.

Q27. The Hamiltonian of a system is given by $H(x, p) = -[p^2 + V^2(x)]^{1/2}$, where x and p are generalized coordinate and momentum respectively and $V(x) \geq 0$. The corresponding Lagrangian is

- (a) $-V(x)\sqrt{1-\dot{x}^2}$ (b) $-V(x)/\sqrt{1-\dot{x}^2}$
(c) $V(x)\sqrt{1-\dot{x}^2}$ (d) $V(x)/\sqrt{1-\dot{x}^2}$

Ans.: (c)

Solution.:

$$\dot{x} = \frac{\partial H}{\partial p} = -\frac{g}{\partial p} \sqrt{p^2 + V^2} = -\frac{2p}{2\sqrt{p^2 + V^2}} \Rightarrow \dot{x}^2 p^2 + \dot{x}^2 V^2 = p^2 \Rightarrow p = \pm \frac{\dot{x}V(x)}{\sqrt{1-\dot{x}^2}} \Rightarrow p = -\frac{\dot{x}V(x)}{\sqrt{1-\dot{x}^2}}$$

[Note: I am choosing -ive sign because it is matching with $p = \frac{\partial L}{\partial \dot{x}}$ later]

$$L = p\dot{x} - H = -\frac{\dot{x}^2 V(x)}{\sqrt{1-\dot{x}^2}} + \sqrt{\frac{\dot{x}^2 V^2}{1-\dot{x}^2} + V^2} \Rightarrow L = \frac{-\dot{x}^2 + 1}{\sqrt{1-\dot{x}^2}} V(x) = V(x)\sqrt{1-\dot{x}^2}$$

Q28. Consider the earth to be a free rigid body symmetric about its north-south (z) axis. If the principal moments of inertia satisfy $I_z = 1.003I_x$, then its angular velocity (in the body fixed frame) would precess about the z -axis with a period of nearly

- (a) 167 days (b) 333 days (c) 556 days (d) 667 days

Ans.: (b)

Solution.: $I_x = I_y \neq I_z$

The body-frame procession frequency of the angular velocity vector is

$$\Omega = \frac{I_z - I_x}{I_x} \omega_z = \frac{1.003I_x - I_x}{I_x} \times \frac{2\pi}{1 \text{ day}} \\ \Rightarrow \frac{2\pi}{T} = \frac{0.003 \times 2\pi}{1 \text{ day}} \Rightarrow T = \frac{1}{0.003} \text{ days} \Rightarrow T \approx 333 \text{ days}$$

PART C

Q51. For a free particle of mass m , consider the following time dependent quantity in phase space

$$Q = \frac{qp}{m} - \frac{p^2 t}{m^2}$$

where q and p are the canonically conjugate position and momentum coordinates respectively.

Then $\frac{dQ}{dt}$ is given by

- (a) 0 (b) $\frac{p^2}{m^2}$ (c) $-\frac{p^2}{m^2}$ (d) $\frac{qp}{mt}$

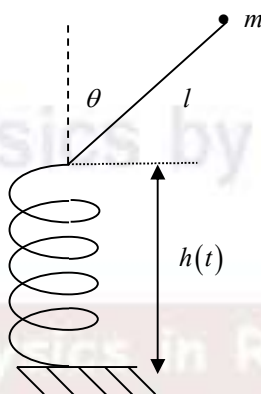
Ans.: (a)

Solution.: Time evolution of Q is given by $\frac{dQ}{dt} = [Q, H] + \frac{\partial Q}{\partial t}$

For a free particle $H = \frac{p^2}{2m}$

$$\frac{dQ}{dt} = \left[\frac{qp}{m} - \frac{p^2 t}{m^2}, \frac{p^2}{2m} \right] - \frac{p^2}{m^2} = \frac{p}{m} \frac{1}{2m} [q, p^2] - \frac{p^2}{m^2} = \frac{p}{2m^2} 2p - \frac{p^2}{m^2} \Rightarrow \frac{dQ}{dt} = 0$$

Q52. A massless rod of length l is hinged at the extreme end of a vertical spring whose other end is fixed to the ground. A point mass m is fixed at end of the rod, as shown in the figure.



Assume harmonic motion of the spring given by $h(t) = h_0(2 + \cos \omega t)$, where $h_0 > l$. The equation of motion of the mass (confined to the plane of the figure) is given by

- (a) $l\ddot{\theta} + \omega^2 h_0 \sin \theta \sin \omega t - g \sin \theta = 0$ (b) $l\ddot{\theta} + \omega^2 h_0 \sin \theta \cos \omega t - g \sin \theta = 0$
(c) $l\ddot{\theta} + \omega^2 h_0 \sin \theta \cos \omega t + g \sin \theta = 0$ (d) $l\ddot{\theta} - \omega^2 h_0 \sin \theta \sin \omega t + g \sin \theta = 0$

Ans.: (b)

Solution.: $x = l \sin \theta \Rightarrow \dot{x} = l \cos \theta \dot{\theta}$

$$y = l \cos \theta + h(t) = l \cos \theta + 2h_0 + h_0 \cos \omega t \Rightarrow \dot{y} = -l \sin \theta \dot{\theta} - h_0 \omega \sin \omega t$$

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) = \frac{1}{2} m \ell^2 \dot{\theta}^2 + \frac{1}{2} m h_0^2 \omega^2 \sin^2 \omega t + m h_0 \omega \ell \dot{\theta} \sin \theta \sin \omega t$$

$$V = mg [h(t) + \ell \cos \theta] = mg [2h_0 + h_0 \cos \omega t + \ell \cos \theta]$$

$$L = T - V = \frac{1}{2} m \ell^2 \dot{\theta}^2 + \frac{1}{2} m h_0^2 \omega^2 \sin^2 \omega t + m h_0 \omega \ell \dot{\theta} \sin \theta \sin \omega t - 2mgh_0 - mgh_0 \cos \omega t - mg\ell \cos \theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m \ell^2 \dot{\theta} + m h_0 \omega \ell \sin \theta \sin \omega t$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = m \ell^2 \ddot{\theta} + m h_0 \omega \ell \dot{\theta} \cos \theta \sin \omega t + m h_0 \omega^2 \ell \sin \theta \cos \omega t$$

$$\frac{\partial L}{\partial \theta} = m h_0 \omega \ell \dot{\theta} \cos \theta \sin \omega t + mg \ell \sin \theta$$

LEM

$$m \ell^2 \ddot{\theta} + m h_0 \omega^2 \ell \sin \theta \cos \omega t - mg \ell \sin \theta = 0 \Rightarrow \ell \ddot{\theta} + h_0 \omega^2 \sin \theta \cos \omega t - g \sin \theta = 0$$

Q53. In its rest frame, a source emits light in a conical beam of width -45° to 45° . An observer is moving towards the source with a speed v . For the observer, the beam width appears to be -30° to 30° . The speed of the observer is closest to

(a) $0.62c$ (b) $0.50c$ (c) $0.82c$ (d) $0.41c$ **Ans.: (d)**

Solution.: $v_{LE}^x = c \cos 45^\circ \Rightarrow v_{LE}^x = -\frac{c}{\sqrt{2}}$. $v_{LO}^x = c \cos 30^\circ \Rightarrow v_{OL}^x = -\frac{\sqrt{3}}{2}c$

$$v_{OE}^x = -v = \frac{v_{OL}^x - v_{EL}^x}{1 - \frac{v_{OL}^x v_{EL}^x}{c^2}} \Rightarrow -v = \frac{\left(-\frac{\sqrt{3}}{2}c\right) - \left(-\frac{1}{\sqrt{2}}c\right)}{1 - \frac{1}{c^2} \left(-\frac{\sqrt{3}}{2}c\right) \left(-\frac{c}{\sqrt{2}}\right)} \Rightarrow -v = \frac{-\sqrt{6} + 2}{2\sqrt{2} - \sqrt{3}}c = \frac{-2.449 + 2}{2.828 - 1.732}c$$

$$\Rightarrow -v = -0.409c \Rightarrow v = 0.41c$$



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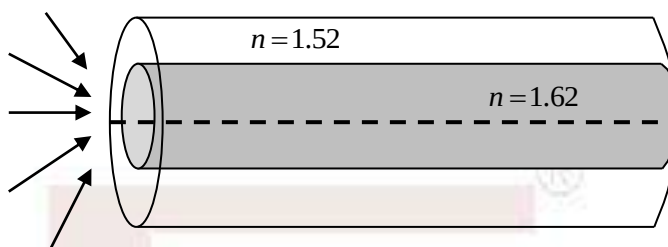
CSIR NET-JRF Physical Sciences Paper July-2025 Solution-Electromagnetic Theory

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PART B

Q34. A 1 km long optical fiber of core and clad refractive indices 1.62 and 1.52, respectively, is laid in a straight line. Several identical light pulses are launched simultaneously from air on the entrance of this fiber from different angles about its axis, as shown below. The diameter of the fiber is small compared to its length. The maximum time difference between the pulses emerging at the other end of the fiber would be closest to



- (a) 355 ns (b) 317 ns (c) 5.40 μ s (d) 5.75 μ s

Ans.: (a)

Solution.: Length $L = 1\text{ km} = 1000\text{ m}$, Core refractive index $n_1 = 1.62$

Cladding refractive index $n_2 = 1.52$

$$\text{Time taken by axial ray } T_{\min} = \frac{L}{v} = \frac{L}{c/n_1} = \frac{Ln_1}{c} = \frac{1000 \times 1.62}{3 \times 10^8} \text{ sec} = 5.4 \mu\text{s}$$

Time taken by ray at maximum acceptance angle

$$T_{\max} = \frac{L}{v \cos(90 - \theta_c)} = \frac{L}{c/n_1 \sin \theta_c} = \frac{Ln_1}{c \sin \theta_c} \text{ where } \theta_c \text{ is critical angle at core cladding interface.}$$

$$\text{So } \sin \theta_c = \frac{n_2}{n_1}. \text{ Thus } T_{\max} = \frac{Ln_1^2}{cn_2} = \frac{1000 \times (1.62)^2}{3 \times 10^8 \times 1.52} = \frac{1000 \times 2.6244}{4.56 \times 10^8} = 5.755 \mu\text{s}.$$

$$\text{Thus } \Delta T = T_{\max} - T_{\min} = 5.755 - 5.400 = 0.355 \mu\text{s} = 355 \text{ ns}$$

