Physics by fiziks

Learn Physics in Right Way

CSIR NET-JRF Physical Sciences Paper Dec.-2023

Solution-Mathematical Physics

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PART B

Q3. Let M be a 3×3 real matrix such that $e^{M\theta}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

where θ is a real parameter. Then M is given by

(1) $\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}$ (2) $\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ (3) $\begin{bmatrix}0 & 0 & 1\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$

Ans: (2)

Solution: $\det e^{M\theta}=1\left(\cos^2\theta+\sin^2\theta\right)=1=e^{\text{Trace}\,M\theta}=e^0=1\Rightarrow Trace\left(M\theta\right)=0$.

So option (3) and (4) is incorrect.

From option (1) : $M\theta=\begin{bmatrix}-\theta & 0 & 0\\ 0 & \theta & 0\\ 0 & 0 & 0\end{bmatrix}\Rightarrow e^{M\theta}=\begin{bmatrix}e^{-\theta} & 0 & 0\\ 0 & e^{\theta} & 0\\ 0 & 0 & 0\end{bmatrix}\Rightarrow \det e^{M\theta}=0$ (incorrect)

So option (2) is correct.

Q4. If z is a complex number, which among the following sets is neither open nor closed?

(1) $\{z|0\le|z-1|\le 2\}$ (2) $\{z||z|\le 1\}$

(3) $\{z|z\in(\mathbf{C}-\{3\})\text{ and }|z|\le 100\}$ (4) $\left\{z|z=re^{i\theta},0\le\theta\le\frac{\pi}{4}\right\}$

Ans: (3)

Q17. The Beta function is defined as $B(x,y)=\int_0^1 t^{x-1}(1-t)^{y-1}\,dt$.

Then $B(x,y+1)+B(x+1,y)$ can be expressed as

(1) $B(x,y-1)$ (2) $B(x+y,1)$

(3) $B(x+y,x-y)$ (4) $B(x,y)$

Ans: (4)

Solution: $B(x,y)=\int\limits_0^1 t^{x-1}(1-t)^{y-1}\,dt$

The $B(x,y+1)+B(x+1,y)=\int\limits_0^1 t^{x-1}(1-t)^{(y+1)-1}\,dt+\int\limits_0^1 t^{(x+1)-1}(1-t)^{y-1}\,dt$

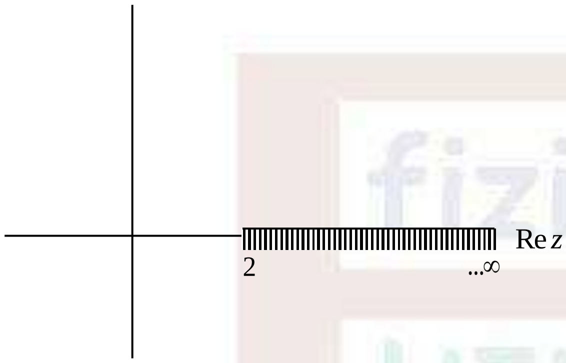
$$=\int\limits_0^1 t^{x-1}(1-t)^{y}\,dt+\int\limits_0^1 t^{x}(1-t)^{y-1}\,dt=\int\limits_0^1 t^{x}(1-t)^{y}\left[t^{-1}+(1-t)^{-1}\right]dt$$

$$= \int_0^1 t^x (1-t)^y \left[\frac{1}{t} + \frac{1}{1-t} \right] dt = \int_0^1 t^x (1-t)^y \left[\frac{1-t+t}{t(1-t)} \right] dt$$

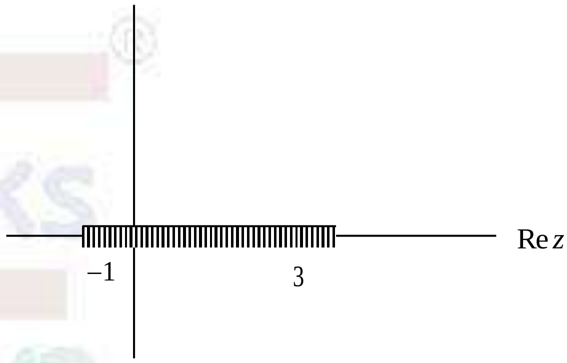
$$= \int_0^1 \frac{t^x (1-t)^y \, dt}{t(1-t)} = \int_0^1 t^{x-1} (1-t)^{y-1} \, dt = B(x, y)$$

Q22. The branch line for the function $f(z) = \sqrt{\dfrac{z^2 - 5z + 6}{z^2 + 2z + 1}}$ is

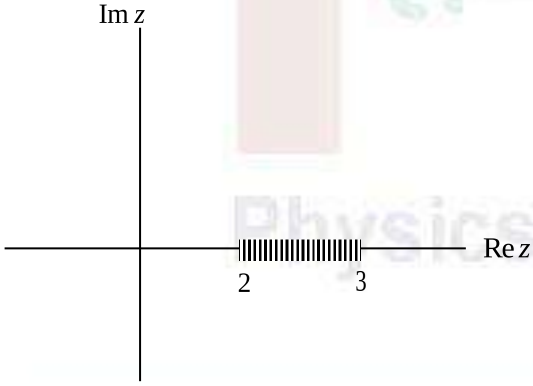
(1)



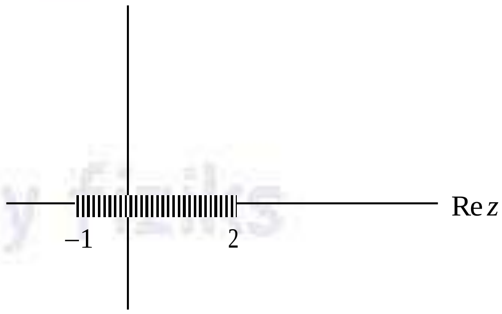
(2)



(3)



(4)



Ans: (3)

Solution:

$$f(z) = \sqrt{\frac{z^2 - 5z + 6}{z^2 + 2z + 1}} = \sqrt{\frac{(z-3)(z-2)}{(z+1)^2}} = \frac{\sqrt{(z-3)(z-2)}}{z+1} = \frac{(z-3)^{1/2}(z-2)^{1/2}}{(z+1)}$$

Thus $z = 3$ and 2 are Branch point singularity.

Then branch line is a line that connects Branch point singularity.

PART C

Q10. The function $f(z)=\dfrac{1}{(z+1)(z+3)}$ is defined on the complex plane. The coefficient of the $(z-z_0)^2$ term of the Laurent series of $f(z)$ about $z_0=1$ is

(1) $\frac{7}{64}$ (2) $\frac{7}{128}$ (3) $\frac{9}{64}$ (4) $\frac{9}{128}$

Ans: (2)

Solution: $f(z)=\sum_{n=\infty}^{\infty}\dfrac{f^n(z_0)}{n!}(z-z_0)^n$

For $n=2$ and $z_0=1$, coefficient is $=\dfrac{f''(z_0)}{2!}$

$$\because\ f(z)=\frac{1}{(z+1)(z+3)}\Rightarrow f'(z)=0-\left[\frac{(z+3)+(z+1)}{(z+1)^2(z+3)^2}\right]=-\frac{1}{(z+1)^2(z+3)}-\frac{1}{(z+1)(z+3)^2}$$

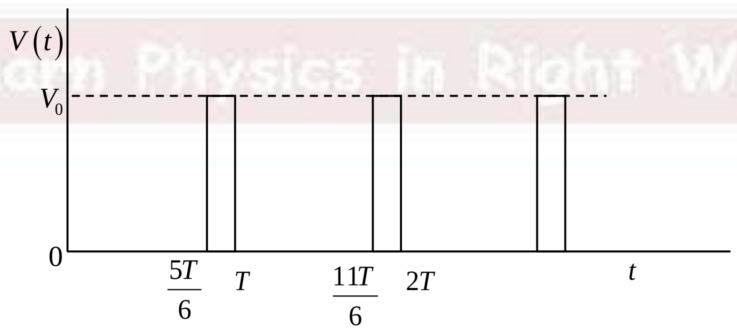
$$\Rightarrow f''(z)=\frac{2(z+1)(z+3)+(z+1)^2}{(z+1)^4(z+3)^2}+\frac{(z+1)\times 2(z+3)+(z+3)^2}{(z+1)^2(z+3)^4}$$

$$\Rightarrow f''(z)=\frac{2(z+3)+(z+1)}{(z+1)^3(z+3)^2}+\frac{2(z+1)+(z+3)}{(z+1)^2(z+3)^3}$$

$$\Rightarrow f''(1)=\frac{2\times4+2}{2^3\cdot4^2}+\frac{2\times2+4}{2^2\times4^3}=\frac{10}{128}+\frac{8}{4\times64}=\frac{10+4}{128}=\frac{7}{64}$$

Thus coefficient $=\dfrac{1}{2}f''(1)=\dfrac{7}{128}$

Q11. An infinite waveform $V(t)$ varies as shown in the figure below



The lowest harmonic that vanishes in the Fourier series of $V(t)$ is

(1) 2 (2) 3 (3) 6 (4) None

Ans: (3)

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Solution:

$$V(t)=\begin{cases}0, & 0<t<\frac{5T}{6}\\ V_0, & \frac{5T}{6}<t<T\end{cases} \qquad p=2L=T\Rightarrow L=T/2$$

$$a_0=\frac{1}{2L}\int_0^{2L}V(t)\,dt=\frac{1}{T}\int_0^{T}V(t)\,dt=\frac{V_0}{T}\int_{5T/6}^{T}dt=\frac{V_0}{6}\neq 0$$

$$\because a_n=\frac{1}{L}\int_0^{2L}V(t)\cos\frac{n\pi t}{L}dt \quad \Rightarrow a_n=\frac{2}{T}\int_{\frac{5T}{6}}^{T}V_0\cos\frac{2n\pi t}{T}dt=\frac{2V_0}{T}\times\frac{T}{2n\pi}\left[\sin 2n\pi-\sin\frac{5n\pi}{3}\right]$$

$$\Rightarrow a_n=\frac{V_0}{n\pi}\left[\sin 2n\pi-\sin\frac{5}{3}n\pi\right]=\begin{cases}0 & \text{for } n=3\\ 0 & \text{for } n=6\end{cases}$$

$$\because b_n=\frac{1}{L}\int_0^{2L}V(t)\sin\frac{n\pi t}{L}dt \quad \Rightarrow b_n=\frac{2}{T}\int_{\frac{5T}{6}}^{T}V_0\sin\frac{2n\pi t}{T}dt=\frac{2V_0}{T}\times\frac{T}{2n\pi}\left[-\cos\frac{2n\pi t}{T}\right]_{\frac{5T}{6}}^{T}$$

$$\Rightarrow b_n=\frac{V_0}{n\pi}\left[\cos\frac{5n\pi}{3}-\cos 2n\pi\right]=\begin{cases}\neq 0 & \text{for } n=3\\ =0 & \text{for } n=6\end{cases}$$

So lowest harmonic that vanishes is for $n=6$.

Q15. Given the data points

x	1	3	5
y	4	28	92

Using Lagrange's method of interpolation, the value of y at $x=4$ is closest to

(1) 54 (2) 55 (3) 53 (4) 56

Ans: (2)

Solution:

Using Lagrange's method of interpolation

$$y(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y(x_2)$$

Given $x_0=1, x_1=3, x_2=5$ and $y(x_0)=4, y(x_1)=28, y(x_2)=92$

$$y(4)=\frac{(4-3)(4-5)}{(1-3)(1-5)}4+\frac{(4-1)(4-5)}{(3-1)(3-5)}28+\frac{(4-1)(4-3)}{(5-1)(5-3)}92$$

$$\Rightarrow y(4)=\frac{(1)(-1)}{(-2)(-4)}4+\frac{(3)(-1)}{(2)(-2)}28+\frac{(3)(1)}{(4)(2)}92\Rightarrow y(4)=-\frac{1}{2}+21+\frac{69}{2}=\frac{-1+42+69}{2}=55$$

Q24. The regular representation of two nonidentity elements of the group of order 3 are given by

(1) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Ans: (3)

Q30. The solution $y(x)$ of the differential equation $y''+\frac{y}{4}=\frac{x}{2}$, where $0\le x\le\pi$, together with the boundary conditions $y(0)=y(\pi)=0$ is

(1) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$ (2) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^n \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2}$

(3) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi}{n}\frac{\sin nx}{\frac{1}{4}-n^2}$ (4) $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2}$

Ans: (4)

Solution:

Characteristic equation $\lambda^2+\frac{1}{4}=0\Rightarrow\lambda=0\pm i\frac{\lambda}{2}$

$C.F.=c_1\cos\frac{x}{2}+c_2\sin\frac{x}{2}$; $P.I.=\frac{1}{D^2+\frac{1}{4}}\cdot\frac{x}{2}=\frac{1}{\frac{1}{4}\left(1+4D^2\right)}\cdot\frac{x}{2}=4\left(1-4D^2\right)\frac{x}{2}=2x$

Thus $y(x)=c_1\cos\frac{x}{2}+c_2\sin\frac{x}{2}+2x$

$\because$ $y(0)=0\Rightarrow 0=c_1+0+0\Rightarrow c_1=0$; $y(\pi)=0\Rightarrow 0=0+c_2\sin{}^{\pi}\!/_2+2\pi\Rightarrow c_2=-2\pi$

Thus $\boxed{y=2x-2\pi\sin\frac{x}{2}}$, $0\le x\le\pi$

Let us find $b_1:b_n=\frac{2}{L}\int_0^L y(x)\sin\frac{n\pi}{L}xdx$ [odd periodic extension of $y(x)$ having $p=L=\pi$]

$\Rightarrow b_1=\frac{2}{\pi}\int_0^{\pi}\left(2x-2\pi\sin\frac{x}{2}\right)\sin xdx$ $\Rightarrow b_1=\frac{4}{\pi}\int_0^{\pi}x\sin xdx-4\int_0^{\pi}\sin\frac{x}{2}\sin xdx=I_1+I_2$

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$$I_1 = \frac{4}{\pi}\int_0^{\pi} x\sin x dx = \frac{4}{\pi}\left[x(-\cos x) - 1(-\sin x)\right]_0^{\pi} = \frac{4}{\pi}\left[-\pi\cos\pi\right] = 4$$

$$I_2 = -4\int_0^{\pi}\sin\frac{x}{2}\sin x dx = -2\left[\int_0^{\pi}\cos\left(\frac{x}{2}-x\right)dx - \int_0^{\pi}\cos\left(\frac{x}{2}+x\right)dx\right]$$

