



Physics by fiziks

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CSIR NET-JRF Physical Sciences Dec.-2025

Complete Solution

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CSIR NET-JRF Physical Sciences Paper Dec.-2025
Solution-Mathematical Physics

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PART B

Q27. The residue of $f(z) = \frac{\cos \pi z}{(1-z^2)^3}$ at $z=1$ is

- (a) $\frac{\pi^2}{16}$ (b) $\frac{3}{16}$ (c) $\frac{3+\pi^2}{16}$ (d) $\frac{3-\pi^2}{16}$

Ans.:(d)

Solution.: $\because f(z) = \frac{\cos \pi z}{(1-z^2)^3} = \frac{\cos \pi z}{(1-z)^3(1+z)^3}$, so $z = \pm 1$ is a pole of order 3.

The residue of $f(z)$ at $z=1$ is $= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} [(z-1)^3 f(z)]$

$$(z-1)^3 f(z) = (z-1)^3 \frac{\cos \pi z}{(1-z)^3(1+z)^3} = -\frac{\cos \pi z}{(1+z)^3}$$

$$\Rightarrow \frac{d}{dz} \left[-\frac{\cos \pi z}{(1+z)^3} \right] = - \left[\frac{-(1+z)^3 \pi \sin \pi z - 3 \cos \pi z (1+z)^2}{(1+z)^6} \right]$$

$$\Rightarrow \frac{d}{dz} \left[-\frac{\cos \pi z}{(1+z)^3} \right] = \frac{\pi \sin \pi z}{(1+z)^3} + \frac{3 \cos \pi z}{(1+z)^4} \Rightarrow \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left[-\frac{\cos \pi z}{(1+z)^3} \right] = \frac{3-\pi^2}{8}$$

$$\text{The residue of } f(z) \text{ at } z=1 \text{ is } = \frac{1}{2!} \frac{3-\pi^2}{8} = \frac{3-\pi^2}{16}$$

Q30. Let $p(x)$ be the probability density function for a positive real variable x , and $g(\alpha) = \int_0^\infty p(x) e^{-\alpha x} dx$. If $g'(\alpha)$ and $g''(\alpha)$ are respectively first and second derivatives of $g(\alpha)$ with respect to α , which of the following gives the variance of x ?

- (a) $g''(0) - [g'(0)]^2$ (b) $g''(0) + [g'(0)]^2$
(c) $[g''(0) - g'(0)]^2$ (d) $\frac{g''(0)}{g'(0)g(0)}$

Ans.:(a)

Solution.: $g(\alpha) = \int_0^\infty p(x) e^{-\alpha x} dx$ and $p(x)$ be the probability density function so

$$g(0) = \int_0^\infty p(x) dx = 1.$$

Differentiate w.r.t α :

$$g'(\alpha) = \int_0^\infty p(x)(-x) e^{-\alpha x} dx \Rightarrow g'(0) = \int_0^\infty p(x)(-x) dx = -\langle x \rangle$$

Again, differentiate w.r.t α :

$$g''(\alpha) = \int_0^\infty x^2 p(x) e^{-\alpha x} dx \Rightarrow g''(0) = \int_0^\infty x^2 p(x) dx = \langle x^2 \rangle$$

$$\text{Now } \text{Var}(x) = \langle x^2 \rangle - \langle x \rangle^2 = g''(0) - [g'(0)]^2 = g''(0) - [g'(0)]^2$$

Q37. If C be the unit circle traversed clockwise, then the integral $\oint_C dz |1+2z|^2$ equals

- (a) $-4\pi i$ (b) $-\pi i$ (c) 0 (d) $-2\pi i$

Ans.:(a)

Solution.: If C be the unit circle the $|z|=1 \Rightarrow z\bar{z}=1 \Rightarrow \bar{z}=\frac{1}{z}$.

$$|1+2z|^2 = (1+2z)(1+2\bar{z}) = (1+2z)\left(1+\frac{2}{z}\right) = 5 + \frac{2}{z} + 2z$$

$$\oint_C dz |1+2z|^2 = \oint_C \left(5 + \frac{2}{z} + 2z\right) dz = 0 + 2(-2\pi i) + 0 = -4\pi i$$

PART C

Q56. A sequence of polynomial $Q_n(x)$ [$n=0,1,2,\dots$] satisfies the recursion relation

$$Q_{n+1}(x) - 2xQ_n(x) + 2nQ_{n-1}(x) = 0 \text{ for all } n \geq 0 \text{ [here } Q_{-1}(x) = 0]$$

The generating function for the polynomials, $g(x,t) = \sum_{n=0}^\infty \frac{t^n}{n!} Q_n(x)$ satisfies

- (a) $\frac{\partial g}{\partial t} = 2(t+x)g$ (b) $\frac{\partial g}{\partial t} = 2(x-t)g$
(c) $\frac{\partial g}{\partial t} = \frac{2(x-t)}{t}g$ (d) $\frac{\partial g}{\partial t} = 2+(x+t)g$

Ans.:(b)

Solution.: Differentiate the generating function $g(x,t) = \sum_{n=0}^\infty \frac{t^n}{n!} Q_n(x)$ then

$$\frac{\partial g}{\partial t} = \sum_{n=1}^\infty \frac{nt^{n-1}}{n!} Q_n(x) = \sum_{n=1}^\infty \frac{t^{n-1}}{(n-1)!} Q_n(x)$$

$$\text{Put } m = n-1: \frac{\partial g}{\partial t} = \sum_{m=0}^\infty \frac{t^m}{m!} Q_{m+1}(x) \text{ or } \frac{\partial g}{\partial t} = \sum_{n=0}^\infty \frac{t^n}{n!} Q_{n+1}(x)$$

$$\text{Now, substitute: } Q_{n+1} = 2xQ_n - 2nQ_{n-1}, \text{ giving } \frac{\partial g}{\partial t} = 2x \sum_{n=0}^\infty \frac{t^n}{n!} Q_n - 2 \sum_{n=0}^\infty \frac{nt^n}{n!} Q_{n-1}$$

$$2x \sum_{n=0}^\infty \frac{t^n}{n!} Q_n = 2xg$$

$$\text{and } \sum_{n=0}^{\infty} \frac{nt^n}{n!} Q_{n-1} = \sum_{n=0}^{\infty} \frac{nt^n}{n!} Q_{n-1} = \sum_{n=1}^{\infty} \frac{t^n}{(n-1)!} Q_{n-1}$$

$$\text{Put } m = n-1: \sum_{n=1}^{\infty} \frac{t^n}{(n-1)!} Q_{n-1} = t \sum_{m=0}^{\infty} \frac{t^m}{m!} Q_m = tg$$

$$\text{Thus } \frac{\partial g}{\partial t} = 2x \sum_{n=0}^{\infty} \frac{t^n}{n!} Q_n - 2 \sum_{n=0}^{\infty} \frac{nt^n}{n!} Q_{n-1} = 2xg - 2tg = 2(x-t)g$$

Q61. In a heap of 20 biased coins, 17 have a 60% probability of showing heads while the other three special coins have a 90% probability of doing so. A coin is selected at random and tossed. If the result is a head, the probability that it was one of the three special coins is best approximated by

- (a) 0.18 (b) 0.14
(c) 0.21 (d) 0.26

Ans.:(c)

$$\text{Solution.} \cdot P(H|17) = 0.6, P(17) = \frac{17}{20} \text{ and } P(H|3) = 0.9, P(3) = \frac{3}{20}$$

$$P(H) = P(H|17)P(17) + P(H|3)P(3) = 0.6 \times \frac{17}{20} + 0.9 \times \frac{3}{20} = \frac{12.9}{20}$$

$$P(3|H) = \frac{P(H|3)P(3)}{P(H)} = \frac{0.9 \times 3/20}{12.9/20} = \frac{2.7}{12.9} \approx 0.21$$