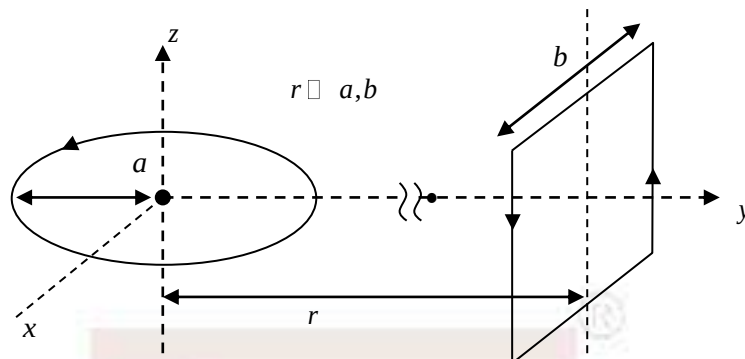
Q35. A plane electromagnetic wave $\vec{E}_I \cos(k_z z + \omega t)$ is incident normally on a perfectly reflecting mirror in vacuum. If the permittivity of free space is ϵ_0 , the force exerted on an area A of the mirror would be

- (a) $A\epsilon_0 |\vec{E}_I|^2 \hat{z}$ (b) $-\frac{A\epsilon_0}{2} |\vec{E}_I|^2 \hat{z}$ (c) $\frac{A\epsilon_0}{2} |\vec{E}_I|^2 \hat{z}$ (d) $-A\epsilon_0 |\vec{E}_I|^2 \hat{z}$

Ans.: (d)

$$\text{Solution.:. } P = \frac{F}{A} = \frac{2I}{c} = \frac{2}{c} \times \frac{1}{2} c \epsilon_0 |\vec{E}_I|^2 \Rightarrow \vec{F} = -A\epsilon_0 |\vec{E}_I|^2 \hat{z}$$

Q36. A circular loop of radius a (in the $x-y$ plane) and a square loop of side b (in the $x-z$ plane) are kept at a distance r . Both carry current I as shown in the figure. If $r \gg a, b$, the torque exerted on the square loop by the circular loop is



(a) $-\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{z}$

(b) 0

(c) $\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{x}$

(d) $-\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{x}$

Ans.: (d)

Solution.: Let $\vec{m}_1 = I(\pi a^2) \hat{z}$ and $\vec{m}_2 = Ib^2 \hat{y}$.

Torque exerted on the square loop by the circular loop

$$\vec{\tau} = \vec{m}_2 \times \vec{B}_1 \text{ where } \vec{B}_1 \text{ is field due } \vec{m}_1.$$

$$\text{So } \vec{B}_1 = \frac{\mu_0}{4\pi r^3} [3(\vec{m}_1 \cdot \hat{r}) \hat{r} - \vec{m}_1] = \frac{\mu_0}{4\pi r^3} [3(m_1 \hat{z} \cdot \hat{y}) \hat{y} - m_1 \hat{z}] = -\frac{\mu_0 m_1}{4\pi r^3} \hat{z}$$

$$\text{Thus } \vec{\tau} = m_2 \hat{y} \times \left(-\frac{\mu_0 m_1}{4\pi r^3} \hat{z} \right) = -\frac{\mu_0 m_1 m_2}{4\pi r^3} \hat{x} = -\frac{\mu_0}{4\pi} \frac{1}{r^3} \pi a^2 b^2 I^2 \hat{x}$$

Q37. In a particular inertial frame, electric field $\vec{E}$ and magnetic field $\vec{B}$ are

$$\vec{E} = E_0 \hat{x}, \quad \vec{B} = \frac{E_0}{2c} \hat{x}$$

Which of the following statements is true?

(a) There exists an inertial frame where $\vec{E} = 0, \vec{B} \neq 0$

(b) There exists no inertial frame where either $\vec{E} = 0$ or $\vec{B} = 0$

(c) There exists an inertial frame where $\vec{B} = 0, \vec{E} \neq 0$

(d) There exists an inertial frame where both $\vec{E} = 0$ and $\vec{B} = 0$

Ans.: (b)

Solution.: For electromagnetic field, two Lorentz invariants are:

$$I_1 = E^2 - c^2 B^2, \quad I_2 = \vec{E} \cdot \vec{B}. \quad \text{Given } \vec{E} = E_0 \hat{x}, \quad \vec{B} = \frac{E_0}{2c} \hat{x}$$

$$\text{Thus } I_1 = E_0^2 - c^2 \left(\frac{E_0}{2c} \right)^2 = E_0^2 - c^2 \frac{E_0^2}{4c^2} = \frac{3E_0^2}{4} \quad \text{and} \quad I_2 = \vec{E} \cdot \vec{B} = E_0 \cdot \frac{E_0}{2c} = \frac{E_0^2}{2c}$$

If $I_1 > 0$: magnetic field can be made zero in some frame.

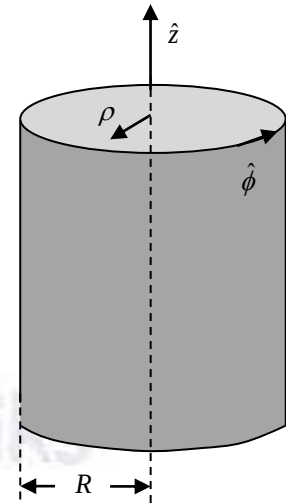
If $I_2 \neq 0$: electric and magnetic fields are not perpendicular this frame.

When $I_2 \neq 0$, there is no frame where $\vec{E} = 0$ because if $\vec{E} = 0$ in some frame, then $I_2 = 0$ in all frame. Similarly, there is no frame where $\vec{B} = 0$ if $I_2 \neq 0$.

PART C

Q48. A long cylinder of radius R carries a magnetization $\vec{M} = k\rho^2 \hat{\phi}$, where k is a constant, ρ is the radial distance from the axis and $\hat{\phi}$ is the azimuthal unit vector (see in the figure). The magnetic field inside and outside the cylinder would be

- (a) $\vec{B}_{\text{inside}} = 0, \vec{B}_{\text{outside}} = \mu_0 k \rho^2 \hat{\phi}$
- (b) $\vec{B}_{\text{inside}} = \mu_0 k \rho^2 \hat{\phi}, \vec{B}_{\text{outside}} = 0$
- (c) $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = \mu_0 k \rho^2 \hat{\phi}$
- (d) $\vec{B}_{\text{inside}} = \vec{B}_{\text{outside}} = 0$



Ans.: (b)

Solution.: Since there is no free current and cylindrical symmetry is present so $\vec{H} = 0$ and

$$\vec{B} = \mu_0 \vec{M} = \mu_0 k \rho^2 \hat{\phi} \quad (\text{inside}) \quad \text{and} \quad \vec{B}_{\text{outside}} = 0.$$

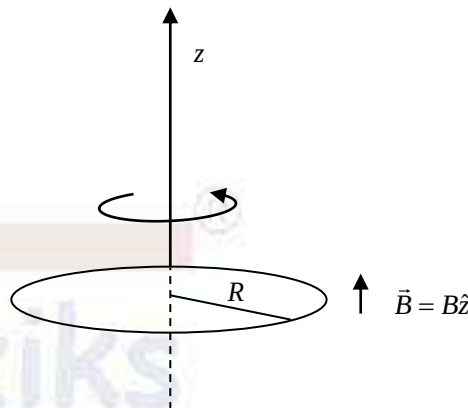
Q57. A thin circular wire loop of mass M , having radius R , carries a static charge Q . The plane of the loop is held perpendicular to a uniform magnetic field $\vec{B}$ along the z -axis passing through its center, as shown in the figure. The loop, initially at rest, can freely rotate about the z -axis. When the magnetic field is switched off the loop starts rotating with an angular frequency

(a) $\frac{QB}{M}$

(b) $\frac{QB}{2M}$

(c) $\frac{\pi QB}{M}$

(d) $\frac{\pi QB}{2M}$



Ans.: (b)

Solution.: When $\vec{B}$ is switched off, there is changing magnetic flux through the loop.

$$\text{i.e. } \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt} \Rightarrow E_\phi(R) \times 2\pi R = -\pi R^2 \frac{dB}{dt} \Rightarrow E_\phi(R) = -\frac{R}{2} \frac{dB}{dt}.$$

Force on a small element of charge dq of the loop: $dF_\phi = dqE_\phi(R)$.

Torque about z -axis: $d\tau_z = R dF_\phi = R dq E_\phi(R)$. Thus, total torque $\tau_z = RQE_\phi(R)$.

$$\Rightarrow \tau_z = RQ \left(-\frac{R}{2} \frac{dB}{dt} \right) = -\frac{QR^2}{2} \frac{dB}{dt}.$$

$$\text{Now } \tau_z = I \frac{d\omega}{dt} = MR^2 \frac{d\omega}{dt} = -\frac{QR^2}{2} \frac{dB}{dt} \Rightarrow M \frac{d\omega}{dt} = -\frac{Q}{2} \frac{dB}{dt} \Rightarrow M \int_0^\omega d\omega = -\frac{Q}{2} \int_{B_0}^0 dB$$

$$\Rightarrow M\omega = \frac{QB_0}{2} \Rightarrow \omega = \frac{QB_0}{2M}$$

Q58. The charge density of the electron cloud of a hydrogen atom is given by

$\rho(\vec{r}) = -\frac{e}{8\pi a^3} \exp(-r/a)$, where a is some characteristic length. The potential energy due to

the interaction between the proton (sitting at the origin) and the electron cloud is given by

(a) $-\frac{e^2}{2\pi\epsilon_0 a}$

(b) $-\frac{e^2}{4\pi\epsilon_0 a}$

(c) $-\frac{e^2}{\pi\epsilon_0 a}$

(d) $-\frac{e^2}{8\pi\epsilon_0 a}$

Ans.: (d)

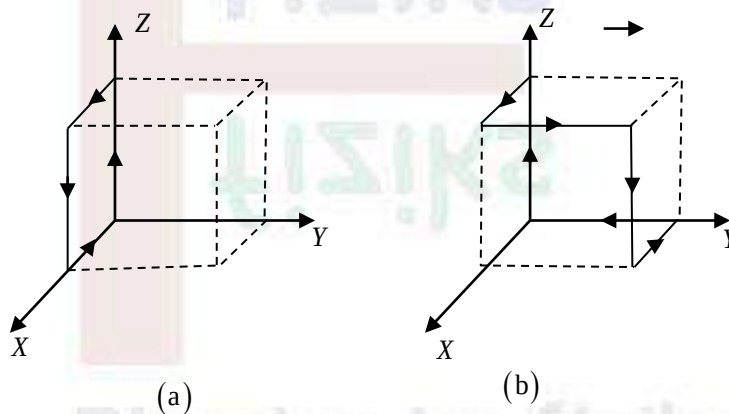
Solution.: For a spherically symmetric $\rho(\vec{r})$, the potential at center is:

$$V_{center} = \frac{1}{4\pi\epsilon_0} \int_0^\infty \frac{\rho(r')}{r'} 4\pi r'^2 dr' = \frac{1}{\epsilon_0} \int_0^\infty r' \rho(r') dr' = \frac{1}{\epsilon_0} \int_0^\infty r' \left[-\frac{e}{8\pi a^3} \exp(-r'/a) \right] dr'$$

$$\Rightarrow V_{center} = -\frac{e}{8\pi a^3 \epsilon_0} \int_0^\infty r' \exp(-r'/a) dr' = -\frac{e}{8\pi a^3 \epsilon_0} \times a^2 = -\frac{e}{8\pi a \epsilon_0}$$

Proton charge $+e$ at the origin, so $U = eV_{center} = e \left(-\frac{e}{8\pi a \epsilon_0} \right) = -\frac{e^2}{8\pi a \epsilon_0}$

Q59. Two identical cubes are shown in figures (a) and (b). The magnitude of the magnetic field at the center of the cube in (a), produced by the currents as shown, is B_0 . The magnitude of the magnetic field at the center of the cube in (b) will be



(a) $\sqrt{3}B_0$

(b) $2B_0$

(c) $\frac{3}{2}B_0$

(d) $\sqrt{2}B_0$

Ans.: (a)

Solution.: In figure (b), we have three mutually perpendicular segments like figure (a).