$\because\ 2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$$\Rightarrow I_2 = -2\left[\int_0^{\pi}\cos\frac{x}{2}dx - \int_0^{\pi}\cos\frac{3x}{2}dx\right] = -2\left[2\sin\frac{x}{2} - \frac{2}{3}\sin\frac{3}{2}x\right]_0^{\pi}$$

$\Rightarrow I_2 = -2\left[2\sin\frac{\pi}{2} - \frac{2}{3}\sin\frac{3\pi}{2}\right] = -2\left[2+\frac{2}{3}\right] = -\frac{16}{3}$. Thus $b_1 = I_1 + I_2 = 4 - \frac{16}{3} = -\frac{4}{3}$

Check from option (4): $\frac{2}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\pi}{2n}\frac{\sin nx}{\frac{1}{4}-n^2} \Rightarrow b_1 = \frac{2}{\pi}(-1)^2\frac{\pi}{2}\frac{1}{\frac{1}{4}-1} = -\frac{4}{3}$

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Solution-Classical Mechanics

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PART B

Q2. A particle moves in a circular orbit under a force field given by $\vec{F}(\vec{r}) = -\frac{k}{r^2}\hat{r}$, where k is a positive constant. If the force changes suddenly to $\vec{F}(\vec{r}) = -\frac{k}{2r^2}\hat{r}$, the shape of the new orbit would be

(1) parabolic (2) circular (3) elliptical (4) hyperbolic

Ans: (1)

Solution: In circular orbit of radius r_0; $V(r) = -\frac{k}{r_0}$ and $E = V_{eff} = \frac{l^2}{2mr_0^2} - \frac{k}{r_0}$

$$\left.\frac{\partial V_{eff}}{\partial r}\right|_{r=r_0} = -\frac{l^2}{mr_0^3} + \frac{k}{r_0^2} = 0 \qquad ...(1)$$

Due to sudden change in force

(i) l will remain same (ii) radius will remain r_0, and

(iii) only tangential component of velocity will be present (no radial component of velocity) at the moment of change. So, total energy of the particle at the moment of change

$$E = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr_0^2} - \frac{k}{2r_0} = 0 + \frac{l^2}{2mr_0^2} - \frac{k}{2r_0} = \frac{r_0}{2}\left(\frac{l^2}{mr_0^3} - \frac{k}{r_0^2}\right) \Rightarrow E = 0 \qquad \text{(From equation (1))}$$

This is the condition of parabolic orbit.

Q10. A particle of unit mass subjected to the 1-dimensional potential

$$V(x) = \frac{2\alpha}{x^3} - \frac{3\beta}{x^2}$$

executes small oscillations about its equilibrium position, where α and β are positive constants with appropriate dimensions. The time period of small oscillations is

(1) $\frac{\pi\alpha^2}{\sqrt{6\beta^5}}$ (2) $\frac{\pi\alpha^2}{\sqrt{3\beta^5}}$ (3) $\frac{2\pi\alpha^2}{\sqrt{3\beta^5}}$ (4) $\frac{2\pi\alpha^2}{\sqrt{6\beta^5}}$

Ans: (4)

Solution: At equilibrium position $\frac{dV}{dx} = -\frac{6\alpha}{x^4} + \frac{6\beta}{x^3} = 0 \Rightarrow x_0 = \frac{\alpha}{\beta}$.

$$\left.\frac{d^2V}{dx^2}\right|_{x_0} = \frac{24\alpha}{x_0^5} - \frac{18\beta}{x_0^4} = \frac{6\beta^5}{\alpha^4}$$

$$\because \omega = \frac{2\pi}{T} = \sqrt{\frac{6\beta^5/\alpha^4}{1}} \Rightarrow T = \frac{2\pi\alpha^2}{\sqrt{6\beta^5}}$$

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Q18. The 1-dimensional Hamiltonian of a classical particle of mass m is

$$H = \frac{P^2}{2m} e^{-x/a} + V(x),$$

where a is a constant with appropriate dimensions. The corresponding Lagrangian is,

(1) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$ (2) $\frac{m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

(3) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$ (4) $\frac{3m}{2}\left(\frac{dx}{dt}\right)^2 e^{-x/a} - V(x)$

Ans: (1)

Solution:

$\because H = \frac{p^2}{2m} e^{-x/a} + V(x)$ thus $\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m} e^{-x/a} \Rightarrow p = m\dot{x} e^{x/a}$

$L = \dot{x}p - H = m\dot{x}^2 e^{x/a} - \frac{p^2}{2m} e^{-x/a} - V(x) = m\dot{x}^2 e^{x/a} - \frac{m^2\dot{x}^2}{2m} e^{2x/a} e^{-x/a} - V(x)$

$L = \frac{1}{2} m\dot{x}^2 e^{x/a} - V(x) \Rightarrow L = \frac{1}{2} m\left(\frac{dx}{dt}\right)^2 e^{x/a} - V(x)$

Q19. The coordinates of the following events in an observer's inertial frame of reference are as follows:

Event 1: $t_1 = 0, x_1 = 0$: A rocket with uniform velocity $0.5c$ crosses the observer at origin along x axis

Event 2: $t_2 = T, x_2 = 0$: The observer sends a light pulse towards the rocket

Event 3: t_3, x_3 : The rocket receives the light pulse

The values of t_3, x_3 respectively are

(1) $2T, cT$ (2) $2T, \frac{c}{2}T$ (3) $\frac{\sqrt{3}}{2}T, \frac{2}{\sqrt{3}}cT$ (4) $\frac{2}{\sqrt{3}}T, \frac{\sqrt{3}}{2}cT$

Ans: (1)

Solution:

In Earth frame, the distance travelled by the rocket in time $T = 0.5cT$. Now a light pulse is emitted at $x = 0$, which will be received by rocket in next t time (Let). So,

$0.5c(T + t) = ct \Rightarrow t = T$

$t_3 = T + t = T + T = 2T$ and $x_3 = 0.5c \times 2T = cT$

Q24. A particle of mass m is moving in a stable circular orbit or radius r_0 with angular momentum L. For a potential energy $V(r)=\beta r^k$ ($\beta>0$ and $k>0$), which of the following options is correct?

(1) $k=3, r_0=\left(\frac{3L^2}{5m\beta}\right)^{1/5}$ (2) $k=2, r_0=\left(\frac{L^2}{2m\beta}\right)^{1/4}$

(3) $k=2, r_0=\left(\frac{L^2}{4m\beta}\right)^{1/4}$ (4) $k=3, r_0=\left(\frac{5L^2}{3m\beta}\right)^{1/5}$

Ans: (2)

Solution: $V_{eff}=\frac{l^2}{2mr^2}+\beta r^k \Rightarrow \frac{\partial V_{eff}}{\partial r}=-\frac{l^2}{mr^3}+k\beta r^{k-1}=0 \Rightarrow r_0^{k+2}=\frac{l^2}{mk\beta}$

$k=2,\ r_0=\left(\frac{l^2}{2m\beta}\right)^{1/4}$ and $k=3, r_0=\left(\frac{l^2}{3m\beta}\right)^{1/5}$

PART C

Q4. A Lagrangian is given by $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}\dot{z}+\dot{z}^2\right)-\alpha(2x+3y+z)$

The conserved momentum is

(1) $m[2\dot{x}+\dot{z}]$ (2) $m[2\dot{x}+\dot{y}+\dot{z}]$

(3) $m\left[\dot{x}+\frac{3}{2}\dot{y}+\frac{1}{2}\dot{z}\right]$ (4) $m[2\dot{x}+3\dot{z}]$

Ans: (2)

Solution: $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}\dot{z}+\dot{z}^2\right)-\alpha(2x+3y+z)$

(i) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0 \Rightarrow m\ddot{x}-(-2\alpha)=0 \Rightarrow \boxed{m\ddot{x}=-2\alpha}$

(ii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}=0 \Rightarrow \frac{1}{2}m\ddot{z}-(-3\alpha)=0 \Rightarrow \boxed{m\ddot{z}=-6\alpha}$

(iii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right)-\frac{\partial L}{\partial z}=0 \Rightarrow \frac{1}{2}m\ddot{y}+m\ddot{z}-(-\alpha)=0 \Rightarrow \frac{1}{2}m\ddot{y}-6\alpha+\alpha=0 \Rightarrow \boxed{m\ddot{y}=10\alpha}$

(a) $\frac{d}{dt}m(2\dot{x}+\dot{z})=(2\ddot{x}+\ddot{z})m=-4\alpha-6\alpha=-10\alpha\neq 0$

(b) $\frac{d}{dt}\left[m(2\dot{x}+\dot{y}+\dot{z})\right]=2m\ddot{x}+m\dot{y}+m\dot{z}\ =-4\alpha+10\alpha-6\alpha=0$

So, $m(2\dot{x}+\dot{y}+\dot{z})$ is a conserved quantity.

Q9. A canonical transformation from the phase space coordinates (q,p) to (Q,P) is generated by the function $\psi(p,Q)=\frac{p^2}{2\omega}\tan 2\pi Q$, where ω is a positive constant. The function $\psi(p,Q)$ is related to $F(q,Q)$ by the Legendre transform $\psi = pq - F$, where F is defined by $dF = pdq - PdQ$. If the solution for (P,Q) is $P(t)=\frac{\omega}{4\pi}t^2, Q(t)=Q_0 = const.$ where t is time, then the solution for (p,q) variables can be written as

(1) $p=\frac{\omega t}{2\pi}\cos 2\pi Q_0$, $q=\frac{t}{2\pi}\sin 2\pi Q_0$ (2) $p=-\frac{\omega t}{2\pi}\cos 2\pi Q_0$, $q=\frac{t}{2\pi}\sin 2\pi Q_0$

(3) $p=\frac{\omega t}{2\pi}\sin 2\pi Q_0$, $q=\frac{t}{2\pi}\cos 2\pi Q_0$ (4) $p=-\frac{\omega t}{2\pi}\sin 2\pi Q_0$, $q=\frac{t}{2\pi}\cos 2\pi Q_0$

Ans: (1)

Solution: For the Legendre transformation $\psi = pq - F$ with $dF = pdq - PdQ$, the transformation equations will be

$$q=\frac{\partial\psi}{\partial p}=\frac{p}{\omega}\tan 2\pi Q \quad ...(1) \quad \text{and} \quad P=\frac{\partial\psi}{\partial Q}=\frac{p^2}{2\omega}2\pi\sec^2 2\pi Q \quad ...(2)$$

From (2): $\frac{\omega}{4\pi}t^2=\frac{p^2}{2\omega}2\pi\sec^2 2\pi Q \Rightarrow p^2=\frac{\omega^2t^2}{4\pi^2}\cos^2 2\pi Q \Rightarrow p=\frac{\omega t}{2\pi}\cos 2\pi Q \quad \because P=\frac{\omega}{4\pi}t^2$

From equation (1): $q=\frac{t}{2\pi}\sin 2\pi Q$

Q26. A particle of mass m is moving in a 3-dimensional potential

$$\phi(r)=-\frac{k}{r}-\frac{k'}{3r^3},\ k,k'>0$$

For the particle with angular momentum l, the necessary condition to have a stable circular orbit is

(1) $kk'<\frac{l^4}{4m^2}$ (2) $kk'>\frac{l^4}{4m^2}$ (3) $kk'<\frac{l^4}{m^2}$ (4) $kk'>\frac{l^4}{m^2}$

Ans: (1)

Solution: $V_{eff}=\frac{l^2}{2mr^2}-\frac{k}{r}-\frac{k'}{3r^3} \Rightarrow \frac{\partial V_{eff}}{\partial r}=-\frac{l^2}{mr^3}+\frac{k}{r^2}+\frac{k'}{r^4}=0 \Rightarrow (mk)r^2-l^2r+mk'=0$

$\Rightarrow r=\frac{l^2\pm\sqrt{l^4-4m^2kk'}}{2mk}$. For real r: $l^4>4m^2kk' \Rightarrow \frac{l^4}{4m^2}>kk'$ or $kk'<\frac{l^4}{4m^2}$

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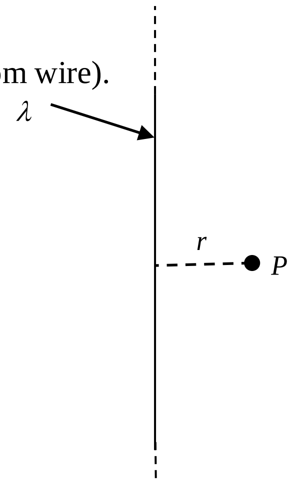
PART B

Q11. A one dimensional infinite long wire with uniform linear charge density λ is placed along the z-axis. The potential difference $\delta V = V(\rho+a) - V(\rho)$, between two points at radial distances $\rho + a$ and ρ from the z-axis, where $a << \rho$, is closest to

(1) $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$ (2) $-\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$ (3) $\frac{\lambda}{2\pi\varepsilon_0}\frac{a}{\rho}$ (4) $\frac{\lambda}{2\pi\varepsilon_0}\frac{a^2}{\rho^2}$

Ans: (2)



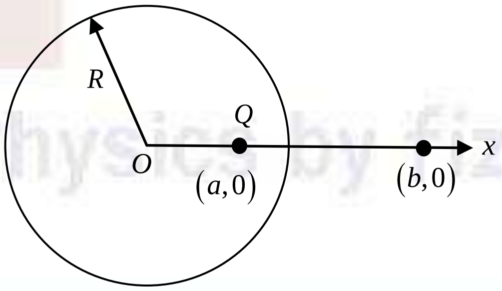
Solution: $\because V(r) = -\frac{\lambda}{2\pi \in_0} ln\left(\frac{r}{\alpha}\right)$ α is some constant (arbitrary distance from wire).

$$\delta V = V(\rho+a) - V(\rho) = -\frac{\lambda}{2\pi \in_0}\left[ln\frac{(\rho+a)}{\alpha} - ln\frac{\rho}{\alpha}\right]$$

$$\Rightarrow \delta V = -\frac{\lambda}{2\pi \in_0} ln\frac{\rho+a}{\rho} = -\frac{\lambda}{2\pi \in_0} ln\left(1+\frac{a}{\rho}\right) \approx -\frac{\lambda}{2\pi \in_0}\frac{a}{\rho}$$

where $ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} ... \approx x$

Q12. A conducting shell of radius R is placed with its centre at the origin as shown below. A point charge Q is placed inside the shell at a distance a along the x-axis from the centre.