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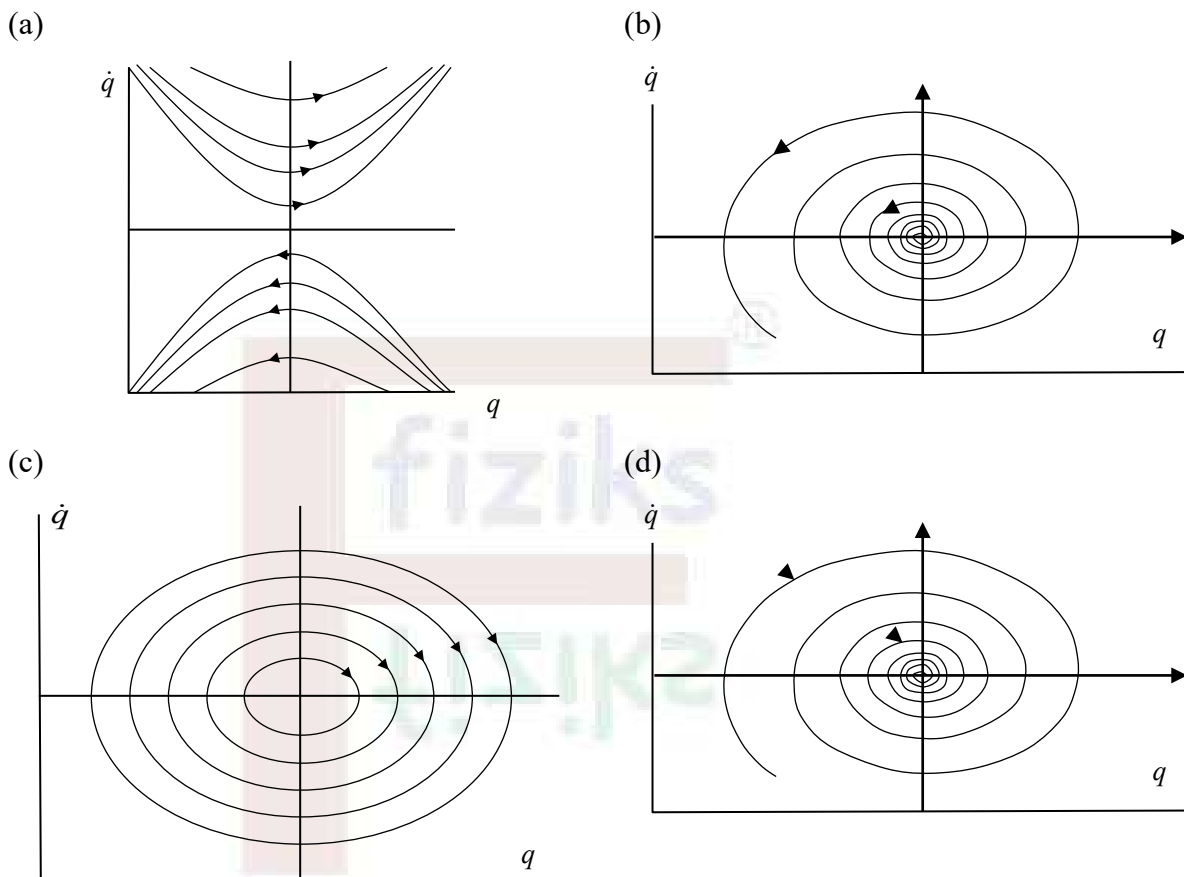
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Solution-Classical Mechanics

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PART B

Q24. Which of the following figures best represents the motion of an oscillator described by the differential equation $\ddot{q} + \dot{q} + q = 0$ in $q - \dot{q}$ plane?



Ans.:(d)

Solution.: Given $\ddot{q} + \dot{q} + q = 0$.

Here, the three terms represent:

$\ddot{q}$: acceleration, $\dot{q}$: damping, q : restoring force

The important point is that the equation contains damping. Therefore, the oscillations gradually decrease in amplitude.

Initially, the oscillator moves with some amplitude. Due to damping, it loses energy. Its oscillations become smaller and smaller. Finally, it comes to rest at $q = 0$, $\dot{q} = 0$.

The oscillator keeps changing its position and velocity, so the graph loops around. Damping reduces the amplitude, so each loop becomes smaller. Eventually, the trajectory reaches the origin in the long-time limit.

Determine the direction: At the point $q > 0$, $\dot{q} = 0$, the equation gives $\ddot{q} = -q < 0$.

So, the velocity starts becoming negative. The trajectory moves downward from the positive q -axis, meaning the spiral moves clockwise.

Q33. The position and velocity vector of a particle changes from $\vec{R}_1$ to $\vec{R}_2$ and $\vec{V}_1$ to $\vec{V}_2$ as time changes from t_1 to t_2 . If $\vec{r}(t)$, $\vec{a}(t)$ are instantaneous position and acceleration vectors of the particle then the integral $I = \int_{t_1}^{t_2} dt (\vec{r}(t) \times \vec{a}(t))$ is

- (a) $\vec{R}_2 \times \vec{V}_1 - \vec{R}_1 \times \vec{V}_2$ (b) $\vec{R}_2 \times \vec{V}_2 - \vec{R}_1 \times \vec{V}_1$
(c) $\vec{R}_1 \times \vec{V}_1 - \vec{R}_2 \times \vec{V}_2$ (d) $\vec{R}_1 \times \vec{V}_2 - \vec{R}_2 \times \vec{V}_1$

Ans.:(b)

Solution.: $I = \int_{t_1}^{t_2} dt (\vec{r}(t) \times \vec{a}(t)) = \int_{t_1}^{t_2} \left(\vec{r} \times \frac{d\vec{v}}{dt} \right) dt$ where $\vec{a} = \frac{d\vec{v}}{dt}$

$$\frac{d}{dt} (\vec{r} \times \vec{v}) = \dot{\vec{r}} \times \vec{v} + \vec{r} \times \dot{\vec{v}} = \vec{v} \times \vec{v} + \vec{r} \times \vec{a} = \vec{r} \times \vec{a} \quad \because \dot{\vec{r}} = \vec{v} \text{ and } \dot{\vec{v}} = \vec{a}$$

$$\Rightarrow I = \int_{t_1}^{t_2} (\vec{r} \times \vec{a}) dt = \int_{t_1}^{t_2} \frac{d}{dt} (\vec{r} \times \vec{v}) dt = [\vec{r} \times \vec{v}]_{t_1}^{t_2} = \vec{R}_2 \times \vec{V}_2 - \vec{R}_1 \times \vec{V}_1$$

Q34. A fly of mass m rests on the edge of a uniform horizontal disc of radius R and mass M . The disc is free to rotate about the vertical axis through its centre. Initially the disc is stationary. The fly starts to walk around the circumference of the disc with speed v relative to the disc. The speed of the fly for a stationary observer is

- (a) $\frac{mv}{M+2m}$ (b) $\frac{Mv}{M-2m}$ (c) $\frac{Mv}{M+2m}$ (d) $\frac{mv}{M-2m}$

Ans.:(c)

Solution.: By conservation of angular momentum about the vertical axis (initial angular momentum is zero):

$$I_{disc} \omega + I_{fly} \omega_{fly} = 0$$

$$\Rightarrow \left(\frac{1}{2} MR^2 \right) \omega + (mR^2) \left(\omega + \frac{v}{R} \right) = 0$$

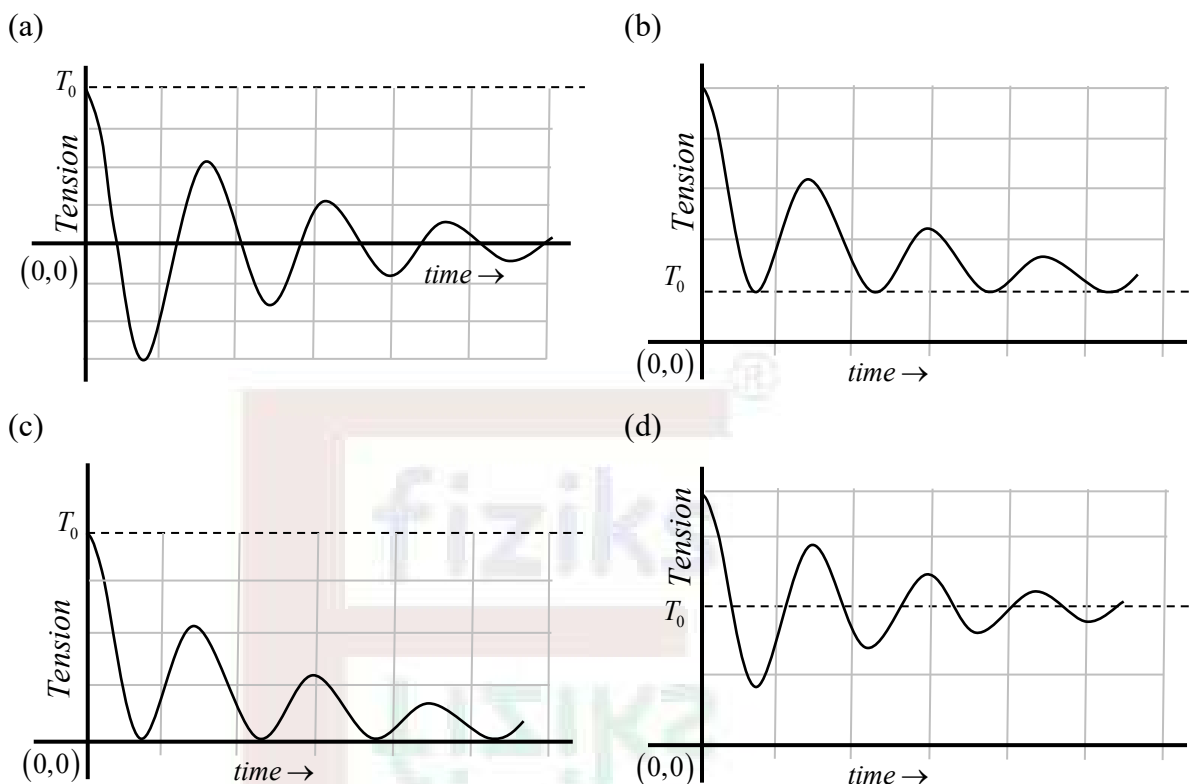
$$\Rightarrow \left(\frac{1}{2} M + m \right) \omega R^2 = -mvR \Rightarrow \omega = -\frac{2mv}{(M+2m)R}$$