So total field magnitude is $B_T = \sqrt{B_0^2 + B_0^2 + B_0^2} = \sqrt{3}B_0$.

Q67. A gas of electrons (with no source of scattering) is placed in an electric field $\vec{E} = Ee^{i\omega t}(\hat{i} + \hat{k})$ and a magnetic field $\vec{B} = B\hat{k}$, where E and B are constants. The frequency at which the conductivity in the z -direction, given by the ratio of the current and the electric field, both in the z -direction, diverges is

(a) 0

(b) $\frac{eB}{m}$

(c) $-\frac{eB}{m}$

(d) $\frac{eB}{2m}$

Ans.: (a)

Solution.: $\vec{E}(t) = Ee^{i\omega t}(\hat{i} + \hat{k})$, $\vec{B} = B\hat{k}$.

Since there is no scattering so electron velocity $\vec{v}$ satisfies $m \frac{d\vec{v}}{dt} = -e[\vec{E}(t) + \vec{v} \times \vec{B}]$.

$$\text{Let } \vec{v}(t) = \vec{v}e^{i\omega t} \Rightarrow \frac{d\vec{v}}{dt} = i\omega\vec{v}e^{i\omega t} \Rightarrow m(i\omega\vec{v}e^{i\omega t}) = -e[Ee^{i\omega t}(\hat{i} + \hat{k}) + \vec{v}e^{i\omega t} \times (B\hat{k})]$$

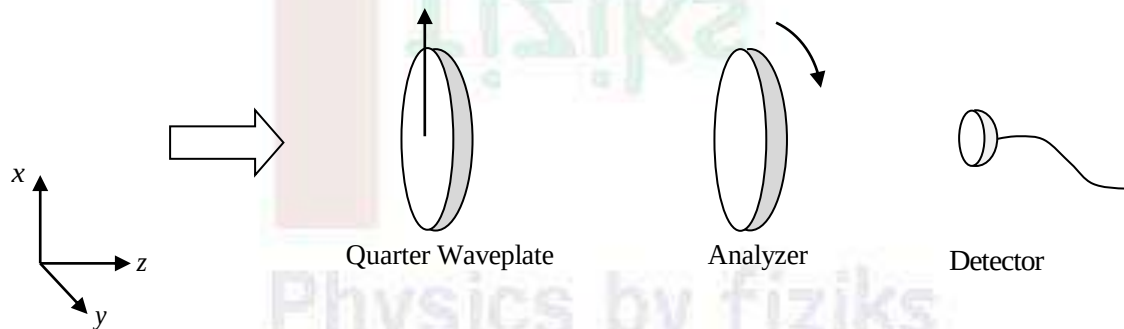
$$\Rightarrow m(i\omega\vec{v}) = -e[E(\hat{i} + \hat{k}) + B(v_y\hat{i} - v_x\hat{j})] \Rightarrow mi\omega v_z = -eE \Rightarrow v_z = -\frac{eE}{mi\omega}$$

Current density: $\vec{J} = -ne\vec{v} \Rightarrow J_z = -nev_z e^{i\omega t}$, $E_z = Ee^{i\omega t}$

$$\text{So, conductivity } \sigma_z = \frac{J_z}{E_z} = \frac{-nev_z e^{i\omega t}}{Ee^{i\omega t}} = \frac{-nev_z}{E} = \frac{-ne}{E} \times \left(-\frac{eE}{mi\omega}\right) = \frac{ne^2 E}{mi\omega}$$

Thus $\sigma_z \rightarrow \infty$ as $\omega \rightarrow 0$.

Q60. A beam of light along the z -axis passes through a quarter wave plate and an analyzer as shown in the figure. The fast axis of the quarter wave plate is aligned with the x -axis. The light intensity is measured by a detector placed after the analyzer.



Consider two scenarios where the incident light beam is (1) circularly polarized and (2) linearly polarized along the x -axis. If the polarization axis of the analyzer is rotated by open full cycle about the z -axis, the number of times the detector measures the maximum intensity in each case would be

- | | |
|---------------------|---------------------|
| (a) (1) 4 and (2) 0 | (b) (1) 2 and (2) 0 |
| (c) (1) 4 and (2) 4 | (d) (1) 2 and (2) 2 |

Ans.: (d)

Q69. A highly collimated laser beam with a diameter of 1 cm and wavelength 500 nm is directed from the earth's surface towards the moon ($\sim 384,000$ km away from the earth). Assuming ideal diffraction limited propagation in vacuum, which of the following best estimates the diameter of the beam upon returning to the earth after reflection from an ideal reflector installed on the moon.

- (a) 200 m (b) 20 m (c) 20 km (d) 200 km

Ans.: (c)





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PART B

Q29. The energy eigenstates of a one-dimensional harmonic oscillator are denoted by $|i\rangle$, where

$i = 0, 1, 2, 3, \dots$. If the momentum operator $\hat{p}$ satisfies $\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|\hat{p}|1\rangle} = \sqrt{2}$, then the value of n is

- (a) 0 (b) 1 (c) 2 (d) 3

Ans.: (d)

Solution.:

Given $\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|\hat{p}|1\rangle} = \sqrt{2}$

In a 1D simple harmonic oscillator, the momentum operator $\hat{p}$ is expressed in terms of the

ladder (annihilation $\hat{a}$ and creation $\hat{a}^\dagger$) operators as: $\hat{p} = -i\sqrt{\frac{m\hbar\omega}{2}}(\hat{a} - \hat{a}^\dagger)$

The action on energy eigenstates is: $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

$$\langle n+1|\hat{p}|n\rangle = -i\sqrt{\frac{m\hbar\omega}{2}}\langle n+1|(\hat{a} - \hat{a}^\dagger)|n\rangle = i\sqrt{\frac{m\hbar\omega}{2}}\sqrt{n+1}$$

$$\langle 2|\hat{p}|1\rangle = i\sqrt{\frac{m\hbar\omega}{2}}\sqrt{2} \quad \text{take } n=1$$

$$\frac{\langle n+1|\hat{p}|n\rangle}{\langle 2|\hat{p}|1\rangle} = \frac{\sqrt{n+1}}{\sqrt{2}} = \sqrt{2}$$

$$\Rightarrow n=3$$

Q30. A system consists of two non-interacting identical spin- $\frac{1}{2}$ particles. The spatial wave

functions for the individual particles are given by $\varphi_1(x)$ and $\varphi_2(x)$. Let x_1 and x_2 denote the positions of the particles respectively. The total wave function of the system (not necessarily normalized) can be

(a) $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2]$

(b) $[\varphi_1(x_1)\varphi_1(x_2) + \varphi_2(x_1)\varphi_2(x_2)][|\uparrow\rangle_1|\uparrow\rangle_2]$

(c) $\varphi_1(x_1)\varphi_2(x_2)|\uparrow\rangle_1|\uparrow\rangle_2$

(d) $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)][|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2]$

Ans.: (a)

Solution.: Identical spin- $\frac{1}{2}$ particles are fermions, so the total wavefunction must be antisymmetric under exchange.

The spatial part $[\varphi_1(x_1)\varphi_2(x_2) - \varphi_2(x_1)\varphi_1(x_2)]$ is antisymmetric, while Spin part:

$$[|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2]$$

is symmetric. Their product is antisymmetric overall.

Q31. A spin- $\frac{1}{2}$ system is prepared in the initial state $|\varphi\rangle = \frac{\sqrt{3}}{2}|\uparrow\rangle + \frac{1}{2}|\downarrow\rangle$ where $|\uparrow\rangle$ & $|\downarrow\rangle$ are eigenstates of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$ respectively. A measurement of $\hat{S}_z$ is followed by a measurement of $\hat{S}_x$ on the system. What is the probability that the measurement of $\hat{S}_x$ yields a value $+\frac{\hbar}{2}$?

- (a) $\frac{1}{2}$ (b) $\frac{2+\sqrt{3}}{4}$ (c) $\frac{2-\sqrt{3}}{4}$ (d) $\frac{3}{8}$

Ans.: (a)

Solution.: Given $|\varphi\rangle = \frac{\sqrt{3}}{2}|\uparrow\rangle + \frac{1}{2}|\downarrow\rangle$ where $|\uparrow\rangle$ & $|\downarrow\rangle$ are eigenstates of $\hat{S}_z$ with eigenvalues $+\frac{\hbar}{2}$ & $-\frac{\hbar}{2}$ respectively.