The electric field at a distance $b > R$ along the x-axis from the centre is

(1) $\frac{Q}{4\pi\varepsilon_0 b^2}\hat{x}$ (2) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} - \frac{aR}{(ab-R^2)^2}\right]\hat{x}$

(3) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{(b-a)^2} + \frac{aR}{(ab-R^2)^2}\right]\hat{x}$ (4) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b^2} - \frac{R^2}{a^2b^2}\right]\hat{x}$

Ans: (1)

Solution: Charge that develops on the inner surface is -Q and on the outer surface of conducting shell is Q. So $\vec{E} = \frac{Q}{4\pi \in_0 b^2}\hat{x}$

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Q13. A small bar magnet is placed in a magnetic field $B(\vec{r}) = B(x)\hat{z}$. The magnet is initially at rest with its magnetic moment along $\hat{y}$. At later times, it will undergo

(1) angular motion in the yz plane and translational motion along $\hat{y}$

(2) angular motion in the yz plane and translational motion along $\hat{x}$

(3) angular motion in the zx plane and translational motion along $\hat{z}$

(4) angular motion in the xy plane and translational motion along $\hat{z}$

Ans: (2)

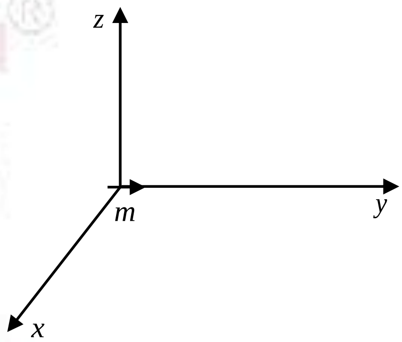
Solution:

Torque $\vec{\tau} = \vec{m} \times \vec{B}$ (Angular momentum in yz-plane)

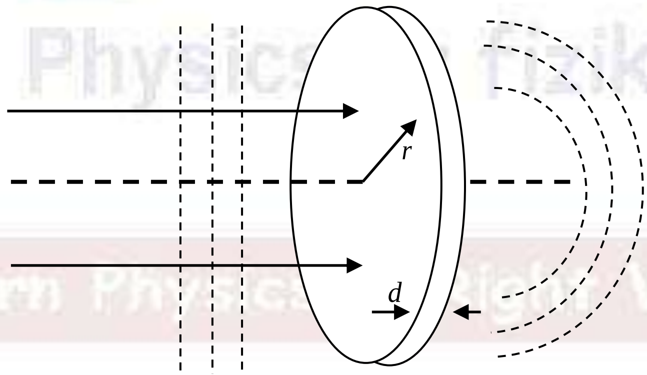
$\vec{m} \cdot \vec{B} = \left[m(\hat{y} + \hat{z})\right] \cdot \left[B(x)\hat{z}\right] = mB(x)$

$\because \vec{F}_{net} = \vec{\nabla}\left(\vec{m} \cdot \vec{B}\right) \Rightarrow \vec{F}_{net} = \vec{\nabla}\left(mB(x)\right) = F_x\hat{x}$

(Translational motion along $\hat{x}$)



Q15. For a flat circular glass plate of thickness d, the refractive index $n(r)$ varies radially, where r is the radial distance from the centre of the plate. A coherent plane wavefront is normally incident on this plate as shown in the figure below.



If the emergent wavefront is spherical and centered on the axis of the plate, then $n(r) - n(0)$ should be proportional to

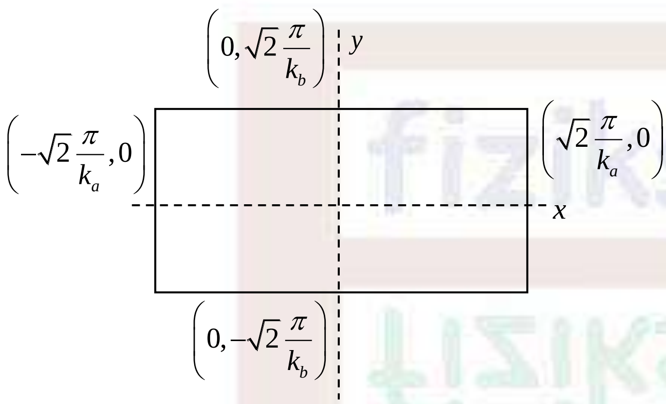
(1) $r^{1/2}$ (2) r (3) r^2 (4) $r^{3/2}$

Ans: (3)

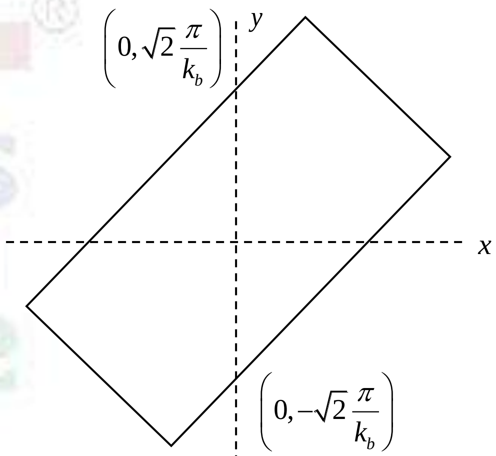
PART C

Q5. A 2-dimensioanl resonant cavity supports a TM mode built from a function $\psi(x,y,t)=\sin\left(\vec{k}_a\cdot\vec{r}-\omega t\right)+\sin\left(\vec{k}_b\cdot\vec{r}-\omega t\right)+\sin\left(\vec{k}_a\cdot\vec{r}+\omega t\right)+\sin\left(\vec{k}_b\cdot\vec{r}+\omega t\right)$ where $\vec{k}_a$ and $\vec{k}_b$ lie in the xy-plane and make angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ with the x-axis, respectively. If $0<\left|\vec{k}_a\right|<\left|\vec{k}_b\right|$, then which of the following closely describes the outline of the cavity?

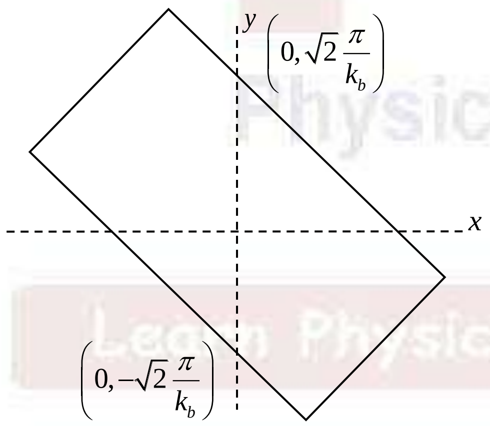
(1)



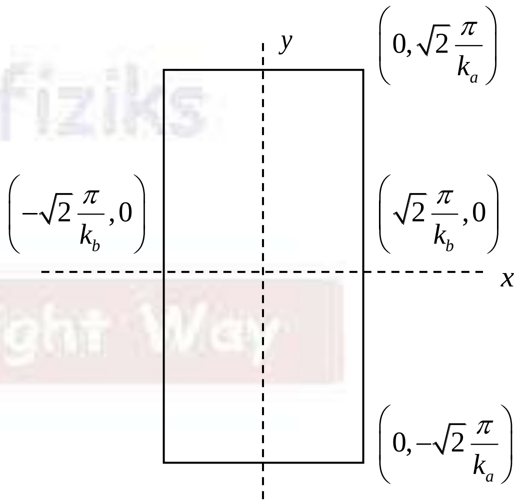
(2)



(3)



(4)



Ans: (2)

Q7. The permittivity of a medium $\varepsilon(\vec{k},\omega)$, where ω and $\vec{k}$ are the frequency and wavevector, respectively, has no imaginary part. For a longitudinal wave, $\vec{k}$ is parallel to the electric field such that $\vec{k}\times\vec{E}=0$, while for a transverse wave $\vec{k}\cdot\vec{E}=0$. In the absence of free charges and free currents, the medium can sustain

(1) longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)>0$

(2) transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)<0$

(3) longitudinal waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)=0$

(4) both longitudinal and transverse waves with $\vec{k}$ and ω when $\varepsilon(\vec{k},\omega)>0$

Ans: (3)

Q18. A transmission line has the characteristic impedance of $(50+1j)\Omega$ and is terminated in a load resistance of $(70-7j)\Omega$ (where $j^2=-1$). The magnitude of the reflection coefficient will be closest to

(1) $\frac{5}{7}$ (2) $\frac{1}{2}$ (3) $\frac{1}{6}$ (4) $\frac{1}{7}$

Ans: (3)

Solution:

The magnitude of the reflection coefficient $\Gamma_L=\left|\frac{Z_L-Z_S}{Z_L+Z_S}\right|=\left|\frac{(70-7j)-(50+1j)}{(70-7j)+(50+1j)}\right|=\left|\frac{20-8j}{20-6j}\right|\approx\frac{1}{6}$

Q19. The radius of a sphere oscillates as a function of time as $R+a\cos\omega t$, with $a<R$. It carries a charge Q uniformly distributed on its surface at all times. If P is the time averaged radiated power through a sphere of radius r, such that $r>>R+a$ and $r>>\frac{c}{\omega}$, then

(1) $P\propto\frac{Q^2\omega^4a^2}{c^3}$ (2) $P\propto\frac{Q^2\omega^4}{c}$

(3) $P=0$ (4) $P\propto\frac{Q^2\omega^6a^4}{c^5}$

Ans: (3)

Solution: Since sphere carries a charge Q uniformly distributed on its surface at all times. Dipole moment of this configuration is always zero. So average radiated power through a sphere of radius r is zero.

Q21. The collision time of the electrons in a metal in the Drude model is τ and their plasma frequency is ω_p. If this metal is placed between the plates of a capacitor, the time constant associated with the decay of the electric field inside the metal is

(1) $\tau + \dfrac{1}{\omega_p}$ (2) $\omega_p \tau^2$

(3) $\dfrac{1}{\omega_p^2 \tau}$ (4) $\dfrac{\tau}{1+\omega_p \tau}$

Ans: (3)

Solution:

$$\omega_p^2 = \frac{ne^2}{\varepsilon_0 m} = \frac{e}{\varepsilon_0}\frac{ne}{m} = \frac{e}{\varepsilon_0}\frac{\mu}{\tau} = \frac{e}{\varepsilon_0}\frac{1}{\tau}\frac{v_d}{E} \Rightarrow E = \frac{ev_d}{\varepsilon_0 \tau \omega_p^2} \Rightarrow E \propto \frac{1}{\tau \omega_p^2} \qquad \because \mu = \frac{ne\tau}{m} = \frac{v_d}{E}$$

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PART B

Q8. The normalized wave function of an electron is

$$\psi(\vec{r}) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\varphi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\varphi)\chi_+\right],$$

where Y_l^m are the normalized spherical harmonics and $\chi_\pm$ denote the wavefunction for the two spin states with eigenvalues $\pm\frac{1}{2}\hbar$. The expectation value of the z component of the total angular momentum in the above state is

(1) $-\frac{3}{4}\hbar$ (2) $\frac{3}{4}\hbar$ (3) $-\frac{9}{8}\hbar$ (4) $\frac{9}{8}\hbar$

Ans: (2)

Solution:

$$\psi(r) = R(r)\left[\sqrt{\frac{3}{8}}Y_1^0(\theta,\phi)\chi_- + \sqrt{\frac{5}{8}}Y_1^1(\theta,\phi)\chi_+\right]$$

Expectation value of the total z-component of the angular momentum is $\langle \hat{J}_z \rangle = \langle \hat{L}_z \rangle + \langle \hat{S}_z \rangle$

$\langle \hat{L}_z \rangle = \sum L_z P(L_z) = 0\hbar\left(\frac{3}{8}\right) + 1\hbar\left(\frac{5}{8}\right) = \frac{5\hbar}{8}$ and $\langle S_z \rangle = \sum S_z P(S_z) = -\frac{1}{2}\hbar\left(\frac{3}{8}\right) + \frac{1}{2}\hbar\left(\frac{5}{8}\right) = \frac{\hbar}{8}$

$\therefore \langle J_z \rangle = \frac{5\hbar}{8} + \frac{\hbar}{8} = \frac{3}{4}\hbar$

Thus correct option is (2)

Q21. The Schrodinger wave function for a stationary state of an atom in spherical polar coordinates (r,θ,ϕ) is

$$\psi = Af(r)\sin\theta\cos\theta e^{i\phi}$$

where A is the normalization constant. The eigenvalue of $\hat{L}_z$ for this state is

(1) $2\hbar$ (2) $\hbar$ (3) $-2\hbar$ (4) $-\hbar$

Ans: (2)

Solution: $\psi = Af(r)\sin\theta\cos\theta e^{i\phi}$

To get m_ℓ value, compare ψ with $e^{im_\ell\phi}$. Thus, we get $m_\ell = 1$

Therefore, $\langle \hat{L}_z \rangle = m_\ell\hbar = \hbar$

Correct option is (2)

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Q25. The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$, is

$$\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right]$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

(1) $3\in, 2\in, -\in$

(2) $6\in, 5\in, 2\in$

(3) $3\in, 2\in, \in$

(4) $-3\in, -2\in, \in$

Ans: (1)

Solution:

$$\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right] = \frac{\in}{\hbar^2}\left[\hat{L}_1\hat{L}_2 + \hat{L}_2^2 - \left(L_{1z} + L_{2z}\right)^2\right]$$

where $\hat{L} = \hat{L}_1 + \hat{L}_2 \Rightarrow \hat{L}^2 = \hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_1\hat{L}_2 + \hat{L}_2\hat{L}_1 = \hat{L}_1^2 + \hat{L}_2^2 + 2\hat{L}_1\hat{L}_2 \qquad \because \left[\hat{L}_1, \hat{L}_2\right] = 0$

$\Rightarrow \hat{L}_1\hat{L}_2 = \frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right)$ and $\hat{L}_z = \hat{L}_{1z} + \hat{L}_{2z}$

Therefore,

$$\hat{H} = \frac{\in}{\hbar^2}\left[\frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right) + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left[\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} - \frac{\hat{L}_2^2}{2} + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left(\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} + \frac{\hat{L}_2^2}{2} - \hat{L}_z^2\right)$$

Now $\left\langle\hat{H}\right\rangle = \frac{\in}{\hbar^2}\left[\frac{\left\langle\hat{L}^2\right\rangle - \left\langle\hat{L}_1^2\right\rangle + \left\langle\hat{L}_2^2\right\rangle}{2} - \left\langle\hat{L}_z^2\right\rangle\right] = \frac{\in}{\hbar^2}\left[\ell(\ell+1)\hbar^2 - \ell_1(\ell_1+1)\hbar^2 + \ell_2(\ell_2+1)\hbar^2 - m_\ell^2\hbar^2\right]$

for $\ell_1 = \ell_2 = 1$ and $\ell = 2$

$$\left\langle\hat{H}\right\rangle = \frac{\in}{\hbar^2}\left[\frac{6\hbar^2 - 2\hbar^2 + 2\hbar^2}{2} - m_\ell^2\hbar^2\right] = \in\left[3 - m_\ell^2\right]$$