The absolute speed of the fly for a stationary observer is

$$v_{abs} = \omega_{fly} R = \left(\omega + \frac{v}{R} \right) R = \left(-\frac{2mv}{(M+2m)R} + \frac{v}{R} \right) R = \left(-\frac{2m}{(M+2m)} + 1 \right) v$$

$$\Rightarrow v_{abs} = \frac{Mv}{M+2m}$$

Q39. A bow has a taut string of tension T_0 (when it is at rest). The string is pulled and released at time $t = 0$. Which plot best represents the tension in the bow string as a function of time?



Ans.:(b)

Solution.:

- When a bow string is pulled, its tension increases above the resting tension T_0 .
- Upon release at $t = 0$, the string oscillates while remaining under tension. The minimum tension during its vibration corresponds to the equilibrium rest tension T_0 , and the tension oscillates around a value above T_0 , gradually damping back down to T_0 .

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PART C

Q49. Find the curve that extremizes the functional

$$I(y) = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 12xy \right] dx$$

for the given boundary conditions $y(0) = 0$ and $y(1) = 1$

- (a) $y = x^3$ (b) $y = x^2$
(c) $y = 2x^2 - x$ (d) $y = 3x^3 - 2x^2$

Ans.:(a)

Solution.: Using the Euler-Lagrange equation $\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$ where $F = (y')^2 + 12xy$

$$\Rightarrow 12x - \frac{d}{dx} (2y') = 0 \Rightarrow y'' = 6x \Rightarrow y' = 3x^2 + c_1 \Rightarrow y = x^3 + c_1x + c_2$$

$$\because y(0) = 0 \Rightarrow c_2 = 0 \Rightarrow y = x^3 + c_1x$$

$$\because y(1) = 1 \Rightarrow 1 = 1 + c_1 \Rightarrow c_1 = 0 \Rightarrow y = x^3$$

Q50. The Hamiltonian of a simple pendulum consisting of mass m attached to a massless string

of length l is $H = \frac{P_\theta^2}{2ml^2} + mgl(1 - \cos \theta)$. If L denotes the Lagrangian, then $\frac{dL}{dt}$ is

- (a) $\frac{g}{l} P_\theta \cos \theta$ (b) $-\frac{g}{l} P_\theta \sin \theta$
(c) $-\frac{2g}{l} P_\theta \sin \theta$ (d) $\frac{g}{l} P_\theta \cos(2\theta)$

Ans.:(c)

Solution.: Hamilton's equation gives $\dot{\theta} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{ml^2}$ $\because H = \frac{P_\theta^2}{2ml^2} + mgl(1 - \cos \theta)$

Using the Legendre transformation, $L = P_\theta \dot{\theta} - H = \frac{P_\theta^2}{ml^2} - \frac{P_\theta^2}{2ml^2} - mgl(1 - \cos \theta)$

$$\Rightarrow L = \frac{P_\theta^2}{2ml^2} - mgl(1 - \cos \theta) \Rightarrow \frac{dL}{dt} = \frac{P_\theta}{ml^2} \dot{P}_\theta - mgl \sin \theta \dot{\theta}$$

The second Hamilton equation gives $\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = -mgl \sin \theta$

$$\Rightarrow \frac{dL}{dt} = \frac{P_\theta}{ml^2} (-mgl \sin \theta) - mgl \sin \theta \left(\frac{P_\theta}{ml^2} \right) = -\frac{g}{l} P_\theta \sin \theta - \frac{g}{l} P_\theta \sin \theta = -\frac{2g}{l} P_\theta \sin \theta$$

Q51. The Lagrangian of a two-particle system is given by

$$L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2) - \frac{1}{2}m\omega^2\left(q_1^2 + q_2^2 + \frac{1}{2}q_1q_2\right)$$

The normal mode frequencies (in units of ω) are

- (a) $\sqrt{\frac{5}{3}}, \frac{1}{2}$ (b) $\sqrt{\frac{5}{6}}, \sqrt{\frac{3}{2}}$ (c) $\sqrt{\frac{6}{5}}, \sqrt{2}$ (d) $\sqrt{\frac{5}{6}}, \sqrt{2}$

Ans.:(b)

Solution.:

$$\therefore L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2) - \frac{1}{2}m\omega^2\left(q_1^2 + q_2^2 + \frac{1}{2}q_1q_2\right)$$

$$\text{Kinetic matrix: } T = m \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix},$$

$$\text{Potential matrix: } V = m\omega^2 \begin{bmatrix} 1 & 1/4 \\ 1/4 & 1 \end{bmatrix}$$

Normal mode frequencies satisfy: $\det(V - \omega_n^2 T) = 0$, let $\lambda = \frac{\omega_n^2}{\omega^2}$

$$\Rightarrow \begin{vmatrix} 1-\lambda & \frac{1}{4}-\frac{\lambda}{2} \\ \frac{1}{4}-\frac{\lambda}{2} & 1-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)^2 - \left(\frac{1}{4}-\frac{\lambda}{2}\right)^2 = 0 \Rightarrow 1 + \lambda^2 - 2\lambda - \left(\frac{1}{16} + \frac{\lambda^2}{4} - \frac{\lambda}{4}\right) = 0$$

$$\Rightarrow 16 + 16\lambda^2 - 32\lambda - 1 - 4\lambda^2 + 4\lambda = 0 \Rightarrow 12\lambda^2 - 28\lambda + 15 = 0$$

$$\Rightarrow \lambda = \frac{28 \pm \sqrt{(28)^2 - 4 \times 12 \times 15}}{2 \times 12} = \frac{28 \pm \sqrt{64}}{2 \times 12} = \frac{28 \pm 8}{2 \times 12} = \frac{36}{24}, \frac{20}{24} = \frac{3}{2}, \frac{5}{6}$$

$$\therefore \lambda = \frac{\omega_n^2}{\omega^2} \Rightarrow \omega_n = \sqrt{\lambda}\omega \Rightarrow \omega_1 = \sqrt{\frac{3}{2}}\omega \text{ and } \omega_2 = \sqrt{\frac{5}{6}}\omega$$

Q63. In a high energy scattering experiment involving two identical particles, each of rest mass m_0 , one particle is initially at rest, while the other one is incident upon it with energy E and momentum p . The total energy of the two-particle system in the centre-of-mass frame, in the limit $E \gg m_0c^2$, is approximately given by

- (a) E (b) $2E$
(c) $\sqrt{\frac{Em_0c^2}{2}}$ (d) $\sqrt{2Em_0c^2}$

Ans.:(d)

Solution.: The invariant center-of-mass energy E_{CM} is given by:

$$E_{CM}^2 = (E_1 + E_2)^2 - (p_1 + p_2)^2 c^2$$

For a fixed target: $E_2 = m_0 c^2$, $p_2 = 0$

$$\Rightarrow E_{CM}^2 = (E + m_0 c^2)^2 - p^2 c^2 = E^2 + (m_0 c^2)^2 + 2E(m_0 c^2) - [E^2 - (m_0 c^2)^2]$$

$$\Rightarrow E_{CM}^2 = 2m_0 c^2 (E + m_0 c^2) \approx 2Em_0 c^2 \quad \because E \gg m_0 c^2$$

$$\Rightarrow E_{CM} = \sqrt{2Em_0 c^2}$$

Q65. The Lagrangian $L = L(x, y, \dot{x}, \dot{y})$ is invariant under the transformation $x \rightarrow x + \epsilon y$ and $y \rightarrow y + \epsilon x$, for any infinitesimal real parameter ϵ . If P_x, P_y denote canonically conjugate momenta corresponding to x, y respectively, then the corresponding conserved quantity is

(a) $yP_x - xP_y$

(b) $yP_x + xP_y$

(c) $xP_x + yP_y$

(d) $xP_x - yP_y$

Ans.:(b)

Solution.:

$x \rightarrow x + \epsilon y$ and $y \rightarrow y + \epsilon x$.