Probability of obtaining $|\uparrow\rangle$: $P(\uparrow) = \left|\frac{\sqrt{3}}{2}\right|^2 = \frac{3}{4}$

Probability of obtaining $|\downarrow\rangle$: $P(\downarrow) = \left|\frac{1}{2}\right|^2 = \frac{1}{4}$

The state corresponding to $+\frac{\hbar}{2}$ for $\hat{S}_x$ is: $|+\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

If the state collapsed to $|\uparrow\rangle$: $P\left(+\frac{\hbar}{2}|\uparrow\rangle\right) = \left|\langle + |_x \uparrow\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

If the state collapsed to $|\downarrow\rangle$: $P\left(+\frac{\hbar}{2}|\downarrow\rangle\right) = \left|\langle + |_x \downarrow\rangle\right|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

Using the law of total probability: $P\left(+\frac{\hbar}{2}\right) = P(\uparrow)P\left(+\frac{\hbar}{2}|\uparrow\rangle\right) + P(\downarrow)P\left(+\frac{\hbar}{2}|\downarrow\rangle\right)$

$$\Rightarrow P\left(+\frac{\hbar}{2}\right) = \frac{3}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{2}$$

Q32. A particle of mass m is in the third energy eigenstate of an infinite potential well of width a . The time interval in which the phase of this wave function changes by 2π is

- (a) $\frac{4ma^2}{3\pi\hbar}$ (b) $\frac{4ma^2}{9\pi\hbar}$ (c) $\frac{8ma^2}{3\pi\hbar}$ (d) $\frac{8ma^2}{9\pi\hbar}$

Ans.: (b)

Solution.: Energy for n^{th} level: $E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2} \Rightarrow E_3 = \frac{9\pi^2 \hbar^2}{2ma^2}$

Time-dependent wave function factor: $e^{\frac{iE_n t}{\hbar}}$

The phase change in time interval Δt is $\Delta\phi = \frac{E_n \Delta t}{\hbar}$ so

$$\Delta\phi = \frac{E_3 \Delta t}{\hbar} = 2\pi \Rightarrow \left(\frac{9\pi^2 \hbar^2}{2ma^2} \right) \frac{\Delta t}{\hbar} = 2\pi \Rightarrow \Delta t = \frac{4ma^2}{9\pi\hbar}$$

Q33. The Hamiltonian of the 1-dimensional quantum harmonic oscillator is given by

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$. The expectation value of $[D, H]$ in the ground state, where

$D = \frac{1}{2\hbar}(xp + px)$, is (in units of $\hbar\omega$)

- (a) i (b) $\frac{1}{2}$ (c) $\frac{-3i}{2}$ (d) 0

Ans.: (d)

Solution.: We need to calculate $\langle 0 | [D, H] | 0 \rangle$ where $|0\rangle$ is the ground state.

$$[D, H] = DH - HD$$

Since the ground state is an energy eigenstate, $H|0\rangle = E_0|0\rangle$

Because H is Hermitian $\langle 0 | H = E_0 \langle 0 |$.

$$\text{So, } \langle 0 | [D, H] | 0 \rangle = \langle 0 | DH - HD | 0 \rangle = \langle 0 | DH | 0 \rangle - \langle 0 | HD | 0 \rangle$$

$$\Rightarrow \langle 0 | [D, H] | 0 \rangle = E_0 \langle 0 | D | 0 \rangle - E_0 \langle 0 | D | 0 \rangle = 0$$

This result holds for any operator D when evaluated as a commutator expectation in an energy eigenstate.

PART C

Q54. $|n\rangle$ denotes the eigenvector of the number operator for a particle of mass m in a one-dimensional potential $V = \frac{1}{2}m\omega^2 x^2$ ($n = 0, 1, 2, \dots$).

For the state vector $|\varphi(x, t=0)\rangle = \frac{1}{\sqrt{3}}|1\rangle + \sqrt{\frac{2}{3}}|2\rangle$, $\langle \hat{x}(t) \rangle$ is

(a) $\frac{2\sqrt{2}}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$

(b) $\frac{4}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos \omega t$

(c) $\frac{2\sqrt{2}}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos 2\omega t$

(d) $\frac{4}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos 2\omega t$

Ans.: (b)

Solution.: For a system governed by a time-independent Hamiltonian H , any initial state $|\psi(0)\rangle = \sum_n c_n |n\rangle$ evolves in time as:

$$|\psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |n\rangle \quad \text{where } E_n = \left(n + \frac{1}{2}\right)\hbar\omega \Rightarrow E_1 = \frac{3}{2}\hbar\omega, E_2 = \frac{5}{2}\hbar\omega,$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{3}} e^{-iE_1 t/\hbar} |1\rangle + \sqrt{\frac{2}{3}} e^{-iE_2 t/\hbar} |2\rangle$$

The corresponding bra state is: $\langle\psi(t)| = \frac{1}{\sqrt{3}} e^{+iE_1 t/\hbar} \langle 1| + \sqrt{\frac{2}{3}} e^{+iE_2 t/\hbar} \langle 2|$

In quantum mechanics, the position operator $\hat{x}$ is related to the annihilation operator $\hat{a}$ and creation operator $\hat{a}^\dagger$ by: $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger)$

The action on energy eigenstates is: $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$ and $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

Now $\hat{x}|1\rangle = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger)|1\rangle = \sqrt{\frac{\hbar}{2m\omega}} (\sqrt{1}|0\rangle + \sqrt{2}|2\rangle)$

and $\hat{x}|2\rangle = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger)|2\rangle = \sqrt{\frac{\hbar}{2m\omega}} (\sqrt{2}|1\rangle + \sqrt{3}|3\rangle)$

Note: $\langle 1|\hat{x}|1\rangle = 0$, $\langle 2|\hat{x}|2\rangle = 0$

$$\langle 1|\hat{x}|2\rangle = \sqrt{\frac{\hbar}{2m\omega}} \langle 1|(\sqrt{2}|1\rangle + \sqrt{3}|3\rangle) = \sqrt{2} \sqrt{\frac{\hbar}{2m\omega}}$$

$$\langle 2|\hat{x}|1\rangle = \sqrt{\frac{\hbar}{2m\omega}} \langle 2|(\sqrt{1}|0\rangle + \sqrt{2}|2\rangle) = \sqrt{2} \sqrt{\frac{\hbar}{2m\omega}}$$

$$\begin{aligned}\langle \hat{x}(t) \rangle &= \langle \psi(t) | \hat{x}(t) | \psi(t) \rangle = \left(\frac{1}{\sqrt{3}} e^{+iE_1 t/\hbar} \langle 1 | + \sqrt{\frac{2}{3}} e^{+iE_2 t/\hbar} \langle 2 | \right) \hat{x} \left(\frac{1}{\sqrt{3}} e^{-iE_1 t/\hbar} | 1 \rangle + \sqrt{\frac{2}{3}} e^{-iE_2 t/\hbar} | 2 \rangle \right) \\ \Rightarrow \langle \hat{x}(t) \rangle &= \frac{1}{3} \langle 1 | \hat{x} | 1 \rangle + \frac{\sqrt{2}}{3} e^{-i(E_2-E_1)t/\hbar} \langle 1 | \hat{x} | 2 \rangle + \frac{\sqrt{2}}{3} e^{+i(E_2-E_1)t/\hbar} \langle 2 | \hat{x} | 1 \rangle + \frac{2}{3} \langle 2 | \hat{x} | 2 \rangle \\ \Rightarrow \langle \hat{x}(t) \rangle &= \frac{1}{3} \times 0 + \frac{\sqrt{2}}{3} e^{-i(E_2-E_1)t/\hbar} \sqrt{2} \sqrt{\frac{\hbar}{2m\omega}} + \frac{\sqrt{2}}{3} e^{+i(E_2-E_1)t/\hbar} \sqrt{2} \sqrt{\frac{\hbar}{2m\omega}} + \frac{2}{3} \times 0 \\ \Rightarrow \langle \hat{x}(t) \rangle &= \frac{2}{3} \sqrt{\frac{\hbar}{2m\omega}} \left[e^{-i(E_2-E_1)t/\hbar} + e^{+i(E_2-E_1)t/\hbar} \right] \\ \because E_1 &= \frac{3}{2} \hbar\omega, E_2 = \frac{5}{2} \hbar\omega \Rightarrow \frac{(E_2-E_1)t}{\hbar} = \frac{\hbar\omega t}{\hbar} = \omega t \\ \Rightarrow \langle \hat{x}(t) \rangle &= \frac{2}{3} \sqrt{\frac{\hbar}{2m\omega}} \left[e^{-i\omega t} + e^{+i\omega t} \right] = \frac{4}{3} \sqrt{\frac{\hbar}{2m\omega}} \cos \omega t\end{aligned}$$

Q55. The ground state wavefunction for the hydrogen atom is

$$\psi_0 = \sqrt{\frac{1}{\pi a_0^3}} e^{-\frac{r}{a_0}}, \text{ where } a_0 \text{ is the Bohr radius.}$$

Considering an additional potential H' as a perturbation to the hydrogen atom Hamiltonian,

given by

$$H' = \begin{cases} \frac{e^2}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{R} \right] & \text{for } 0 < r < R \\ 0 & \text{for } r > R \end{cases}$$

where R is the radius of the proton, $R \ll a_0$. The shift in the ground state energy due to H' is

- (a) $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{4R^2}{3a_0^2}$ (b) $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{R}{a_0}$
(c) $-\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{2R^2}{a_0^2}$ (d) $\left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{2R^2}{3a_0^2}$

Ans.: (d)

Solution.: $\psi_0 = \sqrt{\frac{1}{\pi a_0^3}} e^{-\frac{r}{a_0}} \approx \sqrt{\frac{1}{\pi a_0^3}}, \quad r \leq R \ll a_0; \quad e^{-\frac{r}{a_0}} \approx 1$

$$H' = \frac{e^2}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{R} \right], \quad 0 < r < R$$

First-order energy shift $\Delta E^{(1)} = \langle \psi_0 | H' | \psi_0 \rangle = 4\pi \int_0^R |\psi_0(r)|^2 H'(r) r^2 dr$

$$\Delta E^{(1)} \approx 4\pi \left(\frac{1}{\pi a_0^3} \right) \frac{e^2}{4\pi\epsilon_0} \int_0^R \left(\frac{1}{r} - \frac{1}{R} \right) r^2 dr = \frac{4}{a_0^3} \left(\frac{e^2}{4\pi\epsilon_0} \right) \int_0^R \left(r - \frac{r^2}{R} \right) dr$$

$$\Rightarrow \Delta E^{(1)} = \frac{4}{a_0^3} \left(\frac{e^2}{4\pi\epsilon_0} \right) \left[\frac{R^2}{2} - \frac{R^2}{3} \right] = \frac{4}{a_0^3} \left(\frac{e^2}{4\pi\epsilon_0} \right) \left(\frac{R^2}{6} \right)$$

$$\Rightarrow \Delta E^{(1)} = \left(\frac{e^2}{4\pi\epsilon_0 a_0} \right) \frac{2R^2}{3a_0^2}$$

Q56. The probability density of a free particle of mass m at time $t=0$, is given by

$A \exp\left(-\frac{x^2}{2\sigma^2(0)}\right)$. At $t > 0$, its probability density is proportional to $\exp\left(-\frac{x^2}{2\sigma^2(t)}\right)$, where

$\sigma^2(t)$ is

(a) $\sigma^2(0) + \frac{\hbar^2 t^2}{\sigma^2(0)m^2}$

(b) $\sigma^2(0) + \frac{\hbar^2 t^2}{4\sigma^2(0)m^2}$

(c) $\sigma^2(0) + \frac{4\hbar^2 t^2}{\sigma^2(0)m^2}$

(d) $\sigma^2(0) + \frac{2\hbar^2 t^2}{\sigma^2(0)m^2}$

Ans.: (b)

Solution.: For a freely propagating Gaussian wave packet with initial position variance $\sigma^2(0)$ and no initial position-momentum correlation, the variance evolves according to

$$\sigma^2(t) = \sigma^2(0) + \frac{t^2}{m^2} (\Delta p)^2$$

For the standard minimum-uncertainty Gaussian packet, $\Delta x(0) = \sigma(0)$

and $\Delta x(0) \Delta p = \frac{\hbar}{2} \Rightarrow \Delta p = \frac{\hbar}{2\Delta x(0)} = \frac{\hbar}{2\sigma(0)}$

Thus $\sigma^2(t) = \sigma^2(0) + \frac{\hbar^2 t^2}{4\sigma^2(0)m^2}$.