For $\ell = 2$, $m_\ell = -2, -1, 0, +1, +2$ and $m_\ell^2 = 0, 1, 4$

$\therefore$ The possible values of $\langle H\rangle$ are

$\langle H\rangle = \in[3-4] = -\in$ for $m_\ell^2 = 4$

$\langle H\rangle = \in[3-1] = 2\in$ for $m_\ell^2 = 1$

$\langle H\rangle = \in[3-0] = 3\in$ for $m_\ell^2 = 0$

Thus correct option is (1)

PART C

Q2. A quantum particle of mass m is moving in a one dimensional potential

$$V(x)=V_0\theta(x)-\lambda\delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E>>V_0>\lambda$ and $\sqrt{2mE}>>\frac{V_0\hbar}{\lambda}$ is

(1) $\frac{V_0^2}{4E^2}$ (2) $\frac{V_0^2}{8E^2}$ (3) $\frac{m\lambda^2}{2E\hbar^2}$ (4) $\frac{m\lambda^2}{4E\hbar^2}$

Ans: (3)

Solution:

$$V(x)=V_0\theta(x)-\lambda\delta(x)\ \Rightarrow V(x)=\begin{cases}0, & x<0\\ V_0-\lambda\delta(x), & x\ge 0\end{cases}$$

$$\psi(x)=\begin{cases}A\exp(ik_1x)+B\exp(-ik_1x), & x\le 0\\ C\exp ik_2x, & x\ge 0\end{cases}\quad \text{where } k_1=\sqrt{\frac{2mE}{\hbar^2}},k_2=\sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

The wave function is continuous $A+B=C$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\left(V_0-\lambda\delta(x)\right)\psi(x)=E\psi(x)$$

Integrating both sides: $\frac{-\hbar^2}{2m}\int_{-\delta}^{\delta}\frac{\partial^2\psi}{\partial x^2}dx+\int_{-\delta}^{\delta}\left(V_0-\lambda\delta(x)\right)\psi(x)dx=\int_{-\delta}^{\delta}E\psi(x)dx$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\delta}^{\delta}+\int_{-\delta}^{\delta}V_0\psi(x)dx-\int_{-\delta}^{\delta}\lambda\psi(x)\delta(x)dx=\int_{-\delta}^{\delta}E\psi(x)dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\delta}^{\delta}+0-\lambda\psi(0)=0\Rightarrow-\frac{\hbar^2}{2m}\left[Cik_2-\left(Aik_1-Bik_1\right)\right]-\lambda(A+B)=0$$

$$-\frac{\hbar^2}{2m}\left[(A+B)ik_2-\left(Aik_1-Bik_1\right)\right]-\lambda(A+B)=0 \qquad \because C=A+B$$

$$-\frac{\hbar^2}{2m}A\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)=-\frac{\hbar^2}{2m}B\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)\Rightarrow\frac{B}{A}=\frac{\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)}{\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)}$$

$$R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+(k_1-k_2)^2}{\frac{4m^2\lambda^2}{\hbar^4}+(k_1+k_2)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}-\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}+\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}$$

$$\Rightarrow R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1-\sqrt{1-\frac{V_0}{E}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{8mE}{\hbar^2}}=\frac{1}{1+\frac{2E\hbar^2}{m\lambda^2}}=\frac{m\lambda^2}{2E\hbar^2}$$

Since $E>>V_0>\lambda$ and $\sqrt{2mE}>>\frac{V_0\hbar}{\lambda}$.

Q6. An incident plane wave with wavenumber k is scattered by a spherically symmetric soft potential. The scattering occurs only in S- and P-waves. The approximate scattering amplitude an angles $\theta=\frac{\pi}{3}$ and $\theta=\frac{\pi}{2}$ are

$$f\left(\theta=\frac{\pi}{3}\right)\approx\frac{1}{2k}\left(\frac{5}{2}+3i\right)\text{ and }f\left(\theta=\frac{\pi}{2}\right)\approx\frac{1}{2k}\left(1+\frac{3i}{2}\right)$$

Then the total scattering cross-section is closest to

(1) $\frac{37\pi}{4k^2}$ (2) $\frac{10\pi}{k^2}$ (3) $\frac{35\pi}{4k^2}$ (4) $\frac{9\pi}{k^2}$

Ans: (1)

Solution: For s-wave and p-wave scattering the total cross-section is

$$\sigma_t=\frac{4\pi}{k^2}\left[\sum_{\ell=0}^{1}(2\ell+1)\sin^2\delta_\ell\right]\quad\Rightarrow\sigma_t=\frac{4\pi}{k^2}\left[\sin\delta_0+3\sin^2\delta_1\right]\qquad ...(1)$$

The amplitude function is $f(\theta)=\frac{1}{k}\left[\sum_{\ell=0}^{1}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right]$

$$f(\theta)=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0+3e^{i\delta_1}\sin\delta_1\cos\theta\right]\qquad ...(2)$$

Now $f\left(\theta=\frac{\pi}{2}\right)=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0\right]\Rightarrow\frac{1}{2k}\left[1+\frac{3i}{2}\right]=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0\right]\Rightarrow e^{i\delta_0}\sin\delta_0=\frac{1}{2}+\frac{3i}{4}$ ...(3)

Multiply with its complex conjugate

$$\left(e^{i\delta_0}\sin\delta_0\right)\left(e^{-i\delta_0}\sin\delta_0\right)=\left(\frac{1}{2}+\frac{3i}{4}\right)\left(\frac{1}{2}-\frac{3i}{4}\right)\Rightarrow\sin^2\delta_0=\frac{13}{16}\qquad ...(4)$$

Similarly $f\left(\theta=\frac{\pi}{3}\right)=\frac{1}{k}\left[e^{i\delta_0}\sin\delta_0+\frac{3}{2k}e^{i\delta_1}\sin\delta_1\right] \Rightarrow \frac{1}{2k}\left[\frac{5}{2}+3i\right]=\frac{1}{k}\left[\frac{1}{2}+\frac{3i}{4}\right]+\frac{3}{2k}e^{i\delta_1}\sin\delta_1$

$$\Rightarrow \frac{5}{4}+\frac{3i}{2}-\frac{1}{2}-\frac{3i}{4}=\frac{3}{2}e^{i\delta_1}\sin\delta_1 \Rightarrow \frac{3}{4}+\frac{3i}{4}=\frac{3}{2}e^{i\delta_1}\sin\delta_1 \Rightarrow \frac{1}{2}+\frac{i}{2}=e^{i\delta_1}\sin\delta_1$$

Multiply with its complex conjugate $\left(\frac{1}{2}+\frac{i}{2}\right)^2=\sin^2\delta_1 \Rightarrow \sin^2\delta_1=\frac{1}{2}$ ...(5)

Substitute values of $\sin^2\delta_0$ and $\sin^2\delta_1$ in equation (1), we get $\sigma_t=\frac{4\pi}{k^2}\left[\frac{13}{16}+\frac{3}{2}\right]=\frac{37\pi}{4k^2}$

Thus correct option is (1)

Q17. A quantum system is described by the Hamiltonian $H=-J\sigma_z+\lambda(t)\sigma_x$, where $\sigma_i\,(i=x,y,z)$ are Pauli matrices, J and λ are positive constants $(J>>\lambda)$ and

$$\lambda(t)=\begin{cases}0 \text{ for } t<0\\ \lambda \text{ for } 0<t<T\\ 0 \text{ for } t>T\end{cases}$$

At $t<0$, the system is in the ground state. The probability of finding the system in the excited state at $t>>T$, in the leading order in λ is

(1) $\frac{\lambda^2}{8J^2}\sin^2\frac{JT}{\hbar}$ (2) $\frac{\lambda^2}{J^2}\sin^2\frac{JT}{\hbar}$

(3) $\frac{\lambda^2}{4J^2}\sin^2\frac{JT}{\hbar}$ (4) $\frac{\lambda^2}{16J^2}\sin^2\frac{JT}{\hbar}$

Ans: (2)

Solution:

$H=-J\sigma_z+\lambda(t)\sigma_x$ where $\sigma_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ and $\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

Therefore $H=-J\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+\lambda(t)\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$$\therefore\ H=\begin{bmatrix}-J & 0\\ 0 & J\end{bmatrix}+\begin{bmatrix}0 & \lambda\\ \lambda & 0\end{bmatrix}=H_0+V(t')$$

Unperturbed Hamiltonian is $H_0=\begin{bmatrix}-J & 0\\ 0 & J\end{bmatrix}$

The eigenvalues of unperturbed Hamiltonian (H_0) are $E=J,-J$

The ground state energy is $E_i=-J$ and corresponding eigenvector is $|\psi_i\rangle=\begin{pmatrix}1\\0\end{pmatrix}$

The first exited state energy is $E_f = +J$ and corresponding eigenvector is $\left|\psi_f\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Now $P_{if} = \frac{1}{\hbar^2}\left|\int_0^T e^{i\omega_{if}t'}\left\langle\psi_f\left|V(t')\right|\psi_i\right\rangle dt'\right|^2$

where $\omega_{if} = \frac{E_f - E_i}{\hbar} = \frac{2J}{\hbar}$ and $\left\langle\psi_f\left|V(t')\right|\psi_i\right\rangle = (0 \quad 1)\begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = (0 \quad 1)\begin{pmatrix} 0 \\ \lambda \end{pmatrix} = \lambda$

Thus, $P_{if} = \frac{1}{\hbar^2}\left|\int_0^T e^{i2Jt'/\hbar}\lambda dt'\right|^2 = \frac{\lambda^2}{\hbar^2}\left|\left[\frac{e^{2iJt'/\hbar}}{2iJ/\hbar}\right]_0^T\right|^2 = \frac{\lambda^2}{\hbar^2}\times\frac{\hbar^2}{4J^2}\left|\left[e^{2iJT/\hbar} - 1\right]\right|^2$

$$\Rightarrow P_{if} = \frac{\lambda^2}{4J^2}\left[e^{2iJT/\hbar} - 1\right]\left[e^{-2iJT/\hbar} - 1\right] = \frac{\lambda^2}{4J^2}\left[1 + 1 - e^{2iJT/\hbar} - e^{-2iJT/\hbar}\right] = \frac{\lambda^2}{4J^2}\left[2 - 2\cos\left(\frac{2JT}{\hbar}\right)\right]$$

$$\Rightarrow P_{if} = \frac{\lambda^2}{2J^2}\left[1 - \cos\left(\frac{2JT}{\hbar}\right)\right] = \frac{\lambda^2}{2J^2}\times 2\sin^2\left(\frac{JT}{\hbar}\right) \quad \therefore P_{if} = \frac{\lambda^2}{J^2}\sin^2\left(\frac{JT}{\hbar}\right)$$

Thus correct option is (2).

Q23. In a quantum harmonic oscillator problem, $\hat{a}$ and $\hat{N}$ are the annihilation operator and the number operator, respectively. The operator $e^{\hat{N}}\hat{a}e^{-\hat{N}}$ is

(1) $\hat{a}$ (2) $e^{-1}\hat{a}$

(3) $e^{-(\hat{I}+\hat{a})}$ (4) $e^{\hat{a}}$

(where $\hat{I}$ is the identity operator)

Ans: (2)

Solution:

Since $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B} + \left[\hat{A},\hat{B}\right] + \frac{1}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + \frac{1}{3!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + \ldots$

Here $\hat{A} = \hat{N}$ and $\hat{B} = \hat{a}$ also $\hat{N} = a^+a$ and $\left[\hat{N},\hat{a}\right] = -\hat{a}$

$$\therefore e^{\hat{N}}\hat{a}e^{-\hat{N}} = \hat{a} + \left[\hat{N},\hat{a}\right] + \frac{1}{2!}\left[\hat{N},\left[\hat{N},\hat{a}\right]\right] + \frac{1}{3!}\left[\hat{N},\left[\hat{N},\hat{a}\right]\right] + \ldots$$

$$= \hat{a} - \hat{a} + \frac{1}{2!}\left[\hat{N},-\hat{a}\right] + \frac{1}{3!}\left[\hat{N},-\hat{a}\right] + \ldots = \hat{a} - \hat{a} + \frac{1}{2!}\hat{a} + \frac{1}{3!}\hat{a} + \ldots$$

$$\Rightarrow e^{\hat{N}}\hat{a}e^{-\hat{N}} = \hat{a}\left[1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \ldots\right] = \hat{a}e^{-1} \qquad \Rightarrow e^{\hat{N}}\hat{a}e^{-\hat{N}} = e^{-1}\hat{a}$$

Thus correct option is (2)

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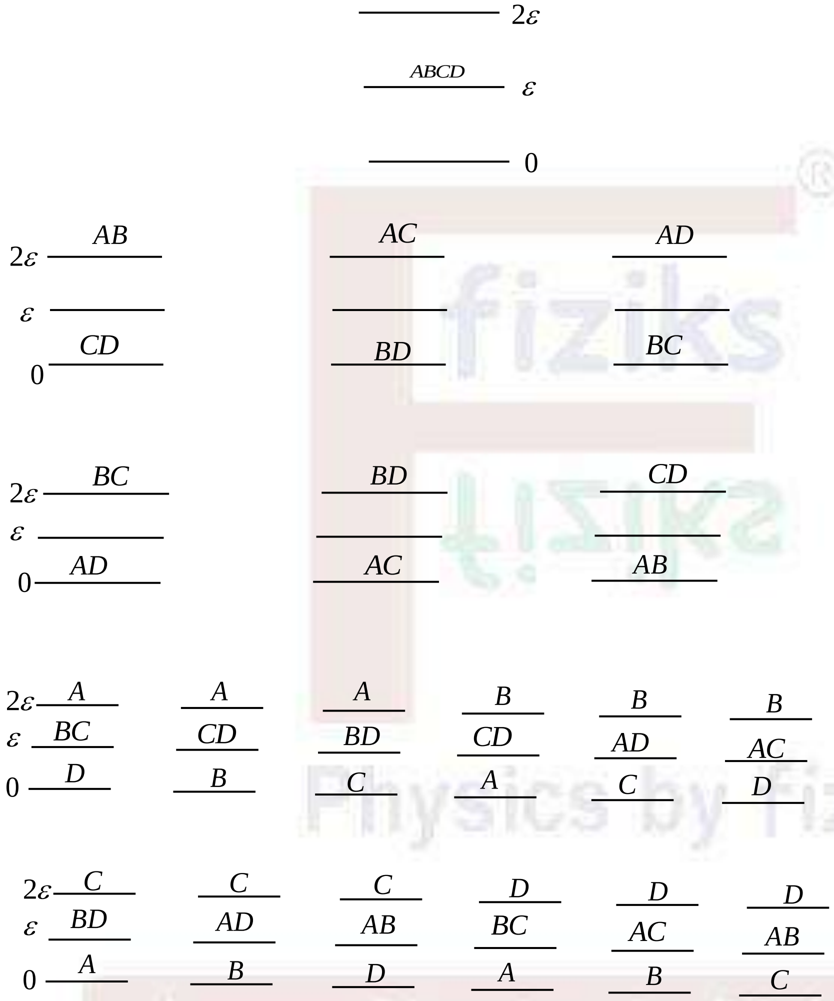
PART B

Q1. Four distinguishable particle fill up energy levels $0, \in, 2\in$. The number of available microstates for the total energy $4\in$ is

(1) 20 (2) 24 (3) 11 (4) 19

Ans: (4)

Solution:



$\therefore$ Total number of microstates so that total energy of a system of 4 distinguishable particle is 4ε is $\Omega = 19$.