Identify the infinitesimal changes: compare with $x \rightarrow x + \epsilon \delta x$ and $y \rightarrow y + \epsilon \delta y$.

The infinitesimal variations of coordinates are: $\delta x = y$, $\delta y = x$

Apply Noether's theorem:

For a continuous symmetry under which the Lagrangian is invariant, the conserved quantity is

$$Q = P_x \delta x + P_y \delta y = P_x y + P_y x$$



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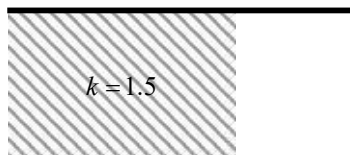
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Solution-Electromagnetic Theory

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PART B

Q21. A fraction $\frac{2}{3}$ of the volume of a parallel plate capacitor is filled with dielectric of relative permittivity $k = 1.5$ (as shown in the figure).



When the filled volume is reduced to $\frac{1}{3}$ of the total volume, the capacitance is smaller by a factor of

- (a) $\frac{7}{8}$ (b) $\frac{5}{6}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Ans.:(a)

Solution.: Case-I: $A_1 d = \frac{2}{3} A d \Rightarrow A_1 = \frac{2}{3} A$ and $A_2 = \frac{1}{3} A$

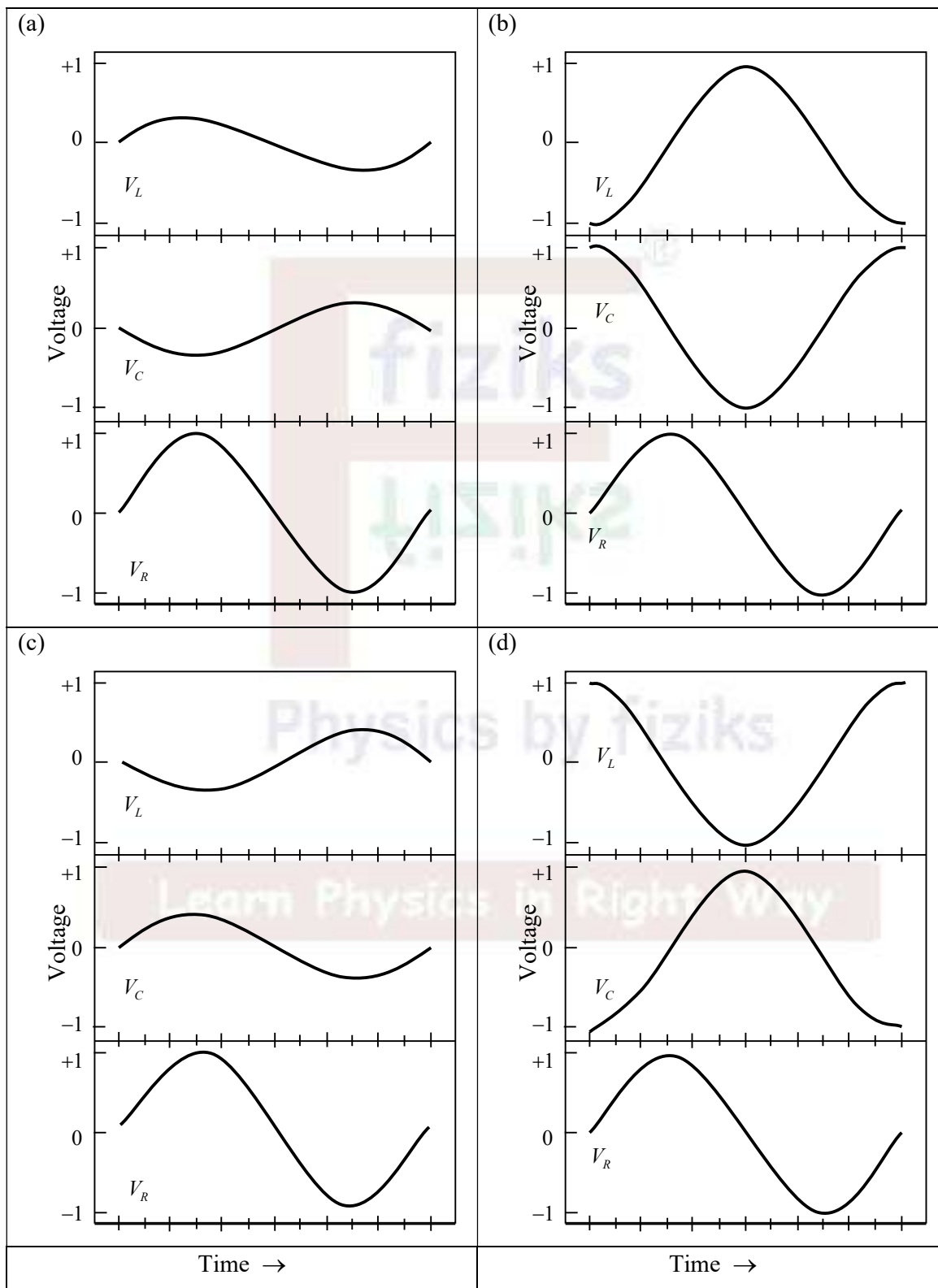
$$C_I = C_1 + C_2 = \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d} = \frac{1.5 \epsilon_0 2A}{3d} + \frac{\epsilon_0 A}{3d} = 4 \frac{\epsilon_0 A}{3d}$$

Case-II: $A'_1 d = \frac{2}{3} A d \Rightarrow A'_1 = \frac{2}{3} A$ and $A'_2 = \frac{1}{3} A$

$$C_{II} = C'_1 + C'_2 = \frac{\epsilon_1 A'_1}{d} + \frac{\epsilon_2 A'_2}{d} = \frac{1.5 \epsilon_0 A}{3d} + \frac{\epsilon_0 2A}{3d} = 3.5 \frac{\epsilon_0 A}{3d} \Rightarrow \frac{C_{II}}{C_I} = \frac{3.5}{4} = \frac{7}{8}$$

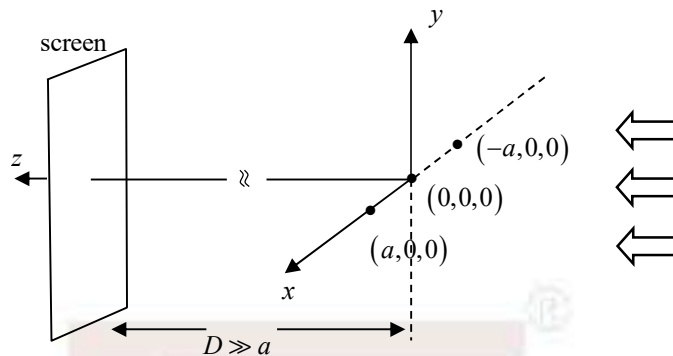
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Q25. In an ideal series LCR circuit, which one of the following best represents the steady-state voltage waveforms V_L, V_C, V_R (only one cycle is shown) across L, C and R as a function of time at resonance frequency