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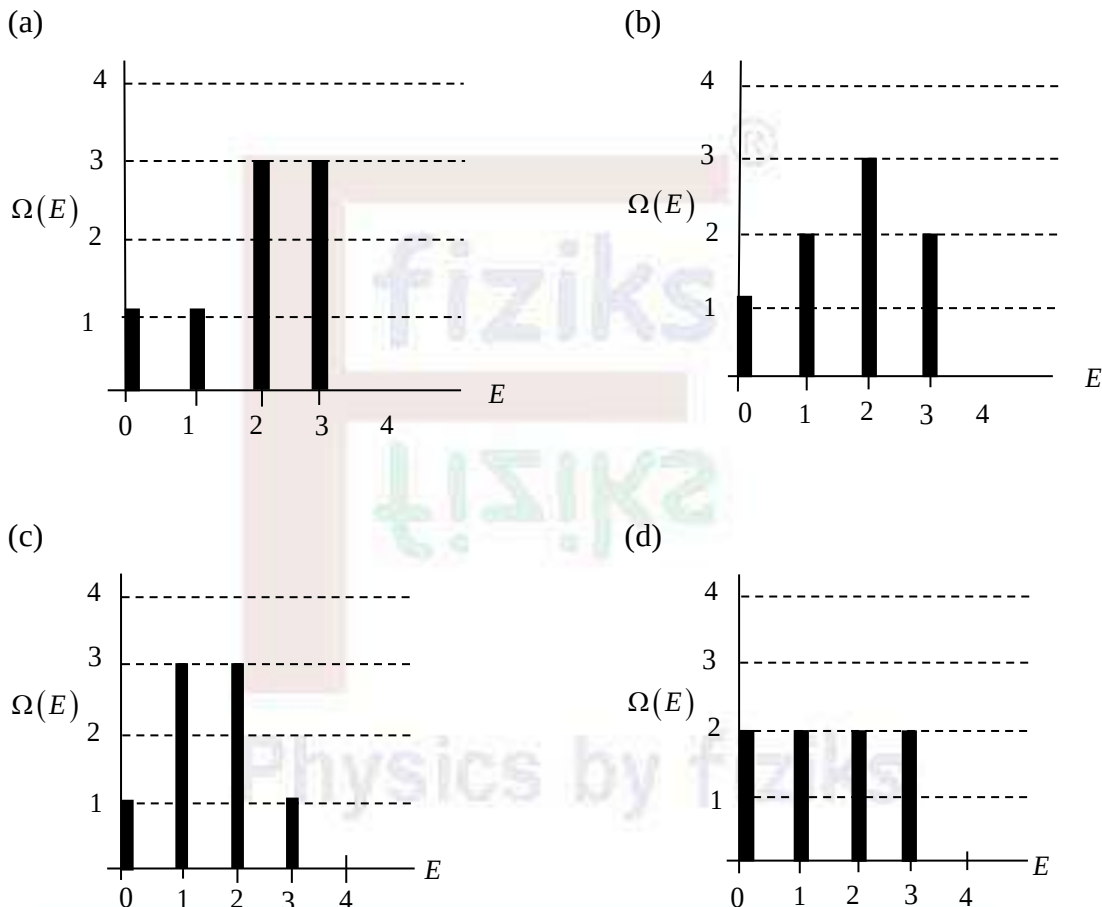
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PART B

Q38. There are two boxes, one at the ground level, and the other at a fixed height h . There are three balls of different colours, each having mass m and radius $r \ll h$. There is no restriction on the number of balls that can be simultaneously put in a given box. For a given value of the total energy E (in units of mgh , g being the acceleration due to gravity), the number of accessible microstates is $\Omega(E)$. The plot of $\Omega(E)$ vs E is



Ans.: (c)

Solution.:

1	_____	1	_____C_____	1	_____B_____	1	_____A_____
0	$\frac{ABC}{E=0}$	0	$\frac{AB}{E=1}$	0	$\frac{AC}{E=1}$	0	$\frac{BC}{E=1}$
1	$\frac{ABC}{E=3}$	1	$\frac{AB}{E=2}$	1	$\frac{AC}{E=2}$	1	$\frac{BC}{E=2}$
0	_____	0	$\frac{C}{E=2}$	0	$\frac{B}{E=2}$	0	$\frac{A}{E=2}$

Q39. The internal energy of a system is given by $U = g(N)V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$, where V is the volume, S is the entropy, N is the number of molecules and R is a constant. The function $g(N)$ is proportional to

- (a) $N^{5/3}$ (b) $N^{1/3}$ (c) $N^{2/3}$ (d) N

Ans.: (a)

Solution.: $U = g(N)V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$

For U to be extensive: $U(\lambda S, \lambda V, \lambda N) = \lambda U(S, V, N)$

$$U(\lambda S, \lambda V, \lambda N) = g(\lambda N)(\lambda V)^{-\frac{2}{3}} \exp\left[\frac{2(\lambda S)}{3(\lambda N)R}\right] = g(\lambda N)\lambda^{-\frac{2}{3}}V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$$

$$\text{Thus } g(\lambda N)\lambda^{-\frac{2}{3}}V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right] = \lambda g(N)V^{-\frac{2}{3}} \exp\left[\frac{2S}{3NR}\right]$$

$$\Rightarrow g(\lambda N)\lambda^{-\frac{2}{3}} = \lambda g(N) \Rightarrow g(\lambda N) = \lambda^{5/3}g(N)$$

$$\text{Let } \lambda = \frac{1}{N}, \Rightarrow g(1) = \left(\frac{1}{N}\right)^{5/3} g(N) \Rightarrow g(N) = g(1)N^{5/3}$$

$$\text{Also check } U \propto g(N)V^{-\frac{2}{3}} \propto N^{5/3}N^{-\frac{2}{3}} \propto N \quad \because V \propto N$$

Q40. Consider one mole of an ideal diatomic gas molecule at temperature T such that $k_B T \ll h\nu$, where ν is the frequency of its vibrational mode. If C_p and C_v are specific heats of

this gas at constant pressure and volume respectively, then the ratio $\gamma = \frac{C_p}{C_v}$, is

- (a) 2 (b) $\frac{7}{2}$ (c) $\frac{5}{3}$ (d) $\frac{9}{7}$

Ans.: (d)

Solution.: Translational d.o.f. = 3, Rotational d.o.f. = 2, Vibrational d.o.f. = 2.

$$C_v = \frac{7}{2}R, C_p = \frac{9}{2}R \Rightarrow \gamma = \frac{C_p}{C_v} = \frac{9}{7}$$

Q41. A refrigerator can be thought to be a reversible engine operating between $T_2 = 20^\circ\text{C}$ and $T_1 = -10^\circ\text{C}$. The work needed to run this is supplied by another engine, that takes in energy at the rate of 500 W and runs with 50% efficiency. If the refrigerator freezes 5 kg of water at 0°C (latent heat $Q_L = 334\text{ kJ/kg}$ for ice) in n hours, then n is closest to

- (a) 0.4 (b) 0.3 (c) 0.1 (d) 0.2

Ans.: (d)

Solution.: Coefficient of performance (COP) for a reversible refrigerator

$$COP = \frac{Q_L}{W} = \frac{T_1}{T_2 - T_1} = \frac{263}{293 - 263} = \frac{263}{30} \approx 8.767 \Rightarrow Q_L = 8.767 \times W$$

Engine supplying work W to the refrigerator:

Engine takes heat input at rate $\dot{Q}_{in} = 500\text{ W}$ runs with efficiency $\eta = 0.5$.

So work output rate of engine $\dot{W} = \eta \times \dot{Q}_{in} = 0.5 \times 500\text{ W} = 250\text{ W}$.

This $\dot{W}$ is the work input to the refrigerator.

Refrigerator heat extraction rate $\dot{Q}_L = 8.767 \times \dot{W} = 8.767 \times 250 = 2191.75\text{ W}$ or J/s

Total heat to be extracted to freeze 5 kg of water at 0°C :

$$Q_{total} = m \times L = 5 \times 334 \times 10^3\text{ J} = 1.67 \times 10^6\text{ J}.$$

$$\text{So time required } t = \frac{Q_{total}}{\dot{Q}_L} = \frac{1.67 \times 10^6\text{ J}}{2191.75} \approx 762\text{ s} \approx 0.2\text{ hours}$$

PART C

Q61. Consider $2N$ Ising spins, s_i ($s_i = \pm 1$) in a one-dimensional lattice with periodic boundary conditions. The Hamiltonian is given by $H = -J \sum_{i=1}^{2N} s_i s_{i+1}$ where J denotes the strength of the nearest-neighbour interactions with $J > 0$. Let F be the fully ferromagnetic state and let A be the lowest energy state with zero magnetization. The energy difference between these two states is

- (a) $\frac{3J}{2}$ (b) $4J$ (c) $\frac{J}{2}$ (d) $2J$

Ans.: (b)

Solution.: Energy of the ferromagnetic state:

If all spins are the same $s_i s_{i+1} = (+1)(+1) = 1$ for every bond.

Number of bonds = $2N$ (since periodic chain of $2N$ sites i.e. $s_{2N+1} = s_1$).

$$\text{So } E_F = -J \sum_{i=1}^{2N} s_i s_{i+1} = -J \sum_{i=1}^{2N} 1 = -2NJ$$

Lowest energy state with zero magnetization:

Zero magnetization means $-J \sum_{i=1}^{2N} s_i = 0$. So, number of spin up = number of spin down spins = N .