Q5. A classical ideal gas is subjected to a reversible process in which its molar specific heat changes with temperature T as $C(T) = C_V + R\dfrac{T}{T_0}$. If the initial temperature and volume are T_0 and V_0 respectively, and the final volume is $2V_0$, then the final temperature is

(1) $T_0/\ln 2$ (2) $2T_0$ (3) $T_0/[1-\ln 2]$ (4) $T_0[1+\ln 2]$

Ans: (4)

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Solution:

Change in entropy is $\Delta S=\int\frac{dQ_{rev}}{T}=\int\frac{C}{T}dT=\int_{T_0}^{T_f}\frac{1}{T}\left(C_V+R\frac{T}{T_0}\right)dT=\int_{T_0}^{T_f}\left(\frac{C_V}{T}+R\frac{1}{T_0}\right)dT$

$\Rightarrow \Delta S=C_V\ln\frac{T_f}{T_0}+\frac{R}{T_0}\left(T_f-T_0\right)$

$\because \Delta S=\int_{T_0}^{T_f}\frac{C_V}{T}dT+\int_{V_0}^{V_f}\frac{R}{V}dV \quad \because \Delta S=\int_{T_0}^{T_f}\frac{C_V}{T}dT+\int_{V_0}^{2V_0}\frac{R}{V}dV=C_V\ln\frac{T_f}{T_0}+R\ln 2$

Comparing both value $C_V\ln\frac{T_f}{T_0}+\frac{R}{T_0}\left(T_f-T_0\right)=C_V\ln\frac{T_f}{T_0}+R\ln 2$

$\Rightarrow \frac{R}{T_0}\left(T_f-T_0\right)=R\ln 2\Rightarrow\frac{R}{T_0}\left(T_f-T_0\right)=R\ln 2\Rightarrow T_f=T_0\left[1+\ln 2\right]$

Q6. A quantum system is described by the Hamiltonian

$$H=JS_z+\lambda S_x$$

where $S_i=\frac{\hbar}{2}\sigma_i$ and $\sigma_i\left(i=x,y,z\right)$ are the Pauli matrices. If $0<\lambda<<J$, then the leading correction in λ to the partition function of the system at temperature T is

(1) $\frac{\hbar\lambda^2}{2Jk_BT}\coth\left(\frac{J\hbar}{2k_BT}\right)$ (2) $\frac{\hbar\lambda^2}{2Jk_BT}\tanh\left(\frac{J\hbar}{2k_BT}\right)$

(3) $\frac{\hbar\lambda^2}{2Jk_BT}\cosh\left(\frac{J\hbar}{2k_BT}\right)$ (4) $\frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right)$

Ans: (4)

Solution:

Given, $H=JS_z+\lambda S_x$ ...(i)

$$H=\frac{\hbar}{2}\left[J\begin{pmatrix}1&0\\0&-1\end{pmatrix}+\lambda\begin{pmatrix}0&1\\1&0\end{pmatrix}\right]=\frac{\hbar}{2}\begin{pmatrix}J&\lambda\\\lambda&-J\end{pmatrix}$$

Obtain the eigen values $\left|H-\lambda' I\right|=0\Rightarrow\begin{vmatrix}J-\lambda'&\lambda\\\lambda&-J-\lambda'\end{vmatrix}=0$

After solving, we get, $-\left(J^2-\lambda'^2\right)-\lambda^2=0$ or $\lambda'^2=\lambda^2+J^2 \Rightarrow\lambda'=\pm\sqrt{\lambda^2+J^2}$

$\therefore$ Energy eigen values are $\frac{\hbar}{2}\left[\pm J\left(1+\frac{\lambda^2}{J^2}\right)^{1/2}\right]=\pm\frac{\hbar J}{2}\left(1+\frac{\lambda^2}{2J^2}\right)$

Here, we retained only first two terms as $\lambda<<J$

The eigen values are $E_1, E_2 = \pm\frac{\hbar}{2}\left(J + \frac{\lambda^2}{2J}\right)$

Now, we can determine the partition function

$$Q = \sum e^{-\beta E_i} = e^{-\beta E_1} + e^{-\beta E_2} = e^{-\frac{\beta J\hbar}{2}} e^{-\frac{\beta\hbar\lambda^2}{4J}} + e^{\frac{\beta J\hbar}{2}} e^{\frac{\beta\hbar\lambda^2}{4J}}$$

$$Q = e^{-\frac{\beta J\hbar}{2}}\left(1 - \frac{\beta\hbar\lambda^2}{4J}\right) + e^{\frac{\beta J\hbar}{2}}\left(1 + \frac{\beta\hbar\lambda^2}{4J}\right) = e^{-\frac{\beta J\hbar}{2}} + e^{\frac{\beta J\hbar}{2}} + \frac{\beta\hbar\lambda^2}{4J}\left(e^{\frac{\beta J\hbar}{2}} - e^{\frac{\beta J\hbar}{2}}\right)$$

$$\Rightarrow Q = \left(e^{-\frac{\beta J\hbar}{2}} + e^{\frac{\beta J\hbar}{2}}\right) + \frac{\beta\hbar\lambda^2}{4J}\left[2\sinh\left(\frac{\beta J\hbar}{2}\right)\right] = \left(e^{-\frac{\beta J\hbar}{2}} + e^{\frac{\beta J\hbar}{2}}\right) + \frac{\hbar\lambda^2}{2Jk_BT}\sinh\left(\frac{J\hbar}{2k_BT}\right) \quad ...(2)$$

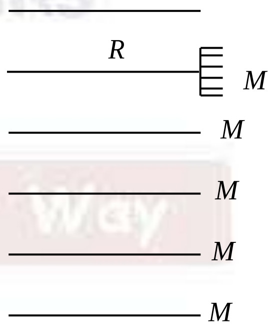
Here, first term in the P.F. is due to unperturbed Hamiltonian whereas the second term arises due to perturbation term and this is our derived result. Note that while expanding $e^{\pm\frac{\beta\hbar\lambda^2}{4J}}$ we have kept only first two terms as $\lambda << J$. Hence (4) is correct.

Q16. Each allowed energy level of a system of non-interacting fermions has a degeneracy M. If there are N fermions and R is the remainder upon dividing N by M, then the degeneracy of the ground state is

(1) R^M (2) 1 (3) M (4) ${}^MC_R$

Ans: (4)

Solution:



It is given to be Fermi system with a degeneracy of M. This means that each energy level will have M fermions up to certain level. Remaining R Fermions have to be filled in M degenerate levels. Therefore, in ground state lower a few levels will contain M each and remaining R have to distributed in M-degeneracy levels so that each will have only 1 Fermions.

This can be achieved in $\Omega = \frac{M!}{R!(M-R)!} = {}^MC_R$ ways.

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Q23. A system of N non-interacting classical spins, where each spin can take values $\sigma = -1, 0, 1$, is placed in a magnetic field h. The single spin Hamiltonian is given by

$$H = -\mu_B h\sigma + \Delta\left(1 - \sigma^2\right),$$

where μ_B, Δ are positive constants with appropriate dimensions. If M is the magnetization, the zero-field magnetic susceptibility per spin $\left.\frac{1}{N}\frac{\partial M}{\partial h}\right|_{h\to 0}$, at a temperature $T = 1/\beta k_B$ is given by

(1) $\beta\mu_B^2$ (2) $\frac{2\beta\mu_B^2}{2+e^{-\beta\Delta}}$ (3) $\beta\mu_B^2 e - \beta\Delta$ (4) $\frac{\beta\mu_B^2}{1+e^{-\beta\Delta}}$

Ans: (2)

Solution:

$\sigma = 1, 0, -1$. $\quad H = -\mu_B h\sigma + \Delta\left(1-\sigma^2\right)$ ...(1)

σ	$H = -\mu_B h\sigma + \Delta\left(1-\sigma^2\right)$
+1	$-\mu_B h$
0	Δ
−1	$+\mu_B h$

It is a three level system problem. Therefore, for a single spin, the Partition Function is

$Q_1 = e^{-\beta\Delta} + e^{\beta\mu_B h} + e^{-\beta\mu_B h} \quad \Rightarrow Q_1 = e^{-\beta\Delta} + 2\cosh\left(\beta\mu_B h\right)$ ...(2)

$\because$ system is given to be N non-interacting classical points,

$\therefore Q_N = \left[e^{-\beta\Delta} + 2\cosh\left(\beta\mu_B h\right)\right]^N$ ...(3)

Helmholtz free energy is $A = -kT\ln Q_N$

$A = -NkT\ln\left[e^{-\beta\Delta} + 2\cosh\left(\beta\mu_B h\right)\right]$ ...(4)

Now, the magnetization is $M = -\left(\frac{\partial F}{\partial h}\right) = \frac{NkT}{e^{-\beta D} + 2\cosh\left(\beta\mu_B h\right)} \times 2\sinh\left(\beta\mu_B h\right) \times \left(\beta\mu_B\right)$

As $h \to 0$, M reduces to $\quad M = \frac{N\mu_B \times \left(2\beta\mu_B h\right)}{e^{-\beta\Delta}+2} = \frac{2N\mu_B^2\beta h}{e^{-\beta\Delta}+2}$

$\therefore \frac{1}{N}\left(\frac{\partial M}{\partial h}\right)_{h\to 0} = \left(\frac{2\mu_B^2\beta}{e^{-\beta\Delta}+2}\right)$

Option (2) is correct.

PART C

Q14. The work done on a material to change its magnetization M in an external field H is $dW = HdM$. Its Gibbs free energy is

$$G(T,H) = -\left(\gamma T + \frac{aH^2}{2T}\right)$$

where $\gamma, a > 0$ are constants. The material is in equilibrium at a temperature $T = T_0$ and in an external field $H = H_0$. If the field is decreased to $\frac{H_0}{2}$ adiabatically and reversibly, the temperature changes to

(1) $2T_0$

(2) $\frac{T_0}{2}$

(3) $\left(\frac{a}{2\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$

(4) $\left(\frac{a}{\gamma}\right)^{\frac{1}{4}}\sqrt{H_0T_0}$

Ans: (2)

Solution: $dU = TdS + HdM$, $G = U + PV - TS - MH$

$\Rightarrow dG = -SdT - MdH$. Thus $S = -\left(\frac{\partial G}{\partial T}\right)_H, M = -\left(\frac{\partial G}{\partial H}\right)_T \Rightarrow \left(\frac{\partial M}{\partial T}\right)_H = \left(\frac{\partial S}{\partial H}\right)_T$

$$\because G(T,H) = -\left(\gamma T + \frac{aH^2}{2T}\right) \Rightarrow M = -\left(\frac{\partial G}{\partial H}\right)_T = \frac{2aH}{2T} = \frac{aH}{T} \Rightarrow \left(\frac{\partial M}{\partial T}\right)_H = -\frac{aH}{T^2} = \left(\frac{\partial S}{\partial H}\right)_T$$

Also $S = -\left(\frac{\partial G}{\partial T}\right)_H = \gamma - \frac{aH^2}{2T^2} \Rightarrow \left(\frac{\partial S}{\partial T}\right)_H = \frac{aH^2}{T^3}$

$$\because \left(\frac{\partial T}{\partial H}\right)_S = -\left(\frac{\partial T}{\partial S}\right)_H\left(\frac{\partial S}{\partial H}\right)_T = -\frac{1}{aH^2/T^3}\left(-\frac{aH}{T^2}\right) = \frac{T^3}{aH^2} \times \frac{aH}{T^2} = \frac{T}{H}$$

$$\Rightarrow \int_{T_0}^{T_f} \frac{1}{T}dT = \int_{H_0}^{H_0/2} \frac{1}{H}dH \Rightarrow \ln\frac{T_f}{T_0} = \ln\left(\frac{1}{2}\right) \Rightarrow T_f = \frac{T_0}{2}$$

Or

$$dG = -SdT + VdP \Rightarrow S = -\left(\frac{\partial G}{\partial T}\right)_P. \quad \text{Thus } S = -\frac{\partial}{\partial T}\left[-\left(\gamma T + \frac{aH^2}{2T}\right)\right] \Rightarrow S = \gamma + \frac{aH^2}{2}\left(-\frac{1}{T^2}\right)$$

Adiabatically reducing the field means adiabatic demagnetization in which $\Delta S = 0$ or $S_F = S_i$

i.e. $\gamma - \frac{a}{2}\left(\frac{H_0}{2}\right)^2\frac{1}{T_f^2} = \gamma - \frac{a}{2}\left(\frac{H_0}{T_0}\right)^2 \Rightarrow \frac{H_0^2}{4T_f^2} = \frac{H_0^2}{T_0^2}$, giving $T_f = \left(\frac{T_0}{2}\right)$

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Q22. A photon inside the sun executes a random walk process. Given the radius of the sun $\approx 7\times10^{8}\,km$ and mean free path of a photon $\approx 10^{-3}\,m$, the time taken by the photon to travel from the centre to the surface of the sun is closest to

(1) 10^{6} sec (2) 10^{24} sec

(3) 10^{12} sec (4) 10^{18} sec

Ans: (3)

Solution: Total time for photon to emerge from the sum is $\tau = \dfrac{R^2}{\ell C}$ ...(1)

where, R is radius of sun, ℓ = mean free path of a photon, c = speed of photon $= 3\times10^{10}$ cm/s

given, $R = 7\times10^{8}\,km = 7\times10^{13}\,cm$, $\ell = 10^{-3}\,m = 10^{-1}\,cm$, $c = 3\times10^{10}\,cm/s$

Ref: Princeton University Press, Chapter 3, Steller Physics

$$\therefore\ \tau = \frac{\left(7\times10^{13}\right)^2}{10^{-1}\times3\times10^{10}} = \frac{49}{3}\times10^{17} = 16.3\times10^{17}\,s \quad \Rightarrow \tau \approx 1.63\times10^{18}\,s$$

$\therefore$ (D) must be correct option for the given information.