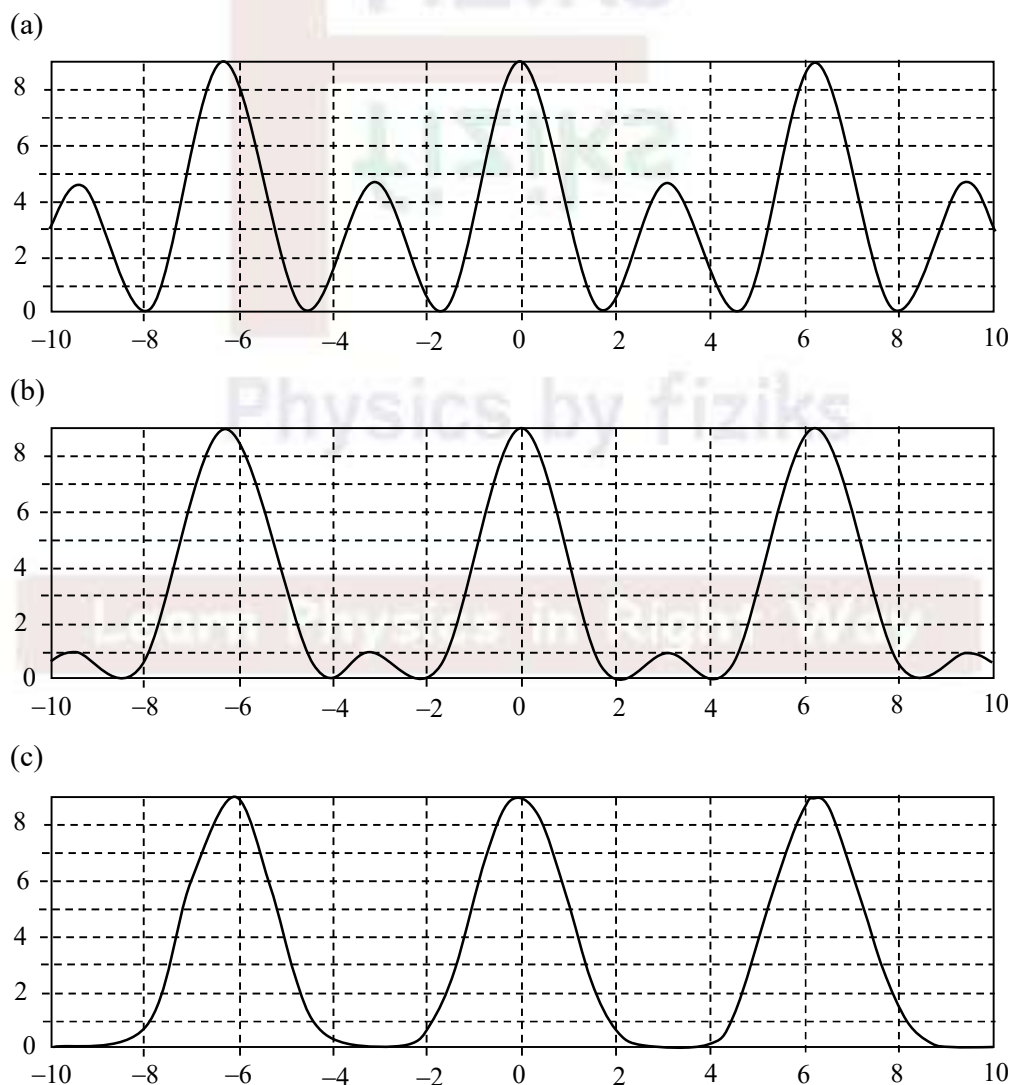


Ans.:(d)

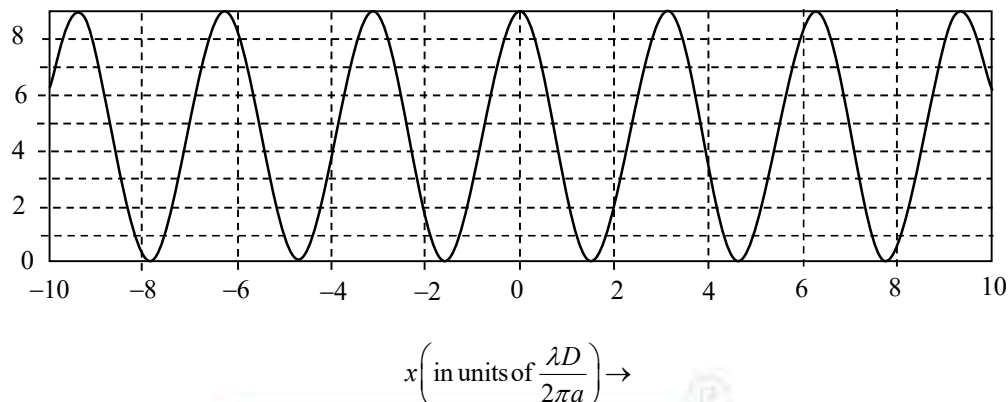
Q32. Three identical pinholes separated by distance a along the x -axis are illuminated by a collimated monochromatic coherent beam of light (wavelength λ) as shown in the figure below.



The intensity (in arbitrary units) pattern of fringes obtained on a screen kept at distance D ($D \gg a$) along the z -axis is best represented by

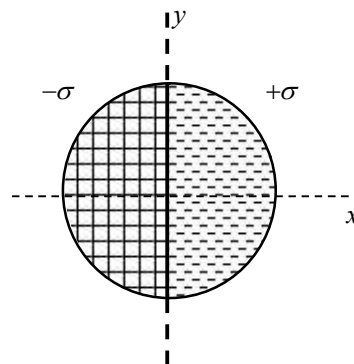


(d)

**Ans.:(b)**

Q38. A circular disc of radius R is made of 2 halves (as shown in the figure), separated by a dielectric of negligible thickness (along the y axis.) If the surface charge density on the right half is $+\sigma$ and that on the left half is $-\sigma$, the dipole moment of the disc is

- (a) $P_x = 0, P_y = \frac{1}{3}\sigma R^3$ (b) $P_x = 0, P_y = \frac{4}{3}\sigma R^3$
 (c) $P_x = \frac{1}{3}\sigma R^3, P_y = 0$ (d) $P_x = \frac{4}{3}\sigma R^3, P_y = 0$

**Ans.:(d)**

Solution.: $P_x = \int_S x \sigma dx dy$, $P_y = \int_S y \sigma dx dy$; $+\sigma$ for $x > 0$ and $-\sigma$ for $x < 0$.

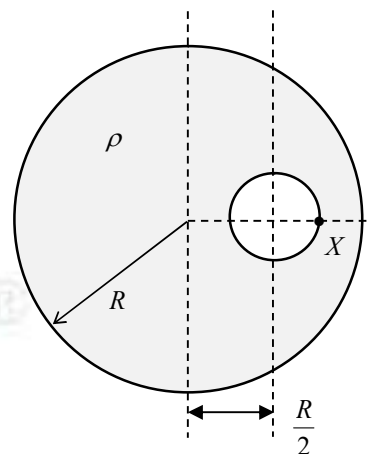
$P_y = \int_S y \sigma dx dy = 0$ since it's an odd integral.

$$P_x = \int_S x \sigma dx dy = \int_{-\pi/2}^{\pi/2} \int_0^R (+\sigma)(r \cos \theta) r dr d\theta + \int_{\pi/2}^{3\pi/2} \int_0^R (-\sigma)(r \cos \theta) r dr d\theta$$

$$\Rightarrow P_x = \sigma \times 2 \times \frac{R^3}{3} + (-\sigma) \times (-2) \times \frac{R^3}{3} = \frac{4\sigma R^3}{3}$$

Q40. A solid sphere of radius R has uniform charge density ρ . A spherical volume of radius $\frac{R}{4}$ is scooped out from the sphere as shown. The electric field at the point marked X is ($\hat{r}$ denotes the unit vector along the radially outward direction)

- (a) $\frac{2\rho R}{9\varepsilon_0} \hat{r}$
 (b) $\frac{\rho R}{6\varepsilon_0} \hat{r}$
 (c) $\frac{\rho R}{3\varepsilon_0} \hat{r}$
 (d) $\frac{\rho R}{9\varepsilon_0} \hat{r}$



Ans.:(b)

Solution.: Let us consider a spherical volume of radius $\frac{R}{4}$ and charge density $-\rho$. Then field at point X due to both sphere charge density $+\rho$ and $-\rho$ is

$$\vec{E}_{+\rho} = \frac{\rho 3R/4}{3\varepsilon_0} \hat{r} \quad \text{and} \quad \vec{E}_{-\rho} = -\frac{\rho R/4}{3\varepsilon_0} \hat{r}.$$

Thus, the resultant field at point X is $\vec{E} = \vec{E}_{+\rho} + \vec{E}_{-\rho} = \frac{\rho 3R/4}{3\varepsilon_0} \hat{r} - \frac{\rho R/4}{3\varepsilon_0} \hat{r} = \frac{\rho R}{6\varepsilon_0} \hat{r}$.

Q42. Two well separated conducting spheres (A and B) of radii 10 cm and 20 cm carry charges $+30\text{ C}$ and -20 C respectively. When they are connected by a thin conducting wire, the final charge on A is Q_A and that on B is Q_B . The values of Q_A and Q_B respectively, are closest to

- (a) 6.7 C and 3.3 C (b) 2.0 C and 8.0 C
 (c) 3.3 C and 6.7 C (d) 8.0 C and 2.0 C

Ans.:(c)

Solution.: When they are connected by a thin conducting wire

$$Q_A + Q_B = 30\text{ C} - 20\text{ C} = 10\text{ C} \quad \text{and} \quad V_A = V_B \Rightarrow \frac{Q_A}{4\pi\varepsilon_0(10)} = \frac{Q_B}{4\pi\varepsilon_0(20)} \Rightarrow Q_B = 2Q_A$$

Hence $3Q_A = 10\text{ C} \Rightarrow Q_A = 3.3\text{ C}$ and $Q_B = 6.7\text{ C}$

PART C

Q58. A monochromatic plane wave is incident normally from a dielectric medium A onto another dielectric medium B . The indices of refraction satisfy $n_A < n_B$. One-fourth of the incident energy is reflected back into medium A . Let $\vec{E}$ be the resultant electric field due to the superposition of the incident wave and the reflected wave. Then, the ratio of the two time-averages $\langle \vec{E}^2 \rangle_{\min} / \langle \vec{E}^2 \rangle_{\max}$ is