In a periodic chain, to have N up and N down spins, the minimal number of domain walls (antiparallel neighboring pairs) is 2.

We can have two blocks: N ups and N downs, then back to ups (periodic boundary means the pattern must be consistent.)

In N up and N down Bonds:

(i) within up block: $N-1$ bonds of type $(+1, +1)$

(ii) within down block: $N-1$ bonds of type $(-1, -1)$

(iii) between block: 2 bonds (one at each boundary between up and down in periodic BC) of type $(+1, -1)$ and $(-1, +1)$. Let us label the sites as $s_1, \dots, s_N = +1$, $s_{N+1}, \dots, s_{2N} = -1$

So $s_N s_{N+1} = (+1)(-1) = -1$, $s_{2N} s_1 = (-1)(+1) = -1$, all other bonds are $+1$.

Thus, number of bonds = $2N$, number $+1$ of bonds = $2N-2$, number -1 of bonds = 2

$$\text{So energy } E_A = -J \sum_{i=1}^{2N} s_i s_{i+1} = -J [(2N-2)(+1) + 2(-1)] = -J(2N-4)$$

Thus the energy difference $E_A - E_F = -J(2N-4) - [-2NJ] = 4J$

Q62. Two discrete time random walkers start from the point $x=0$ at time $t=0$ taking discrete steps of unit length along the x axis. The first walker is unbiased and the second walker is biased to move towards the right with probability p . The probability that they are at a distance of 2 units from each other at both time steps $t=1$ and $t=2$ is

- (a) $\frac{1}{4}$ (b) $\frac{1}{2} - \frac{p}{2}$ (c) $1 - \frac{3p}{4}$ (d) $\frac{p}{2}$

Ans.: (a)

Solution.: We have two discrete time random walkers start from the point $x=0$ at time $t=0$.

Walker 1(Unbiased): $P(\text{step}+1) = \frac{1}{2}$, $P(\text{step}-1) = \frac{1}{2}$

Walker 2(Biased): $P(\text{step}+1) = p$, $P(\text{step}-1) = 1-p$

Let X_t = position of walker 1 at time t , Y_t = position of walker 2 at time t . Initially $X_0 = Y_0 = 0$

Let $D_t = X_t - Y_t$ = signed difference.

We want $P(|D_1|=2 \text{ and } |D_2|=2)$

Let a_i = step of walker 1 at time i : $a_i = \pm 1$ with probability $\frac{1}{2}$ each.

b_i = step of walker 2 at time i : $b_i = +1$ with prob. p , -1 with prob. $1-p$.

Then $D_1 = a_1 - b_1$, $D_2 = (a_1 + a_2) - (b_1 + b_2) = D_1 + (a_2 - b_2)$

Possible $D_1 = a_1 - b_1$ values:

$a_1 = +1, b_1 = -1, D_1 = 2$, Prob. = $\frac{1}{2}(1-p)$; $a_1 = -1, b_1 = +1, D_1 = -2$, Prob. = $\frac{1}{2}p$

$a_1 = +1, b_1 = +1, D_1 = 0$, Prob. = $\frac{1}{2}p$; $a_1 = -1, b_1 = -1, D_1 = 0$, Prob. = $\frac{1}{2}(1-p)$

Total $P(D_1 = 2) = \frac{1}{2}(1-p) + \frac{1}{2}p = \frac{1}{2}$

We also want $D_2 = D_1 + (a_2 - b_2) = \pm 2$

Case-I: $D_1 = 2$

$D_2 = 2 + (a_2 - b_2) = \pm 2 \Rightarrow a_2 = 1, b_2 = 1 \left(P = \frac{1}{2}p \right), a_2 = -1, b_2 = -1 \left(P = \frac{1}{2}(1-p) \right)$

Sum of Probability: $\frac{1}{2}p + \frac{1}{2}(1-p) = \frac{1}{2}$

Case-I overall probability: $P(D_1 = 2) \times P(|D_2| = 2 | D_1 = 2) = \frac{1-p}{2} \times \frac{1}{2} = \frac{1-p}{4}$

Case-II: $D_1 = -2$

$$D_2 = -2 + (a_2 - b_2) = \pm 2 \Rightarrow a_2 = 1, b_2 = 1 \left(P = \frac{1}{2} p \right), a_2 = -1, b_2 = -1 \left(P = \frac{1}{2} (1-p) \right)$$

$$\text{Sum of Probability: } \frac{1}{2} p + \frac{1}{2} (1-p) = \frac{1}{2}$$

$$\text{Case-II overall probability: } P(D_1 = -2) \times P(|D_2| = 2 | D_1 = -2) = \frac{p}{2} \times \frac{1}{2} = \frac{p}{4}$$

$$\text{Thus, total probability} = \frac{1-p}{4} + \frac{p}{4} = \frac{1}{4}$$

Q63. A rigid molecule can have two possible rotational states: $j = 0$ or $j = 1$. Its rotational

energies are given by $\epsilon_j = \frac{\hbar^2}{2I} j(j+1)$, where I is its moment of inertia. For an ensemble of such molecules in thermal equilibrium at temperature T , the ratio of the number of molecules in the $j = 1$ state (N_1), to those in $j = 0$ state (N_0), is $\frac{N_1}{N_0} = 0.003$. The temperature T (in units

of $\frac{\hbar^2}{2Ik_B}$, where k_B is the Boltzmann constant) is closest to

- (a) 0.29 (b) 0.21 (c) 0.15 (d) 0.34

Ans.: (a)

Solution.: Rotational energies are given by $\epsilon_j = \frac{\hbar^2}{2I} j(j+1)$, so $\epsilon_0 = 0$, $\epsilon_1 = \frac{\hbar^2}{I}$

Degeneracy of each level is $g_j = 2j+1$, $g_0 = 1$, $g_1 = 3$.

$$\text{Boltzmann distribution } \frac{N_1}{N_0} = \frac{g_1}{g_0} e^{-(\epsilon_1 - \epsilon_0)/k_B T} \Rightarrow \frac{N_1}{N_0} = 3e^{-\epsilon_1/k_B T} = 3e^{-2E_0/k_B T} \text{ where } \epsilon_1 = \frac{\hbar^2}{I} = 2E_0$$

$$\therefore \frac{N_1}{N_0} = 0.003 = 3e^{-2E_0/k_B T} \Rightarrow -\frac{2E_0}{k_B T} = \ln(0.001) = -3\ln 10 \Rightarrow \frac{E_0}{k_B T} = \frac{3}{2} \ln 10$$

$$\Rightarrow \frac{\hbar^2}{2Ik_B T} = \frac{3}{2} \ln 10 \Rightarrow T = \frac{\hbar^2}{2Ik_B} \frac{2}{3\ln 10} = \frac{\hbar^2}{2Ik_B} \frac{2}{3 \times 2.303} \approx 0.29 \frac{\hbar^2}{2Ik_B}$$

Q64. A thermodynamic system (at temperature T and volume V), is described by its internal energy $U = AT^4V$ and pressure $p = \frac{1}{3}AT^4$, where A is a constant of appropriate dimension.

The Helmholtz free energy of the system is

- (a) $\frac{4}{3}AT^4V$ (b) $\frac{1}{3}AT^4V$ (c) $-\frac{1}{3}AT^4V$ (d) $-\frac{4}{3}AT^4V$

Ans.: (c)

Solution.: Given $U = AT^4V$, $p = \frac{1}{3}AT^4$.

$$\left(\frac{\partial F}{\partial V}\right)_T = -p(T, V) = -\frac{1}{3}AT^4 \text{ since } dF = -SdT - pdV$$

$$\Rightarrow F(T, V) = -\frac{1}{3}AT^4V + f(T) \Rightarrow S(T, V) = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{4}{3}AT^3V - f'(T)$$

$$U = F + TS = \left[-\frac{1}{3}AT^4V + f(T)\right] + T\left[\frac{4}{3}AT^3V - f'(T)\right]$$

$$\Rightarrow U = AT^4V = AT^4V + f(T) - Tf'(T) \Rightarrow f(T) - Tf'(T) = 0$$

$$\text{Let } f(T) = kT \text{ then } f'(T) = k \text{ and check } f(T) - Tf'(T) = kT - kT = 0$$

$$\text{From Third Law as } T \rightarrow 0, S \rightarrow 0 \Rightarrow f'(T) = 0 \Rightarrow k = 0 \Rightarrow f(T) = 0.$$

$$\text{Thus } F(T, V) = -\frac{1}{3}AT^4V$$



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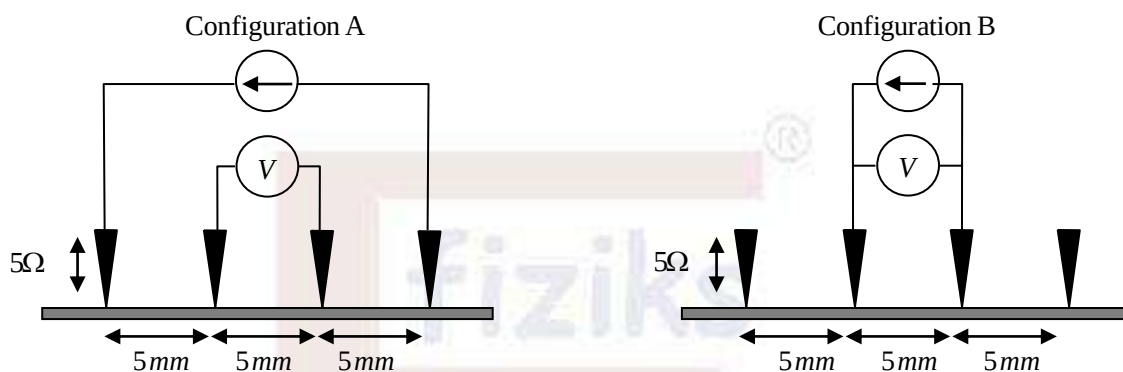
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PART B

Q42. Let R_A and R_B be the resistances of a channel determined (by taking the ratio of the voltage measured and current flowing) using configurations A and B respectively, as shown in the figure. In both configurations, each lead resistance is 5Ω and each contact resistance is 10Ω . The channel has a resistivity of $20\Omega/mm$. Considering the voltmeter and the current source as ideal devices, the ratio R_B/R_A is:



(a) 1.1

(b) 1.2

(c) 1.3

(d) 1.5

Ans.: (c)

Solution.: Let lead+contact resistance $R_C = 15\Omega$ per terminal

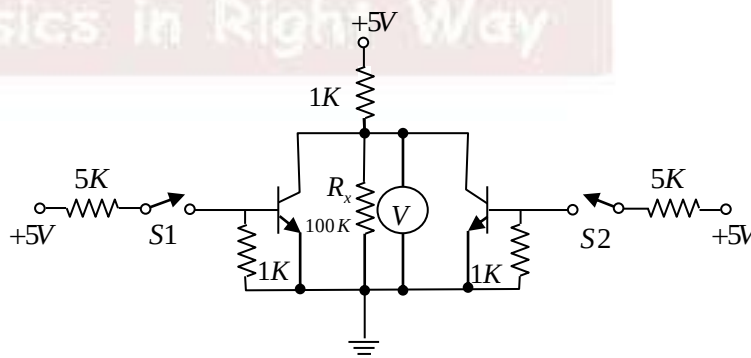
r = resistance per mm of channel = $20\Omega/mm$.