Note: Actually, here R was given to be wrong. Actual value of $R = 7\times10^{8}\,m = 7\times10^{10}\,cm$.

$\tau = \dfrac{\left(7\times10^{10}\right)^2}{10^{-1}\times3\times10^{10}} = \dfrac{49}{3}\times10^{11}\,s \Rightarrow \tau \approx 1.6\times10^{12}\,s$. This gives the correct answer.

Q27. A system of non-relativistic and non-interacting bosons of mass m in two dimensions has a density n. The Bose-Einstein condensation temperature T_c is

(1) $\dfrac{12n\hbar^2}{\pi m k_B}$ (2) $\dfrac{3n\hbar^2}{\pi m k_B}$

(3) $\dfrac{6n\hbar^2}{\pi m k_B}$ (4) 0

Ans: (4)

Solution:

Dimension	Massive Boson $\varepsilon = p^2/2m$, Non-Relativistic	Mass-less Bosons Photon, $\varepsilon = pc$
1-D	No	No
2-D	No	Yes
3-D	Yes	Yes

Therefore, in 2D, BEC is not feasible for non-relativistic particle. Most appropriate option is (4). $T = 0K$ cannot achieved.

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PART B

Q7. In the measurement of a radioactive sample, the measured counts with and without the sample for equal time intervals are $C = 500$ and $B = 100$, respectively. The errors in the measurements of C and B are $|\Delta C| = 20$ and $|\Delta B| = 10$, respectively. The net error $|\Delta Y|$ in the measured counts from the sample $Y = C - B$, is closet to

(1) 22 (2) 10 (3) 30 (4) 43

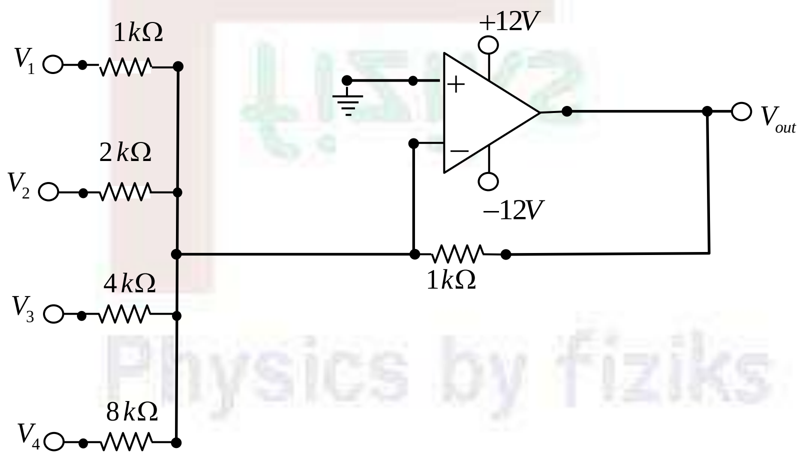
Ans: (1)

Solution:

The net error $|\Delta Y|$ in the measured counts from the sample $Y = C - B$, is

$$|\Delta Y| = \sqrt{|\Delta C|^2 + |\Delta B|^2} = \sqrt{20^2 + 10^2} = \sqrt{500} \approx 22$$

Q9. In the circuit shown below using an ideal opamp, inputs $V_j\,(j = 1,2,3,4)$ may either be open or connected to a $-5V$ battery.



The minimum measurement range of a voltmeter to measure all possible values of V_{out} is

(1) 10 V (2) 30 V (3) 3 V (4) 1 V

Ans: (1)

Solution: $V_0 = -\frac{R_F}{R_1}V_1 - \frac{R_F}{R_2}V_2 - \frac{R_F}{R_3}V_3 - \frac{R_F}{R_4}V_4 \Rightarrow V_0 = -\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right](-5V)$

$$\Rightarrow V_0 = -[1 + 0.5 + 0.25 + 0.125](-5V) = 1.875 \times 5 = 10V$$

Q14. For three inputs A, B and C, the minimum number of 2-input NAND gates required to generate the output $Y = \overline{A + B} + \overline{C}$ is

(1) 3 (2) 4 (3) 7 (4) 6

Ans: (2)

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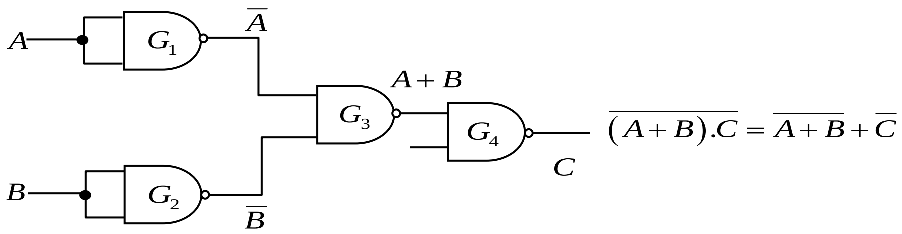
Solution:



So minimum number of 2-input NAND gates required is 4.

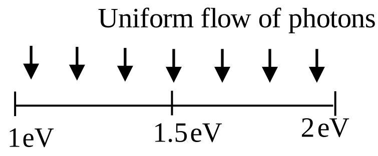
Q20. The light incident on a solar cell has a uniform photon flux in the energy range of 1 eV to 2 eV and is zero elsewhere. The active layer of the cell has a bandgap of 1.5 eV and absorbs 80% of the photons with energies above the bandgap. Ignoring non-radiative losses, the power conversion efficiency (ratio of the output power to the input power) is closest to

(1) 47% (2) 70% (3) 23% (4) 35%

Ans: (1)

Solution:

Flux of photons is uniform. Let us assume total 100 photons are falling. The average energy of the given spectral region is



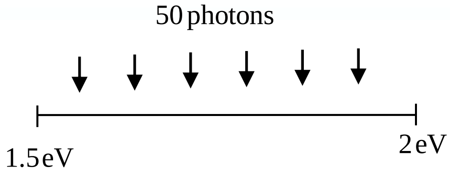
$$\varepsilon_{au} = \frac{1+2}{2} eV = 1.5 eV$$

We are taking the average here, $\because$ flux is uniform.

Therefore, total energy incident per second $= 1.5 eV \times 100 = 150 eV$

Further, bandgap of active layer of scalar cell is = 1.5 eV. Hence, only photons with energy greater than or equal to 1.5 eV will be absorbed. That is out of 100 photons only 50 photons will be absorbed.

Average energy of spectrum between $1.5 - 2 eV$, $\varepsilon_{av} = \frac{2+1.5}{2} = 1.75 eV$



Energy content or energy incident on solar cell. $= 50 \times 1.75 = 87.5 eV$

$\because$ It is given to be 80%, therefore, energy absorbed is $\varepsilon_{ab} = 87.5 \times 0.8 = 70 eV$

$\therefore$ Efficiency $\eta = \frac{70 eV}{150 eV} \times 100 = 46.66 \approx 47\%$

PART C

Q13. Gauge factor of a strain gauge is defined as the ratio of the fractional change in resistance $\left(\frac{\Delta R}{R}\right)$ to the fractional change in length $\left(\frac{\Delta L}{L}\right)$. A metallic strain gauge with a gauge factor 2 has a resistance of 100Ω under unstrained condition. An aluminum foil with Young's modulus $Y = 70\,GN/m^2$ is installed on the metallic gauge. Keeping the foil within its elastic limit, a stress of $0.2\,GN/m^2$ is applied on the foil. The change in the resistance of the gauge will be closest to

(1) 0.14Ω (2) 1.23Ω

(3) 0.28Ω (4) 0.56Ω

Ans: (4)

Solution:

Gauge factor $G_F = \left(\frac{\Delta R}{R}\right)\Big/\left(\frac{\Delta L}{L}\right) \Rightarrow \frac{\Delta R}{R} = G_F\frac{\Delta L}{L}$

Young's modulus $Y = 70\,GN/m^2 = \frac{Stress}{Strain} = \frac{0.2\,GN/m^2}{\Delta L/L} \Rightarrow \frac{\Delta L}{L} = \frac{0.2}{70}$

Thus $\frac{\Delta R}{R} = 2\times\frac{0.2}{70} \Rightarrow \Delta R = 2\times\frac{0.2}{70}\times 100 = 0.57$

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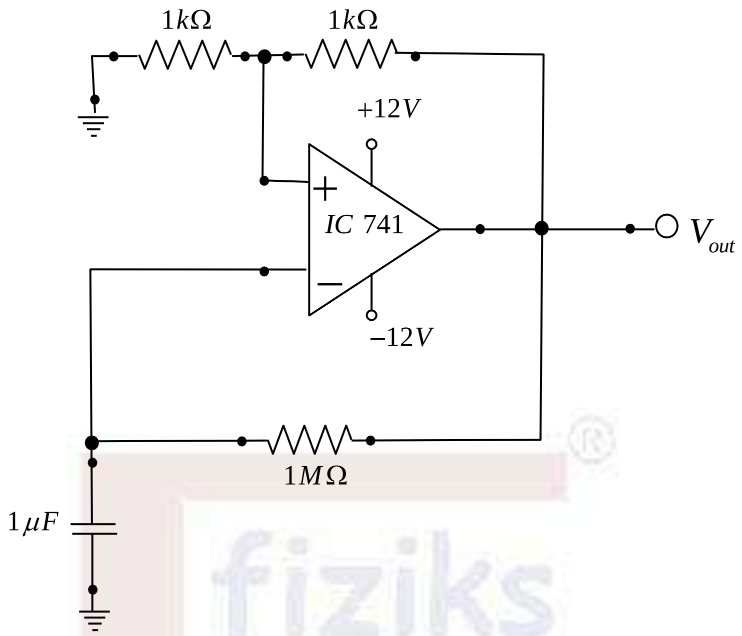
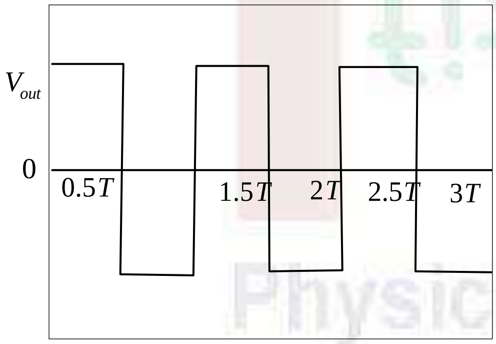
Q20. A circuit with operational amplifier is shown in the figure below.

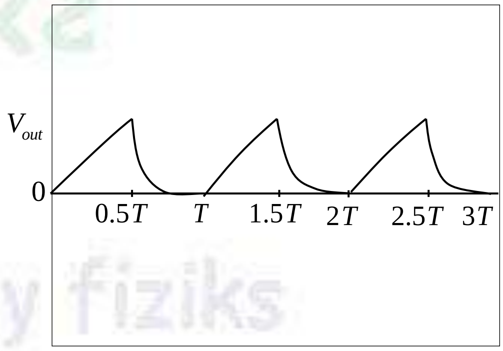


The output voltage waveform V_{out} will be closest to

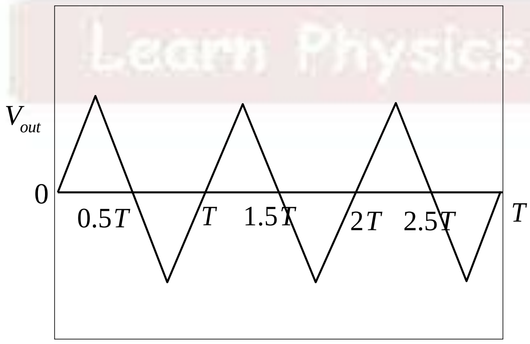
(1)



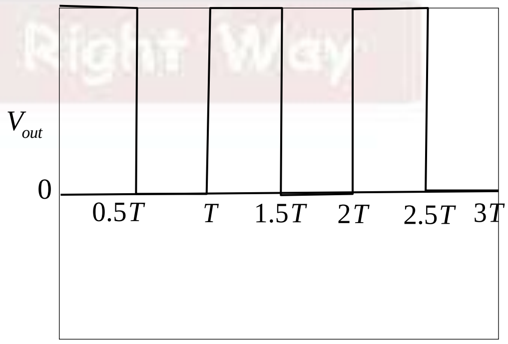
(2)



(3)



(4)



Ans: (1)

Solution:

It’s a square wave generator circuit.