- (a) $\frac{1}{8}$ (b) $\frac{1}{9}$
(c) $\frac{4}{9}$ (d) $\frac{1}{4}$

Ans.:(b)

Solution.: $\frac{U_R}{U_I} = \frac{E_{0R}^2}{E_{0I}^2} = \frac{1}{4} \Rightarrow \frac{E_{0R}}{E_{0I}} = \frac{1}{2} \Rightarrow E_{0R} = \frac{1}{2} E_{0I}$

$$E_{\max} = E_{0R} + E_{0I} = \frac{3}{2} E_{0I} \text{ and } E_{\min} = E_{0R} - E_{0I} = \frac{1}{2} E_{0I}$$

$$\frac{\langle \vec{E}^2 \rangle_{\min}}{\langle \vec{E}^2 \rangle_{\max}} = \frac{(E_{\min})^2}{(E_{\max})^2} = \frac{1/4}{9/4} = \frac{1}{9}$$

Q64. Electromagnetic waves of frequency ω are incident on an electron gas, whose relaxation time is τ . Let σ_{low} and σ_{high} represent the respective electrical conductivities of the gas in low frequency ($\omega\tau \ll 1$) and high frequency ($\omega\tau \gg 1$) limits. The ratio ($\sigma_{\text{low}}/\sigma_{\text{high}}$) is

- (a) inversely proportional to ω^2 (b) directly proportional to ω^2
(c) independent of ω (d) directly proportional to ω

Ans.:(d)

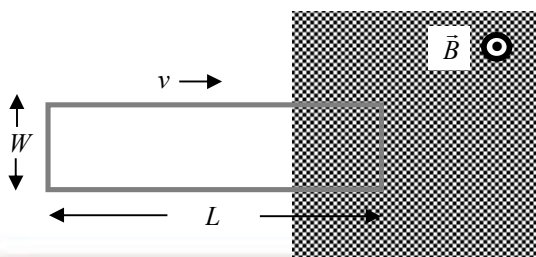
Solution.: For electron gas (Drude model), $\sigma(\omega) = \frac{ne^2\tau}{m(1+i\omega\tau)} \Rightarrow |\sigma| = \frac{ne^2\tau}{m\sqrt{1+\omega^2\tau^2}}$

low frequency ($\omega\tau \ll 1$): $\sigma_{\text{low}} = \frac{ne^2\tau}{m}$

and high frequency ($\omega\tau \gg 1$): $\sigma_{\text{high}} = \frac{ne^2}{m\omega}$

Thus $\frac{\sigma_{\text{low}}}{\sigma_{\text{high}}} = \frac{ne^2\tau}{m} \frac{m\omega}{ne^2} = \omega\tau \propto \omega$

Q75. A long rectangular metallic loop of width W and length $L (\gg W)$ starts entering a region, where there is a uniform magnetic field B perpendicular to the plane of the loop. The resistance of the loop is R and its mass is M . If v_0 is the velocity of the loop just before entering the region, then neglecting the self-inductance effect, the velocity at a later time t is



(a) $v(t) = \frac{v_0}{1 + \frac{B^2 W^2}{MR} t}$

(b) $v(t) = \frac{v_0}{1 + \left(\frac{B^2 W^2}{MR} t \right)^2}$

(c) $v(t) = v_0 e^{-\frac{B^2 W^2}{MR} t}$

(d) $v(t) = \frac{v_0}{1 + \ln \left(1 + \frac{B^2 W^2}{MR} t \right)}$

Ans.:(c)

Solution.: As the loop enters the magnetic field, the area inside the field increases at the rate:

$$\frac{dA}{dt} = Wv$$

Hence induced emf: $\varepsilon = \frac{d\phi}{dt} = B \frac{dA}{dt} = BWv$

Current induced in the loop: $I = \frac{\varepsilon}{R} = \frac{BWv}{R}$

Only the vertical side inside the magnetic field feels horizontal force so force on it $F = BIW$.

$$\Rightarrow F = BIW = B \frac{BWv}{R} W = \frac{B^2 W^2 v}{R}. \text{ This force opposes the motion.}$$

Using Newton's law we can write equation of motion: $F = M \frac{dv}{dt} = -\frac{B^2 W^2}{R} v$

$$\Rightarrow \frac{dv}{v} = -\frac{B^2 W^2}{R} dt \Rightarrow \ln v = -\frac{B^2 W^2}{R} t + C.$$

Using $v = v_0$ at $t = 0$: $C = \ln v_0 \Rightarrow v(t) = v_0 e^{-\frac{B^2 W^2}{MR} t}$



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PART B

Q23. A spin- $\frac{1}{2}$ particle is in a magnetic field $\vec{B} = B_x \hat{x} + B_y \hat{y}$ for which the spin-dependent Hamiltonian is $\hat{H} = -A \hat{S} \cdot \vec{B}$ (A is a positive constant and $\hat{S}$ is the spin-operator). The eigenvalues of the Hamiltonian are

(a) $\pm A \frac{\hbar}{2} (B_x + B_y)$ (b) $\pm A \frac{\hbar}{2} \sqrt{B_x B_y}$

(c) $\pm A \frac{\hbar}{2} (B_x^2 + B_y^2)^{\frac{1}{2}}$ (d) 0

Ans.:(c)

Solution.:

The spin operator for a spin- $\frac{1}{2}$ particle is given by $\hat{S} = \frac{\hbar}{2} \vec{\sigma}$ where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices.

$$\hat{H} = -A \hat{S} \cdot \vec{B} = -\frac{\hbar}{2} A (\sigma_x B_x + \sigma_y B_y) \text{ where } \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\Rightarrow \hat{H} = -\frac{\hbar}{2} A \begin{bmatrix} 0 & B_x - iB_y \\ B_x + iB_y & 0 \end{bmatrix}. \text{ Let } M = \begin{bmatrix} 0 & B_x - iB_y \\ B_x + iB_y & 0 \end{bmatrix}$$

The eigenvalues are determined by solving the secular equation $\det(M - \lambda I) = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & B_x - iB_y \\ B_x + iB_y & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - (B_x^2 + B_y^2) = 0 \Rightarrow \lambda = \pm \sqrt{(B_x^2 + B_y^2)}$$

Including the prefactor, the energy eigenvalues are

$$\pm A \frac{\hbar}{2} \sqrt{(B_x^2 + B_y^2)}.$$

Q26. A quantum mechanical particle in a harmonic potential has the wave function $\frac{1}{\sqrt{2}} [\psi_0(x) + \psi_1(x)]$ at $t=0$, where $\psi_0(x)$ and $\psi_1(x)$ are the wave functions of the ground state and the first excited state respectively. If the frequency of the oscillator is ω , the probability density of finding the particle at x after time $t = \frac{\pi}{\omega}$ is

(a) $\frac{1}{2} |\psi_1(x) - \psi_0(x)|^2$ (b) $\frac{1}{2} |\psi_1(x) + \psi_0(x)|^2$

(c) $\frac{1}{2} |\psi_1(x) - i\psi_0(x)|^2$ (d) $\frac{1}{2} |\psi_1(x)|^2 + \frac{1}{2} |\psi_0(x)|^2$

Ans.:(a)

Solution.:

$$\psi(x, 0) = \frac{1}{\sqrt{2}} [\psi_0(x) + \psi_1(x)]$$

For a quantum harmonic oscillator, the energy eigenvalues are

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega \Rightarrow E_0 = \frac{1}{2} \hbar \omega, E_1 = \frac{3}{2} \hbar \omega$$

Each energy eigenstate acquires a phase factor: $\psi_n(x, t) = \psi_n(x) e^{-iE_n t / \hbar}$

$$\text{Hence, } \psi(x, t) = \frac{1}{\sqrt{2}} [\psi_0 e^{-i\omega t/2} + \psi_1 e^{-i3\omega t/2}]$$

$$\text{At } t = \frac{\pi}{\omega}; \quad \psi\left(x, \frac{\pi}{\omega}\right) = \frac{1}{\sqrt{2}} [\psi_0 e^{-i\pi/2} + \psi_1 e^{-i3\pi/2}] = \frac{1}{\sqrt{2}} [-i\psi_0 + i\psi_1] = \frac{i}{\sqrt{2}} [\psi_1 - \psi_0]$$

$$\text{Probability density } P(x, t) = |\psi(x, t)|^2 = \frac{1}{2} |\psi_1(x) - \psi_0(x)|^2$$

Q35. If $\hat{L}$ is the angular momentum operator for a quantum particle, then $\hat{L} \times \hat{L}$ is

- (a) $\hbar^2$ (b) $-i\hbar\hat{L}$ (c) 0 (d) $i\hbar\hat{L}$

Ans.:(d)

Solution.:

The commutation relations for angular momentum components are

$$[L_x, L_y] = i\hbar L_z, [L_y, L_z] = i\hbar L_x, [L_z, L_x] = i\hbar L_y$$

$$\text{So, } \left(\hat{L} \times \hat{L}\right)_x = L_y L_z - L_z L_y = [L_y, L_z] = i\hbar L_x$$

$$\text{Similarly, } \left(\hat{L} \times \hat{L}\right)_y = i\hbar L_y, \quad \left(\hat{L} \times \hat{L}\right)_z = i\hbar L_z$$

$$\text{Thus } \hat{L} \times \hat{L} = i\hbar \hat{L}$$

Q41. A quantum particle of mass m is moving in a potential

$$V(x, y) = \frac{m\omega^2}{8} [5(x^2 + y^2) + 8xy]$$

The lowest energy eigenstate with degeneracy has an energy

- (a) $\frac{7}{2} \hbar \omega$ (b) $\frac{3}{2} \hbar \omega$ (c) $4 \hbar \omega$ (d) $\frac{5}{2} \hbar \omega$

Ans.:(d)

Solution.: Given $V(x, y) = \frac{m\omega^2}{8} [5(x^2 + y^2) + 8xy]$

The term $8xy$ couples the x and y motions. To separate the motion, introduce rotated

coordinates: $u = \frac{x+y}{\sqrt{2}}, v = \frac{x-y}{\sqrt{2}}$

$\Rightarrow x^2 + y^2 = u^2 + v^2$ and $xy = \frac{u^2 - v^2}{2}$

$\Rightarrow V = \frac{m\omega^2}{8} \left[5(u^2 + v^2) + 8 \left(\frac{u^2 - v^2}{2} \right) \right] = \frac{m\omega^2}{8} [9u^2 + v^2] = \frac{1}{2} m \left(\frac{3\omega}{2} \right)^2 u^2 + \frac{1}{2} m \left(\frac{\omega}{2} \right)^2 v^2$

The coupling term has disappeared, so the system becomes two independent oscillators.

Compare with the standard oscillator potential: $V = \frac{1}{2} m\omega_u^2 u^2 + \frac{1}{2} m\omega_v^2 v^2$

So, $\omega_u = \frac{3\omega}{2}, \omega_v = \frac{\omega}{2}$.

For two independent oscillators, the energy $E_{n_u, n_v} = \left(n_u + \frac{1}{2} \right) \hbar \omega_u + \left(n_v + \frac{1}{2} \right) \hbar \omega_v$

$\Rightarrow E_{n_u, n_v} = \left(n_u + \frac{1}{2} \right) \hbar \frac{3\omega}{2} + \left(n_v + \frac{1}{2} \right) \hbar \frac{\omega}{2} = \hbar \omega \left[\frac{3}{2} n_u + \frac{1}{2} n_v + 1 \right] = \frac{\hbar \omega}{2} [3n_u + n_v + 2]$

The ground state $(n_u, n_v) = (0, 0)$ has energy $E_{0,0} = \frac{\hbar \omega}{2} [0 + 0 + 2] = \hbar \omega$ but it is nondegenerate.

Now $E_{1,0} = \frac{\hbar \omega}{2} [3 + 0 + 2] = \frac{5}{2} \hbar \omega, E_{0,3} = \frac{\hbar \omega}{2} [0 + 3 + 2] = \frac{5}{2} \hbar \omega$

Both states have the same energy. This is the lowest energy at which degeneracy occurs.

Q44. Commutator of two matrices A and B is defined as $[A, B] = AB - BA$ and the anti-commutator as $\{A, B\} = AB + BA$. If $\{A, B\} = 0$. Then we can express $[A, BC]$ as

- | | |
|------------------|----------------|
| (a) $B\{A, C\}$ | (b) $-B[A, C]$ |
| (c) $-B\{A, C\}$ | (d) $[A, B]C$ |

Ans.:(c)

Solution.: $\because \{A, B\} = AB + BA = 0 \Rightarrow AB = -BA$

$[A, BC] = ABC - BCA = -BAC - BCA = -B(AC + CA) = -B\{A, C\}$

PART C

Q55. For a particle in the angular momentum state $|l = 4, m_l = 2\rangle$, the expectation value of the operator $L_x L_y$ is

- (a) $-\hbar^2$ (b) $\hbar^2$
(c) $-i\hbar^2$ (d) $i\hbar^2$

Ans.:(d)

Solution.: Since $L_{\pm} = L_x \pm iL_y \Rightarrow L_x = \frac{L_+ + L_-}{2}$, $L_y = \frac{L_+ - L_-}{2i}$

$$\Rightarrow L_x L_y = \frac{L_+ + L_-}{2} \cdot \frac{L_+ - L_-}{2i} = \frac{1}{4i} [L_+^2 - L_-^2 - L_+ L_- + L_- L_+]$$

We need $\langle l, m | L_x L_y | l, m \rangle$

The terms L_+^2 and L_-^2 change m by two units. Their expectation values in the same $|l, m\rangle$ state vanish.

$$\text{Hence, } \langle L_x L_y \rangle = \frac{1}{4i} [-\langle L_+ L_- \rangle + \langle L_- L_+ \rangle]$$

$$L_+ L_- = L^2 - L_z^2 + \hbar L_z \Rightarrow \langle L_+ L_- \rangle = \hbar^2 [l(l+1) - m^2 + m]$$

$$\text{And } L_- L_+ = L^2 - L_z^2 - \hbar L_z \Rightarrow \langle L_- L_+ \rangle = \hbar^2 [l(l+1) - m^2 - m]$$

$$\langle L_x L_y \rangle = \frac{1}{4i} [-\langle L_+ L_- \rangle + \langle L_- L_+ \rangle] = \frac{1}{4i} (-2m\hbar^2) = \frac{im\hbar^2}{2} = i\hbar^2 \quad \because m = 2$$

Q66. Consider the one-dimensional motion of a particle of positive charge q confined to an infinite potential well

$$V(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq \pi \\ \infty & \text{otherwise} \end{cases}$$

which is subjected to a perturbing electric field $\vec{E} = E_0 \hat{x}$. The shift in the ground state energy, to the first order in q , is

- (a) $\frac{q\pi E_0}{2}$ (b) $-\frac{q\pi E_0}{2}$
(c) $q\pi E_0$ (d) $-q\pi E_0$

Ans.:(b)

Solution.: The potential energy of a charge in an electric field is $V'(x) = -qE_0x$.

The negative sign follows from the relation between potential energy and electric potential for a uniform field directed along $+x$.

The first-order energy correction is $E_0^{(1)} = \langle \psi_1 | V' | \psi_1 \rangle = -qE_0 \langle x \rangle$

Here, $\psi_1 = \sqrt{\frac{2}{\pi}} \sin x$ denotes the ground-state wavefunction of the well.

The expectation value of position is $\langle x \rangle = \int_0^\pi x |\psi_1|^2 dx = \int_0^\pi x \frac{2}{\pi} \sin^2 x dx$

Using the symmetry of the ground-state probability density about $x = \frac{\pi}{2}$, $\langle x \rangle = \frac{\pi}{2}$.

$$\Rightarrow E_0^{(1)} = -qE_0 \langle x \rangle = -qE_0 \frac{\pi}{2}$$

Q74. A one-dimensional quantum harmonic oscillator with frequency ω is in its ground state. Its

normalised wave function is given by $\psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{2\hbar}x^2\right]$. The frequency is suddenly increased to 2ω . The probability of finding the particle in its new ground state is

(a) $\frac{2\sqrt{2}}{3}$

(b) $\left(\frac{2\sqrt{2}}{3}\right)^{\frac{1}{2}}$

(c) $\frac{2}{3}$

(d) $\left(\frac{3}{2\sqrt{2}}\right)^{\frac{1}{2}}$

Ans.:(a)

Solution.: The oscillator initially has frequency ω and is in its ground state:

$$\psi_i(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{2\hbar}x^2\right]$$

After the change in frequency, the normalized ground state is

$$\psi_f(x) = \left(\frac{2m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left[-\frac{m\omega}{\hbar}x^2\right]$$

The sudden change means that the wavefunction does not immediately change at the instant of the switch. The probability of the new ground state is therefore determined by the overlap between the initial wavefunction and the new ground-state wavefunction.