Let distance between voltage probe = d mm

Let R_A and R_B be the resistances of a channel determined (by taking the ratio of the voltage measured and current flowing) using configurations A and B respectively.

$$R_A = rd, R_B = rd + 2R_C \Rightarrow \frac{R_B}{R_A} = 1 + \frac{2R_C}{rd} = 1 + \frac{2 \times 15}{20 \times 5} = 1.3$$

Q43. The circuit, composed of npn transistors of high β , resistors and switches, is shown in the figure. The biasing is sufficient to turn on the transistors when respective switches S1 and S2 are closed.



The voltage across the resistor

$R_x = 100k\Omega$ is

(a) $\sim 5V$ when both S1 and S2 are closed

(b) $\sim 5V$ when either S1 or S2 are closed

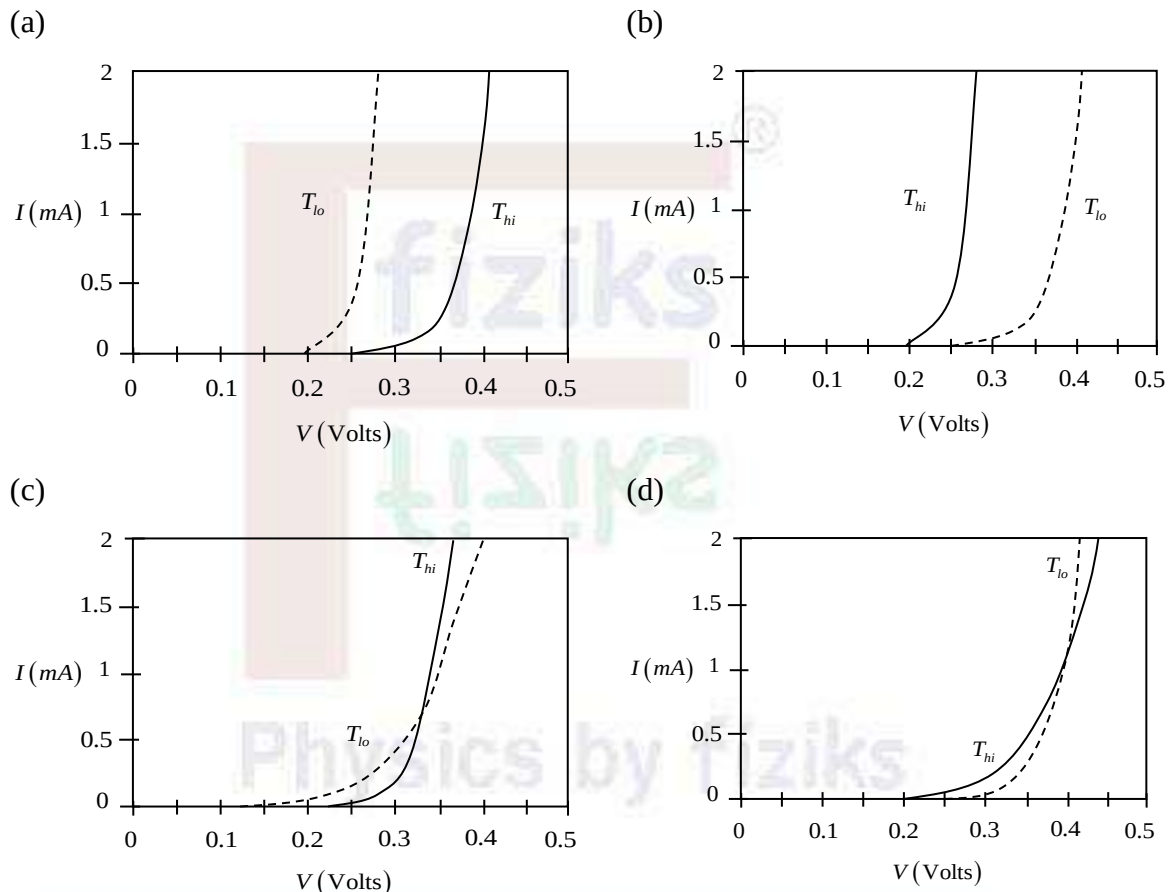
(c) $\sim 5V$ when both S1 and S2 are open

(d) $\sim 0V$ when both S1 and S2 are open

Ans.: (c)

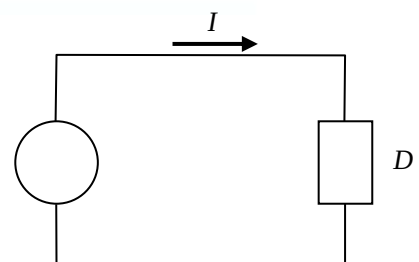
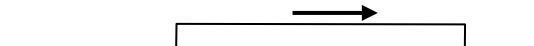
Solution.: Closing either switch turns on that transistor, pulling the collector voltage down to nearly 0V. Only when both transistors are OFF, output across R_x is high ($\approx 5V$)

Q44. A Silicon $p-n$ junction diode is operated under forward bias at two temperatures $T_{hi} \approx 300K$, (shown by solid line) and $T_{lo} \approx 200K$, (shown by dotted line). Which of the following plots best represents the $I-V$ characteristics of the diode?



Ans.: (b)

Q45. Consider the device D shown in the figure below. Its current-voltage characteristic is given by $I = aV + bV^2$, where I is the current, V is the input voltage, and a and b are constants. The device is used to mix a voltage signal $V = V_{DC} + V_{AC}$, where $V_{AC} = V_0 \cos \omega t$. V_{DC} and V_0 are constants. The frequency components present in the current I are



- (a) 0 and ω
 (c) 0 and 2ω
- (b) 0, ω and 2ω
 (d) ω and 2ω

Ans.: (b)**Solution.:** Given $I = aV + bV^2$, $V_{AC} = V_0 \cos \omega t$, V_{DC} and V_0 are constants.

$$\Rightarrow I = aV + bV^2 = a(V_{DC} + V_0 \cos \omega t) + b(V_{DC} + V_0 \cos \omega t)^2$$

$$(V_{DC} + V_0 \cos \omega t)^2 = V_{DC}^2 + V_0^2 \cos^2 \omega t + 2V_{DC}V_0 \cos \omega t = V_{DC}^2 + V_0^2 \left(\frac{1 + \cos 2\omega t}{2} \right) + 2V_{DC}V_0 \cos \omega t$$

So, frequency components present in the current I are 0, ω and 2ω .**PART C****Q68.** The minimum number of two input NOR gates required to obtain the following output for three digital inputs A, B and C

$$Y = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

would be

(a) 4

(b) 3

(c) 5

(d) 6

Ans.: (a)

$$\text{Solution.: } Y = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$$

$$(\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C}) = \bar{A} + \bar{A}B + \bar{A}\bar{C} + \bar{B}\bar{A} + \bar{B}C + \bar{C}\bar{A} + \bar{C}B$$

$$\Rightarrow (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C}) = \bar{A}(1 + B) + \bar{A}\bar{C} + \bar{B}\bar{A} + (\bar{B} + B)\bar{C} = \bar{A} + \bar{C} + \bar{A}\bar{C} + \bar{B}\bar{A}$$

$$\Rightarrow (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C}) = \bar{A}(1 + \bar{B}) + \bar{C}(1 + \bar{A}) = \bar{A} + \bar{C}$$

$$\text{Thus } Y = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C) = (\bar{A} + \bar{C})(\bar{A} + \bar{B} + C)$$

$$\Rightarrow Y = \bar{A} + \bar{A}\bar{B} + \bar{A}C + \bar{C}\bar{A} + \bar{C}B = \bar{A} + \bar{A} + \bar{B}\bar{C} = \bar{A} + \bar{B}\bar{C} = \bar{A} + \overline{B + C} = \overline{\overline{\bar{A} + B + C}}$$

$$(i) \bar{A} = A \text{ NOR } A$$

$$(ii) \overline{B + C} = B \text{ NOR } C$$

$$(iii) \overline{\bar{A} + B + C} = \bar{A} \text{ NOR } \overline{B + C}$$

$$(iv) \overline{\overline{\bar{A} + B + C}} = \text{Invert}(\bar{A} \text{ NOR } \overline{B + C})$$



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PART C

Q70. Consider a laser cooling experiment where atoms are slowed down by an inelastic process of absorption and subsequent emission of photons. If light of wavelength 776.5 nm is used to slow down potassium atoms (mass number 39) with initial speed 130ms^{-1} , the number of such absorption and emission cycles needed to bring the atoms to rest is closest to

- (a) 10^3 (b) 10^2 (c) 10^5 (d) 10^4

Ans.: (d)

Solution.: Each absorption of a photon (from the cooling laser, opposite to the atom's motion) gives the atom a momentum kick $p_{ph} = h/\lambda$ opposite to its velocity. Simultaneous re-emission is in a random direction so on average it gives zero net momentum. Therefore, the average momentum lost by an atom per absorption-emission cycle is h/λ . The number of cycles required is the atom's initial momentum divided by the momentum change per cycle.