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Solution-Atomic and Molecular Physics

Be Part of Disciplined Learning

PART C

Q1. In the rotational-vibrational spectrum of an idealized carbon monoxide (CO) molecule, ignoring rotational-vibrational coupling, two transitions between adjacent vibrational levels with wavelength λ_1 and λ_2, correspond to the rotational transition from $J'=0$ to $J''=1$ and $J'=1$ to $J''=0$, respectively. Given that the reduced mass of CO is $1.2\times 10^{-26}\,kg$, equilibrium bond length of CO is 0.12 nm and vibrational frequency is 5×10^{13} Hz, the ratio of $\frac{\lambda_1}{\lambda_2}$ is closest to

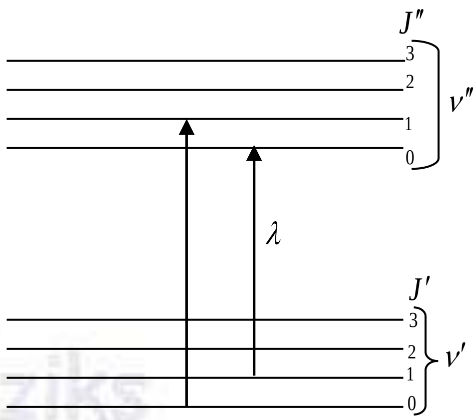
(1) 0.9963 (2) 0.0963 (3) 1.002 (4) 1.203

Ans: (1)

Solution:

$$B=\frac{h}{8\pi^2 c\mu r^2}=\frac{6.6\times 10^{-34}}{\left[8\times(3.14)^2\times 3\times 10^8\times 1.2\times 10^{-26}\times\left(0.12\times 10^{-9}\right)^2\right]}$$

$$\Rightarrow B=162.1434$$



$$v_{os}=c\omega_e \Rightarrow \omega_e=\frac{v_{os}}{c}=\frac{5\times 10^{13}}{3\times 10^8}\Rightarrow \omega_e=\frac{5}{3}\times 10^5=166666.667$$

$$\text{Total wave number}=\left(v+\frac{1}{2}\right)\omega_e+BJ\left(J+1\right)$$

$$\Delta\bar{v}=\omega_e+B\left[J''\left(J''+1\right)-J'\left(J'+1\right)\right]$$

$$\Delta\bar{v}_1=\omega_e+2B,\ \Delta\bar{v}_2=\omega_e-2B \quad \Rightarrow \frac{\Delta\bar{v}_1}{\Delta\bar{v}_2}=\frac{\omega_e+2B}{\omega_e-2B}$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2}=\frac{\omega_e-2B}{\omega_e+2B}=\frac{166666.67-2\times 1.62.1434}{166666.67+2\times 162.1434} \qquad \Rightarrow \frac{\lambda_1}{\lambda_2}=0.9961$$

Q8. The ionization potential of hydrogen atom is 13.6 eV and λ_H and λ_D denote longest wavelengths in Balmer spectrum of hydrogen and deuterium atoms, respectively. Ignoring the fine and hyperfine structures, the percentage difference $y=\frac{\lambda_H-\lambda_D}{\lambda_H}\times 100$, is closest to

(1) 1.0003% (2) -0.03% (3) 0.03% (4) -1.0003%

Ans: (3)

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Solution: $\frac{1}{\lambda_H}=R_H\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$ and $\frac{1}{\lambda_D}=R_D\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$ $\Rightarrow \frac{\lambda_D}{\lambda_H}=\frac{R_H}{R_D}=\frac{R_\infty}{1+\frac{m}{m_P}}\times\frac{1+\frac{m}{2m_P}}{R_\infty}$

$$\Rightarrow \frac{\lambda_D}{\lambda_H}=\frac{1+\frac{1}{2\times1836}}{1+\frac{1}{1836}}=\frac{3673}{2\times1837}=0.99973 \quad \Rightarrow \frac{\lambda_H-\lambda_D}{\lambda_H}\times100=[1-0.99973]\times100=0.027$$

Q25. A solar probe mission detects a fractional wavelength shift $(\Delta\lambda/\lambda)$ of the spectral line $\lambda=630\,nm$ within a sunspot to be of the order of 10^{-5}. Assuming this shift is caused by the normal Zeeman effect (i.e., neglecting other physical effects), the estimated magnetic field (in tesla) within the observed sunspot is closest to

(1) 3×10^{-5} (2) 300

(3) 0.3 (4) 3×10^{5}

Ans: (3)

Solution:

$\Delta\nu'=46.7B \quad m^{-1}$, $\Delta\lambda=\lambda^2\Delta\nu'=\lambda^2 46.7B$

$$B=\frac{\Delta\lambda}{\lambda}\cdot\frac{1}{46.7\lambda}=10^{-5}\times\frac{1}{46.7\times\left(630\times10^{-9}\right)}$$

$$B=\frac{10^4}{46.7\times630}=0.34\ \text{Tesla}$$

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Solution-Solid State Physics

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PART C

Q16. The lattice constant of the bcc structure of sodium metal is 4.22Å. Assuming the mass of the electron inside the metal to be the same as free electron mass, the free electron Fermi energy is closest to

(1) 3.2 eV (2) 2.9 eV (3) 3.5 eV (4) 2.5 eV

Ans: (1)

Solution: Structure is BCC so, effective number of atom is equal to 2.

$$\therefore \quad n = \frac{N_{eff}}{a^3} = \frac{2}{\left(4.22\times10^{-10}\right)^3} = \frac{2}{75.15\times10^{-30}} \Rightarrow n = 0.0268\times10^{30} \qquad \text{here } a = 4.22\text{Å}$$

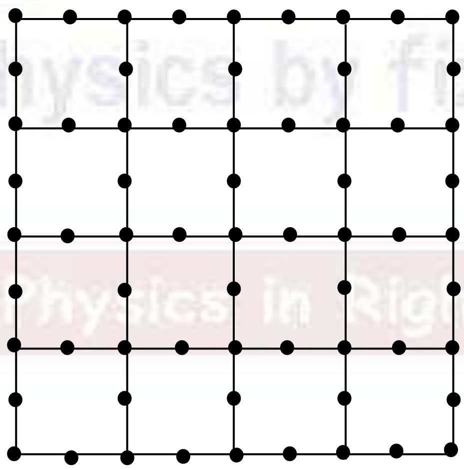
$$\because E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

$$\Rightarrow E_F = \frac{\left(6.67\times10^{-34}\right)^2}{8\times\left(3.14\right)^2\times9.1\times10^{-31}}\left[3\times3.14\times3.14\times0.0266\times10^{30}\right]^2$$

$$E_F = \frac{44.48\times10^{-68}}{717.77\times10^{-31}}\left(0.8530\right)\times10^{20} \Rightarrow E_F = \frac{37.94\times10^{-48}\times10^{31}}{717.77\times1.6\times10^{-19}}eV \Rightarrow E_F = \frac{3794}{1148.432}eV$$

$$\Rightarrow E_F = 3.3eV$$

Q28. In the section of an infinite lattice shown in the figure below, all sites are occupied by identical hard circular discs so that the resulting structure is tightly packed.



The packing fraction is

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{4}$

(3) $\frac{3\pi}{16}$ (4) $\frac{9\pi}{16}$

Ans: (3)

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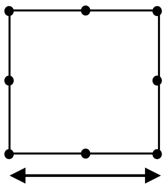
Solution:

Packing fraction is $P.F. = \dfrac{n_{eff} \times \pi r^2}{A}$



where $n_{eff} = \dfrac{1}{4}n_c + \dfrac{1}{2}n_f + n_i = \dfrac{1}{4}\times 4 + \dfrac{1}{2}\times 4 + 0 = 1 + 2 = 3$

and $A = a^2 = 16r^2$

$\therefore \; P.F = \dfrac{3\times \pi r^2}{16r^2} = \dfrac{3\pi}{16}$

Thus correct option is (3).

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Solution-Nuclear and Particle Physics

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PART C

Q3. Atmospheric neutrinos are produced from the cascading decays of cosmic pions $\left(\pi^{\pm}\right)$ to stable particles. Ignoring all other neutrino sources, the ratio of muon neutrino $\left(\nu_{\mu}+\bar{\nu}_{\mu}\right)$ flux to electron neutrino $\left(\nu_{e}+\bar{\nu}_{e}\right)$ flux in atmosphere is expected to be closest to

(1) 2:3 (2) 1:1 (3) 1:2 (4) 2:1

Ans: (4)

Solution:

$$\pi^{+}\rightarrow\nu_{\mu}+\bar{\nu}_{\mu}$$

The decay of a pion into $\nu_{\mu}+\bar{\nu}_{\mu}$ is mediated by weak (force) interaction which is governed by certain conservation laws, particularly conservation of lepton number and conservation of helicity. In the decay of pion, lepton number is conserved. In this case, the helicity is also conserved because helicity of π^{+} is zero and the helicity of $\left(\nu_{\mu}+\bar{\nu}_{\mu}\right)$ is also zero.

$$\pi^{+}\rightarrow\nu_{e}+\bar{\nu}_{e}$$

In this decay, the conservation of helicity is violated because helicity of ν_{e} is opposite to the helicity of $\bar{\nu}_{e}$. So its decay rate is low with a factor dependent on the masses of the electron and muon. So, option (d) is correct.

Q12. In a shell model description, neglecting Coulomb effects, which of the following statements for the energy and spin-parity is correct for the first excited state of $A=12$ isobars ${}^{12}_{5}B, {}^{12}_{6}C, {}^{12}_{7}N$?

(1) same for ${}^{12}_{5}B, {}^{12}_{6}C$ and ${}^{12}_{7}N$

(2) different for each ${}^{12}_{5}B, {}^{12}_{6}C$ and ${}^{12}_{7}N$

(3) same for ${}^{12}_{6}C$ and ${}^{12}_{7}N$, but different for ${}^{12}_{5}B$

(4) same for ${}^{12}_{5}B$ and ${}^{12}_{7}N$ but different for ${}^{12}_{6}C$

Ans: (4)

Solution: Excited states are same for ${}^{12}_{5}B$ and ${}^{12}_{7}N$ because their neutron and proton numbers are just interchanged.

(1) $n_P=5, n_n=7$ for ${}^{12}_{5}B$ (2) $n_P=7, n_n=5$ for ${}^{12}_{7}N$

Excited states are different for ${}^{12}_{6}C$ because its proton number and neutron number are different $\left(n_P=6, n_n=6\right)$.

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Q29. The ground state of ${}^{207}_{82}Pb$ nucleus has spin-parity $J^{\pi} = \left(\frac{1}{2}\right)^{-}$, while first excited state has $J^{\pi} = \left(\frac{5}{2}\right)^{-}$. For the transition from the first excited state to the ground state, possible multipolarities of emitted electromagnetic radiation are

(1) E2, E3 (2) M2, M3

(3) M2, E3 (4) E2, M3

Ans: (4)

Solution:

$\Delta\pi = \text{No}; \quad |J_i - J_f| < L < (J_i + J_f)$

$L = \underset{E2}{\underset{\downarrow}{2}}, \underset{M3}{\underset{\downarrow}{3}}$

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Solution-General Aptitude

Be Part of Disciplined Learning

PART A

Q1. All the four entries in column A must be matched with all those in column B. Each correctly matched option gets one mark and no mark is awarded otherwise. Which of the following mark(s) CANNOT be scored?

(1) 3 (2) 1 (3) 2 (4) 4

Ans: (1)

Solution:

If all the three entries are correctly matched, then fourth will automatically be matched to correct one. Hence, score 3, is not possible at all.

Q2. Four children had 27 apples among them. No child had less than 5 apples. If no two children had the same number of apples, then which of the following could NOT be the number of apples a child had?

(1) 5 (2) 6 (3) 8 (4) 9

Ans: (3)

Solution:

Total no. of apples = 27, Total no. of children = 4

Condition:

(a) each child gets ≥ 5 apples. (b) No. two children get same no. of apples.

Child :	c_1	c_2	c_3	c_4
	5	5	5	5
(+7) :	0	1	2	4
	5	6	7	9

Hence, only way to distribute 27 apples among 4 children keeping in mind conditions imposed is: (5, 6, 7, 9) in any order. Hence, 8 is the no. a child cannot have.

Q3. In 1979, Ramesh's age was the sum of the digits of his year of birth. In 2017, on his birthday, what was his age?

(1) 49 (2) 57 (3) 60 (4) 64

Ans: (3)

Solution:

Ramesh's age in 1979 = Sum of digits of his year of birth

Let year of birth $=19xy$

Age in 1979 $=1979-19xy=(70-10x)+(9-y)$

Digital sum of year of birth $=1+9+x+y=10+x+y$

Given, $(70-10x)+(9-y)=10+x+y \Rightarrow 69-10x-y=x+y$ or $11x+2y=69$

$$x=\frac{69-2y}{11}=\frac{55+(14-2y)}{11}=5+\frac{14-2y}{11}$$

Only possible value for $y=7 \quad \therefore \quad x=5$ and $y=7$

So, year of birth = 1957

Age in 2017 $=(2017-1957)=60$

Q4. If $a<x<b$, then for which of the following relations does $0<y<1$ always hold?

1. $y=\frac{a-x}{b+a}$
2. $y=\frac{x-a}{b-a}$
3. $y=\frac{x-b}{b-a}$
4. $y=\frac{b-x}{a+b}$

Ans: (2)

Solution:

$0<y<1$ or, $0<\frac{x-a}{b-a}<1$ or, $0<x-a<b-a$ or, $a<x<b$, which is the stated condition.

Only choice which will always hold is (2).

Q5. A person's viral load measured in some unit was 15, 25, 50, 200, 300, 150 and 30 on days 1 to 7, respectively. The maximum relative change took place between

1. day 3 to day 4
2. day 4 to day 5
3. day 5 to day 6
4. day 6 to day 7

Ans: (1)

Solution:

Viral load on	D_1	D_2	D_3	D_4	D_5	D_6	D_7
	15	25	50	200	300	150	30

Rel. charges from D_3 to $D_4=\frac{200-50}{50}=3$

$$D_4 \text{ to } D_5=\frac{300-200}{200}=\frac{1}{2}$$

$$D_5 \text{ to } D_6=\frac{150-300}{300}=-\frac{1}{2}$$

$$D_6 \text{ to } D_7=\frac{30-150}{150}=\frac{-120}{150}$$

Hence, maximum relative change = 3 which is from D_3 to D_4.

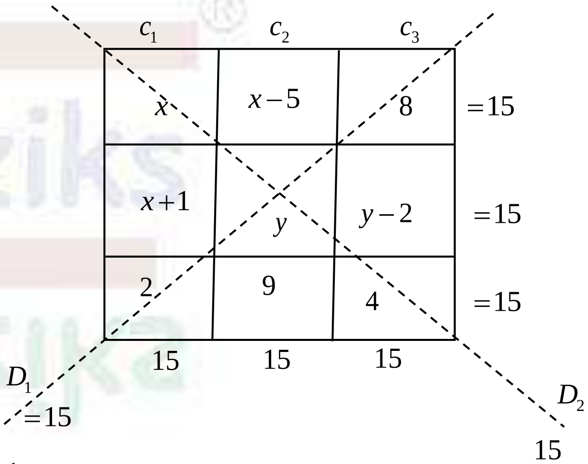
Q6. What is the value of x in the given magic square, (i.e, a square grid in which the sum of the numbers in rows, columns and diagonals is the same)?

x	$x-5$	8
$x+1$	y	$y-2$
2	9	4

(1) 6 (2) 4 (3) 3 (4) 1

Ans: (1)

Solution:



In diagonal D_1:

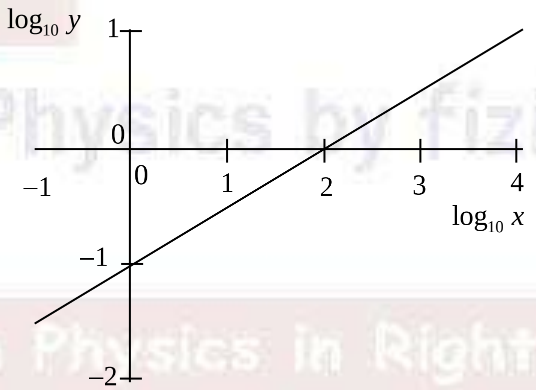
$8+y+2=15 \quad \therefore y=5$

In C:

$x+(x+1)+2=15$

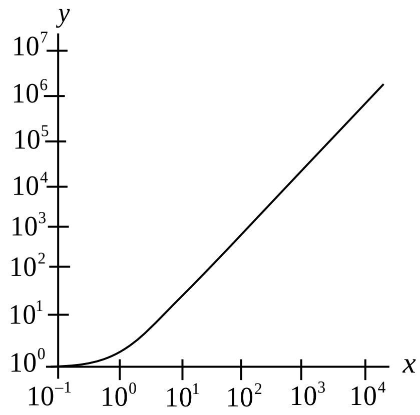
$2x=12 \Rightarrow x=6$

Q7. In the figure $\log_{10} y$ is plotted against $\log_{10} x$

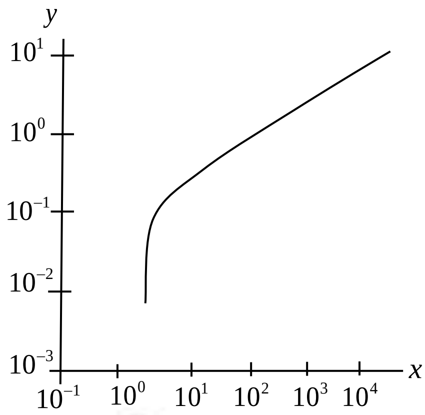


When y is plotted against x, then the plot in the provided range is

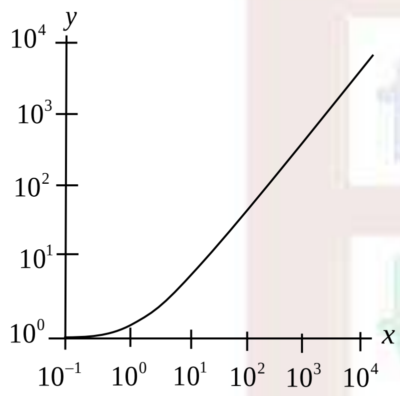
(A)



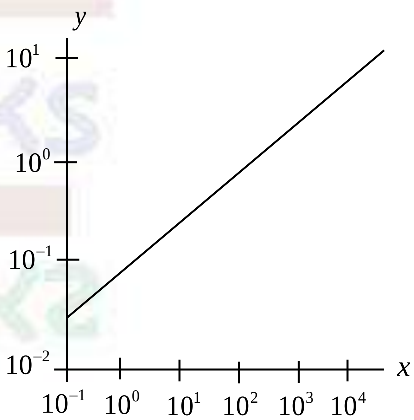
(B)



(C)



(D)



(1) A (2) B (3) C (4) D

Ans: (4)

Solution:

From the given graph:

coordinates are: $\left(10^0, 10^{-1}\right)$; $\left(10^2, 1\right)$; $\left(10^3, <10\right)$; $\left(10^4, 10\right)$

From the plots A, B, C, D plot D is the correct one.

Q8. Radius of sphere is measured with 5% uncertainty. What is the uncertainty in the volume, determined from this radius?

(1) 5% (2) 6.6% (3) 125% (4) 15%

Ans: (4)

Solution: Radius is measured with 5% uncertainly

Let radius $= r$ & uncertainly $= \frac{\Delta r}{r} \times 100 = 5\%$

then uncertainly in volume $= \frac{4}{3}\pi \left(r + \Delta r\right)^3 - \frac{4}{3}\pi r^3$

$$=\frac{\cancel{r^3}+3r^2\cdot\Delta r+3r\cdot(\Delta r)^2+(\Delta r)^3-\cancel{r^3}\times\frac{4}{3}\pi r^3}{r^3}=3\cdot\left(\frac{\Delta r}{r}\right)+3\cdot\left(\frac{\Delta r}{r}\right)^2\approx 3\cdot\frac{\Delta r}{r}$$

$\therefore$ % change $=3\cdot\left(\frac{\Delta r}{r}\right)\times 100$

Given, $\frac{\Delta r}{r}\times 100=5\%$ $\quad\therefore$ uncertainty in volume $=3\times 5=15\%$

Q9. In a market, you can buy a mango for Rs.10, a lemon for Rs.1 and 8 chillies for Rs.1. How many of these items do you need to buy to get a mix of 100 items for exactly Rs. 100?

(1) 6 mangoes, 22 lemons, 72 chillis (2) 7 mangoes, 21 lemons, 72 chillis

(3) 1 mango, 9 lemons, 80 chillis (4) 8 mangoes, 12 lemons, 80 chillis

Ans: (2)

Solution:

Cost of a mango = ₹10, a lemon = ₹1, 8 chillies = ₹1

Total items = 100, Total cost = 100

Let you buy x mango, y lemon and z chillies.

Then $10x+y+\frac{z}{8}=100$; $x+y+z=100$

$9x+\left(\frac{z}{8}-z\right)=0 \Rightarrow 9x=\frac{7z}{8}$ or, $72x=7z$

$$72\times 7=7\times 72$$
$$\downarrow \qquad \downarrow$$
$$x \qquad z$$

for $x=7, z=72, y=100-(72+7)=21$

Q10. A letter is drawn at random from the following string of letters.

R A M U K Y A J N A S

What is the probability that it is NOT a vowel?

(1) 1/2 (2) 6/11 (3) 7/11 (4) 8/11

Ans: (3)

Solution:

String of letters: R A M U K Y A J N A S

Vowel: A U A A

Probability of picking a vowel $=\frac{4}{11}$. Probability that it is not a vowel $=1-\frac{4}{11}=\frac{7}{11}$

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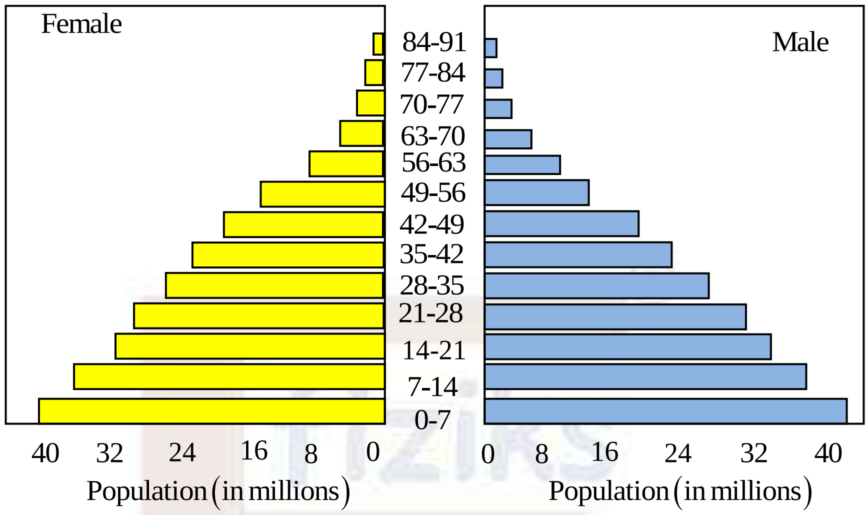
Q11. The figure shows age-wise bar graph of male and female population of two countries. Which one of the following is likely to be true?

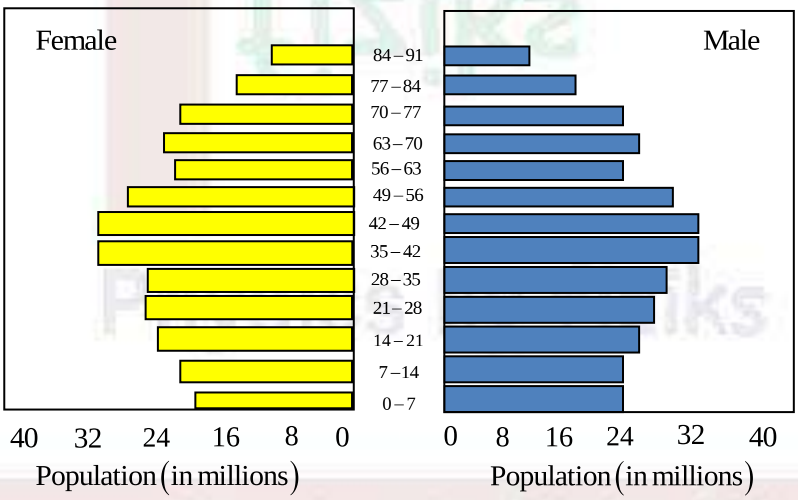
Country P

Age group



Country Q

Age group



1. Country Q has higher life expectancy

2. Country P has higher per-capita income

3. The population of country P is decreasing more rapidly than Q

4. Country P has better health facilities

Ans: (1)

Solution:

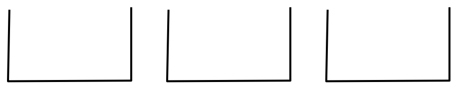
Obvious from bar-graph that population in higher age brackets in Q is more than P.

Q12. What is the minimum number of pourings needed to get 4 litre of milk from a fully filled 8 litre can, using ungraduated empty 5 and 3 litre cans? No milk should be wasted.

(1) 4 (2) 5 (3) 6 (4) 8

Ans: (3)

Solution:



	8	5	3
	8	0	0
1st	3	5	0
2nd	3	2	3
3rd	6	2	0
4th	6	0	2
5th	1	5	2
6th	1	4	3

Q13. In how many ways can a menu be made from 5 dishes, if the menu contains either 3 or 4 dishes?

(1) 2 (2) 3 (3) 7 (4) 15

Ans: (4)

Solution:

Total no. of dishes = 5, Menu from 3 or 4 dishes

No. of menu choosing 3 dishes $= {}^5C_3 = 10$

No. of menu choosing 4 dishes $= {}^5C_4 = 5$

$\therefore$ Total no. of menu = 10+5 = 15

Q14. SCRIPT : DIRECTOR :: ?? : CHEF

Choose the most appropriate option from the following to fill the blank

(1) MENU (2) RECIPE

(3) RESTAURANT (4) MEAL

Ans: (2)

Solution:

A "Director" works with "Script"

So, A "Chef" will work with "Recipe"

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Q15. The sum of the two positive integers is 14. Then their product CANNOT be divisible by

(1) 12 (2) 13 (3) 14 (4) 49

Ans: (3)

Solution:

Sum of two positive integer = 14

a	b	$a*b$
1	13	13
2	12	24
3	11	33
4	10	40
5	9	45
6	8	48
7	7	49

Hence, their product cannot be divisible by 14.

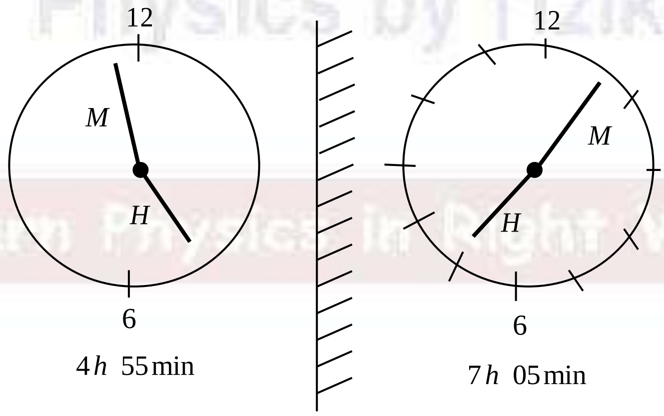
Q16. The time seen in a mirror placed opposite a numberless analog (with hands) wall clock is 4h 55min. What approximately is the correct time?

(1) 4 h 55 min (2) 5 h 05 min

(3) 7 h 05 min (4) 1 h 35 min

Ans: (3)

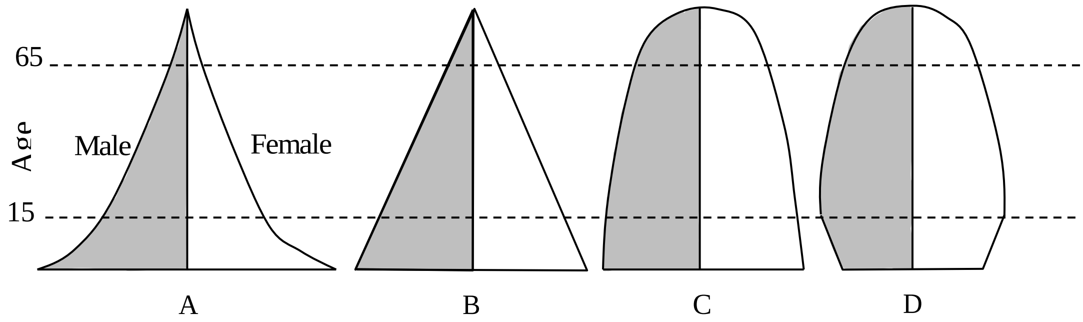
Solution:



From the figure it is obvious that 4h 55 min will have mirror image time 7h 05 min.

$$\begin{array}{r} H \quad \text{min} \\ 12:00 \\ -4:55 \\ \hline 7:05 \end{array}$$

Q17.



The above figures show population pyramids to four countries A, B, C and D. The country showing the most stable population is

(1) C (2) A (3) B (4) D

Ans: (1)

Solution:

The most stable population among all population pyramids is of C.

Q18. For every 5 chocolates that Ramesh gets, Suresh gets 3 chocolates. Geeta gets 3 chocolates for every 2 chocolates that Suresh gets. If Geeta has 18 chocolates, then the sum of chocolates with Ramesh and Suresh is

(1) 16 (2) 30 (3) 32 (4) 38

Ans: (3)

Solution:

5 chocolates $\leftarrow$ Ramesh (R), 3 chocolates $\leftarrow$ Suresh (S)

2 chocolates $\leftarrow$ Suresh (S), 3 chocolates $\leftarrow$ Geeta (G)

S:G = $[2:3]\times 6 = 12:18$

R:S = $[5:3]\times 4 = 20:12$

Hence, sum of chocolates with R and S $= 20+12 = 32$

Q19. A truck from a post office is sent to collect post from a plane as per schedule. The plane lands ahead of schedule, therefore its contents are transported by a rickshaw. The rickshaw meets the truck 30 minutes after the arrival of plane, and the post is transferred. The truck returns to the post office 20 minutes early. How early did the plane arrive? (Assume all transactions are instantaneous).

(1) 10 minutes (2) 20 minutes

(3) 30 minutes (4) 40 minutes

Ans: (4)

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Solution:

Time taken by truck to meet $=\frac{20}{2}=10\,\text{min}$ and this time is 10 min early.

Also, rickshaw meets the truck after 30 min of plane arrival.

So, plane arrives $(30+10)=40\,\text{min}$ early.

Q20. A bird keeps flying continuously between two trains, that are following each other on a straight track. The train behind is slower than the one ahead by 1.5 km/h. If the speed of the bird is 20 km/h, what distance would the bird cover in an hour?

(1) 20 km (2) 30 km (3) 50 km (4) 60 km

Ans: (3)

Solution:

Speed of bird = 20 km/h.

So, irrespective of the distances between bird and speeds of trains, will cover in 1 hour

$=20\times1=20\,km$