Number of cycles

$$N = \frac{mv_0}{h/\lambda} = \frac{mv_0\lambda}{h} = \frac{(39 \times 1.66 \times 10^{-27})(130)(776.5 \times 10^{-9})}{(6.6 \times 10^{-34})} \Rightarrow N = 9.9 \times 10^3 \approx 10^4$$

Q71. An atom is subjected to a weak magnetic field $B = 0.1T$. A spectral line of wavelength 184.9nm corresponding to a $J = 1$ to $J = 0$ transition splits into three components. The highest and the lowest components are separated by $3.2 \times 10^{-4}\text{nm}$. The magnetic moment of the atom in $J = 1$ state (in units of Bohr magneton) is

- (a) 2.82 (b) 0.71 (c) 1.41 (d) 4.23

Ans.: (c)

Solution.: $U_{m_j} = g_J m_J \mu_B B \Rightarrow \Delta U = g_J (+1 - (-1)) \mu_B B = 2g_J \mu_B B$

$$|\Delta U| = \frac{hc}{\lambda^2} \Delta \lambda \Rightarrow 2g_J \mu_B B = \frac{hc}{\lambda^2} \Delta \lambda$$

$$g_J = \frac{hc \Delta \lambda}{2\mu_B B \lambda^2} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)(3.2 \times 10^{-4} \times 10^{-9})}{2 \times (9.27 \times 10^{-24}) \times 0.1 \times (1.849 \times 10^{-7})^2} \Rightarrow g_J \approx 1$$

$$\text{Thus } \mu = g_J \sqrt{J(J+1)} \mu_B = 1 \sqrt{1(1+1)} \mu_B = \sqrt{2} \mu_B \Rightarrow \mu = 1.414 \mu_B$$

Q72. In a rotational-vibrational spectrum of $HCl(H^{35}Cl)$, the first R -branch line and the first P -branch line are observed at $\lambda^{-1} = 2906\text{cm}^{-1}$ and $\lambda^{-1} = 2865\text{cm}^{-1}$, respectively. The equilibrium bond length of this molecule would be closest to

- (a) $0.2\text{\AA}$ (b) $1.3\text{\AA}$ (c) $13\text{\AA}$ (d) $2.1\text{\AA}$

Ans.: (b)

Solution.:

First R -line is $R(0)[J' = 1 \rightarrow J'' = 0]$: $\bar{\nu}_{R(0)} = \nu_0 + 2B'$

First P -line is $P(1)$: $\bar{\nu}_{P(1)} = \nu_0 - 2B''$

$$\Rightarrow \bar{\nu}_{R(0)} - \bar{\nu}_{P(1)} = 2B' + 2B'' = 2(B' + B'') \Rightarrow 2906 - 2865 = 2(B' + B'')$$

$$\Rightarrow B' + B'' = 20.5\text{cm}^{-1} \Rightarrow B_{av} = \frac{B' + B''}{2} = 10.25\text{cm}^{-1} \Rightarrow \frac{h}{8\pi^2 Ic} = 10.25 \times 100\text{m}^{-1}$$

$$\mu = \frac{m_H m_{Cl}}{m_H + m_{Cl}} = \frac{1 \times 35}{1 + 35} \times 1.66 \times 10^{-27} \Rightarrow \mu = 1.614 \times 10^{-27}\text{kg}$$

$$\therefore I = \mu r^2 \Rightarrow r = \sqrt{\frac{h}{8\pi^2 \mu c B}} = \sqrt{\frac{6.6 \times 10^{-34}}{8 \times (3.14)^2 \times 1.614 \times 10^{-27} \times (3 \times 10^8) \times 1025}}$$

$$\Rightarrow r = 1.3 \times 10^{-10}\text{m} \Rightarrow r = 1.3\text{\AA}$$

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Solution-Solid State Physics

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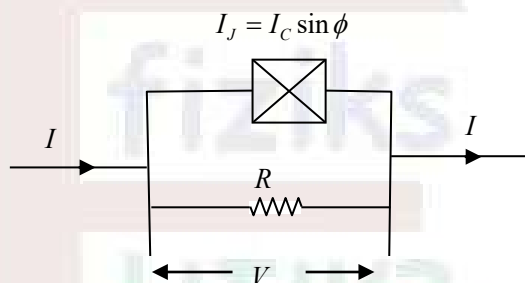
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PART C

Q65. The current $I_J(t)$ through a Josephson junction (shown by the crossed box in the figure) and the voltage $V(t)$ across it, are given by

$$I_J(t) = I_C \sin \phi(t); \quad \frac{d\phi(t)}{dt} = \frac{2eV(t)}{\hbar}$$

where I_C is the critical current of the junction and $\phi(t)$ is the phase difference across the junction. A resistor R is connected in parallel to the junction and a constant current $I > I_C$ flows through the combination as shown.



The energy dissipated in R in the time ϕ changes by 2π is

- (a) $\frac{\hbar}{2e} I$ (b) $\frac{\hbar}{2e} I_C$ (c) $\frac{\hbar}{2e} (I - I_C)$ (d) $\frac{\hbar}{2e} (I + I_C)$

Ans.: (a)

Solution.:

The total constant current I entering the parallel combination splits into the Josephson junction current $I_J(t)$ and the resistor current $I_R(t)$:

$$I = I_J(t) + I_R(t) \Rightarrow I_R(t) = I - I_J(t) = I - I_C \sin \phi(t)$$

The voltage across the parallel combination (and hence across the resistor R) is given by:

$$V(t) = \frac{\hbar}{2e} \frac{d\phi(t)}{dt}$$

The energy dissipated in the resistor R is given

$$\begin{aligned} E &= \int_0^{2\pi} V(t) I_R(t) dt = \int_0^{2\pi} \frac{\hbar}{2e} \frac{d\phi(t)}{dt} (I - I_C \sin \phi(t)) \frac{dt}{d\phi} d\phi \\ \Rightarrow E &= \frac{\hbar}{2e} \int_0^{2\pi} (I - I_C \sin \phi(t)) d\phi = \frac{\hbar}{2e} \left[I(2\pi) - I_C (-\cos \phi(t)) \right]_0^{2\pi} \\ \Rightarrow E &= \frac{\hbar}{2e} I(2\pi) = \frac{1}{2e} \frac{\hbar}{2\pi} I(2\pi) = \frac{\hbar}{2e} I \end{aligned}$$

Q66. A semiconductor has the dispersion relation $E = E_0 - A \cos(\alpha k_x)$, where A and α are positive constants. The effective electron mass close to the minimum energy is

- (a) $\frac{\hbar^2}{A^2 \alpha}$ (b) $\frac{1}{4} \frac{\hbar^2}{A^2 \alpha}$ (c) $\frac{\hbar^2}{A \alpha^2}$ (d) $\frac{1}{2} \frac{\hbar^2}{A \alpha^2}$

Ans.: (c)

Solution.:

The effective mass m^* is given by: $m^* = \frac{\hbar^2}{d^2 E / dk_x^2}$

$$\text{Given } E = E_0 - A \cos(\alpha k_x) \quad \frac{dE}{dk_x} = A \alpha \sin(\alpha k_x) \Rightarrow \frac{d^2 E}{dk_x^2} = A \alpha^2 \cos(\alpha k_x)$$

$$\text{At the Band Minimum } (k_x = 0): \quad \left. \frac{d^2 E}{dk_x^2} \right|_{k_x=0} = A \alpha^2$$

$$\Rightarrow m^* = \frac{\hbar^2}{A \alpha^2}$$

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PART C

Q73. If the binding energies per nucleon of the nuclei $X (A=240)$ and $Y (A=120)$ are 7.6 MeV and 8.5 MeV respectively, the energy released in the symmetric fission, $X \rightarrow Y + Y$ is

- (a) 94 MeV (b) 9.4 MeV (c) 108 MeV (d) 216 MeV

Ans.: (d)

Solution.: $\Delta E = 2BE_y - BE_x = 2(120 \times 8.5) - 240 \times 7.6 = 2040 - 1824 = 216 \text{ MeV}$

Q74. When a neutron of 1 keV kinetic energy impinges on a ^{12}C target, the total scattering cross section is 1000 barns. The approximate value of the phase shift δ_0 is

- (a) 18° (b) 108° (c) 90° (d) 36°

Ans.: (d)

Solution:

$$E_n = 1 \text{ keV} = 10^{-3} \text{ MeV}, \sigma = 1000 \text{ barns} = 1000 \times 10^{-28} \text{ m}^2 = 1000 \times 100 \text{ fm}^2 = 10^5 \text{ fm}^2$$

As target ^{12}C is very heavy, we can take the δ -wave (low energy) approximation.

Reduced mass $\mu \approx m_n$.

For s -wave, the total cross-section is $\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0$ where k is the neutron wave number

$$k = \sqrt{\frac{2\mu E}{\hbar^2}} = \sqrt{\frac{E}{\hbar^2/2\mu}} = \sqrt{\frac{10^{-3} \text{ MeV}}{20.7 \text{ MeV fm}^2}} \Rightarrow k \approx 6.95 \times 10^{-3} \text{ fm}^{-1}$$

$$\sin^2 \delta_0 = \frac{\sigma k^2}{4\pi} = \frac{10^5 (6.95 \times 10^{-3})^2}{4 \times 3.14} \approx 0.38 \Rightarrow \sin \delta_0 = 0.62 \Rightarrow \delta_0 \approx 38^\circ$$

The nearest option is (d)

Q75. The ρ -mesons are $J^P = 1^-$ particles that decay strongly into pions. The ratio of the particle

decay widths $\frac{\Gamma(\rho^0 \rightarrow \pi^0 \pi^0)}{\Gamma(\rho^+ \rightarrow \pi^+ \pi^0)}$ is closest to

- (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) 2

Ans.: (c)

Solution.: ρ -mesons decay rates are proportional to I_3

$$\frac{\Gamma(\rho^0 \rightarrow \pi^0 \pi^0)}{\Gamma(\rho^+ \rightarrow \pi^+ \pi^0)} = \frac{0}{+1} = 0$$

Ind Method

(i) $\rho^+ \rightarrow \pi^+ \pi^0$

$$I \quad \vec{1} \quad \vec{1} + \vec{1} = 0, 1, 2$$

$$I_3 \quad +1 \quad +1 \quad 0$$

Both $\vec{I}$ and I_3 are conserved, so this decay is allowed and indeed is the dominant decay of ρ^+

(ii) $\rho^0 \rightarrow \pi^0 \pi^0$

$$I_3 \quad 0 \quad 0 + 0$$

$$\vec{I} \quad \vec{1} \quad \vec{1} + \vec{1} = \vec{0}, \vec{2}$$

$I = \vec{1}$ state is not allowed as two neutral pions are identical bosons. Their isospin state is symmetric under exchange. The combination of two π^0 corresponds only to $I = 0$ and $I = 2$ components (no $I = 1$ is possible)

So, Isospin I is not conserved in $\rho^0 \rightarrow \pi^0 \pi^0$. So $\rho^0 \rightarrow \pi^0 \pi^0$ is forbidden (or suppressed)

Therefore $\frac{\Gamma(\rho^0 \rightarrow \pi^0 \pi^0)}{\Gamma(\rho^+ \rightarrow \pi^+ \pi^0)} \approx 0$

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