The transition amplitude is

$$c_0 = \int_{-\infty}^{\infty} \psi_f^*(x) \psi_i(x) dx = \frac{(m^2 \omega^2)^{\frac{1}{8}} (2^2 m^2 \omega^2)^{\frac{1}{8}}}{(\pi \hbar)^{1/2}} \int_{-\infty}^{\infty} \exp\left[-\frac{3m\omega}{2\hbar} x^2\right] dx$$

$$\Rightarrow c_0 = \frac{(m^2 \omega^2)^{\frac{1}{8}} (2^2 m^2 \omega^2)^{\frac{1}{8}}}{(\pi \hbar)^{1/2}} \sqrt{\frac{\pi}{3m\omega/2\hbar}} = \frac{(m^2 \omega^2)^{\frac{1}{8}} (2^2 m^2 \omega^2)^{\frac{1}{8}}}{(\pi \hbar)^{1/2}} \sqrt{\frac{2\pi \hbar}{3m\omega}} = \frac{2^{1/4} 2^{1/2}}{\sqrt{3}} \Rightarrow c_0 = \frac{2^{3/4}}{\sqrt{3}}$$

The required probability is $P = |c_0|^2 = \frac{2^{3/2}}{3} = \frac{2\sqrt{2}}{3}$



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PART B

Q22. A 1-dimensional random walker's displacement is always positive and is equally likely to be anywhere in the range $[L, L+b]$. After N statistically independent steps the mean distance covered by the walker is

- (a) NL (b) $N\sqrt{L^2 + b^2}$ (c) $N\left(L + \frac{b}{2}\right)$ (d) $NL + b\sqrt{N}$

Ans.:(c)

Solution.: Each step length x is always positive and uniformly distributed in $[L, L+b]$.

Since every step is forward, the distance covered after N steps is just $S_N = x_1 + x_1 + \dots + x_N$.

The mean distance is $\langle S_N \rangle = N\langle x \rangle$.

Mean of one step in $[L, L+b]$: $\langle x \rangle = \frac{L + (L+b)}{2} = L + \frac{b}{2}$

After N independent steps $\langle S_N \rangle = N\left(L + \frac{b}{2}\right)$

Q28. A classical mono-atomic ideal gas is in thermal equilibrium at temperature T . The velocity of a molecule of this gas, of mass m , is $\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$. The value of the ensemble average $\langle v_x^2 v_y^2 \rangle$ is

- (a) $\left(\frac{k_B T}{2m}\right)^2$ (b) $\left(\frac{k_B T}{m}\right)^2$ (c) $\left(\frac{3k_B T}{2m}\right)^2$ (d) $\left(\frac{2k_B T}{m}\right)^2$

Ans.:(b)

Solution.: $\because \langle v_x^2 \rangle = \langle v_y^2 \rangle = \frac{k_B T}{m} \Rightarrow \langle v_x^2 v_y^2 \rangle = \langle v_x^2 \rangle \cdot \langle v_y^2 \rangle = \left(\frac{k_B T}{m}\right) \cdot \left(\frac{k_B T}{m}\right) = \left(\frac{k_B T}{m}\right)^2$

Q29. B, C and F are three systems which have particles of same mass and same number density kept at the same low temperature T . Here C is a classical ideal gas, F is a free Fermi gas and B is a free Bose gas, with pressures P_C, P_F and P_B respectively. Then

- (a) $P_B > P_C > P_F$ (b) $P_F > P_C > P_B$
(c) $P_C > P_F > P_B$ (d) $P_C > P_B > P_F$

Ans.:(b)

Solution.: At low temperature T : $P_F > P_C > P_B$

Q45. Five indistinguishable atoms are sitting on the distinguishable vertices of a pentagon. The atoms can be in one of the two states: g with energy 0, and e with energy E . However neighbouring atoms cannot both be in the e state. The partition function of this system at temperature T , is

(a) $1 + 5e^{-\frac{E}{k_B T}} + 2e^{-\frac{2E}{k_B T}}$

(b) $1 + 5e^{-\frac{E}{k_B T}} + 3e^{-\frac{2E}{k_B T}}$

(c) $1 + 5e^{-\frac{E}{k_B T}} + 10e^{-\frac{2E}{k_B T}}$

(d) $1 + 5e^{-\frac{E}{k_B T}} + 5e^{-\frac{2E}{k_B T}}$

Ans.:(d)

Solution.:

Case-I: Zero atoms in e state

So all 5 atoms are in g state i.e. configuration (g, g, g, g, g) .

Number of ways = 1 and total energy = 0.

Contribution to partition function = $e^{-0} = 1$

Case-II: One atom in e state

Exactly 1 atom in e state and the remaining 4 atoms are in g state.

Number of ways in which we can choose 1 of the 5 distinguishable vertices = ${}^5C_1 = 5$

Total energy = E .

Contribution to partition function = $5e^{-\beta E}$

Case-III: Two atom in e state

Exactly 2 atom in e state and the remaining 3 atoms are in g state.

Number of ways in which we can choose 2 of the 5 distinguishable vertices = ${}^5C_2 = 10$

Forbidden configurations: $(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)$

Allowed configurations = 5

Total energy = $2E$.

Contribution to partition function = $5e^{-2\beta E}$

The partition function of this system at temperature T , is $Z = 1 + 5e^{-\frac{E}{k_B T}} + 5e^{-\frac{2E}{k_B T}}$

PART C

- Q47.** A spherical gaseous ball of radius 15 m was formed with a temperature $T = 3 \times 10^5\text{ K}$. The gas expands adiabatically and its temperature drops to $5 \times 10^3\text{ K}$. Given $\gamma = \frac{5}{3}$ for this gas, the radius of the ball becomes approximately
- (a) 212 m (b) 86 m (c) 137 m (d) 116 m

Ans.:(d)

Solution.: Initial radius, $R_1 = 15\text{ m}$; initial temp. $T_1 = 3 \times 10^5\text{ K}$; final temp. $T_2 = 5 \times 10^3\text{ K}$

Adiabatic index $\gamma = \frac{5}{3}$.

For adiabatic process: $TV^{\gamma-1} = \text{constant} \Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_1 R_1^{3(\gamma-1)} = T_2 R_2^{3(\gamma-1)} \quad \because V \propto R^3$

$$\Rightarrow T_1 R_1^2 = T_2 R_2^2 \quad \because \gamma = \frac{5}{3}$$

$$\Rightarrow R_2^2 = \frac{T_1}{T_2} R_1^2 = \frac{3 \times 10^5}{5 \times 10^3} \times (15)^2 = 13500 \Rightarrow R_2 = \sqrt{13500} = 116.2\text{ m}$$

- Q52.** Consider a one-dimensional lattice (with lattice spacing a) along X-axis with sites labelled by $x = 0, 1, 2, \dots, L$. A particle carrying a charge $-q$ can occupy any one of these sites. An electric field of strength E is applied in the positive x -direction. The average energy of the particle at a temperature T (in the limit $L \rightarrow \infty$) is ($\beta = \frac{1}{k_B T}$)

- (a) $\frac{Eq a}{e^{\beta Eq a} - 1}$ (b) $\frac{Eq a}{1 + e^{\beta Eq a}}$ (c) $\frac{Eq a}{2}$ (d) $-Eq a$

Ans.:(a)

Solution.: Potential energy U_n of the charge $-q$ at site n where $n = 0, 1, 2, \dots, L$ is

$$U_n = -q \times V(x) = -q \times (-Ena) = qEan = n\varepsilon \quad \text{where } \varepsilon = qEa.$$

The canonical partition function for a single particle capable of occupying these states is

$$Z = \sum_{n=0}^L e^{-\beta U_n} = \sum_{n=0}^L e^{-\beta n\varepsilon} = \sum_{n=0}^L (e^{-\beta\varepsilon})^n. \quad \text{In the limit } L \rightarrow \infty: Z = \sum_{n=0}^{\infty} (e^{-\beta\varepsilon})^n = \frac{1}{1 - e^{-\beta\varepsilon}}$$

$$\text{Average energy } \langle U \rangle = -\frac{\partial}{\partial \beta} \ln Z = -\frac{\partial}{\partial \beta} \ln(1 - e^{-\beta\varepsilon})^{-1} = \frac{\partial}{\partial \beta} \ln(1 - e^{-\beta\varepsilon}) = \frac{\varepsilon e^{-\beta\varepsilon}}{1 - e^{-\beta\varepsilon}}$$

$$\Rightarrow \langle U \rangle = \frac{\varepsilon}{e^{\beta\varepsilon} - 1} = \frac{qEa}{e^{\beta qEa} - 1}$$

Q60. The excitations of a three-dimensional solid are bosonic in nature and their energy dispersion is given by $\epsilon_k \propto k^2$, in the long wavelength limit. If the chemical potential of the system is zero, the temperature dependence of specific heat of the system at low temperature is proportional to

- (a) T^3 (b) $T^{\frac{3}{2}}$
(c) $T^{\frac{5}{2}}$ (d) $T^{\frac{1}{2}}$

Ans.:(b)

Solution.:

For bosonic excitations with long-wavelength dispersion $\epsilon_k \propto k^2$

In 3D, the density of states behaves as $g(\epsilon) \propto \epsilon^{1/2}$

With chemical potential $\mu = 0$, the bosonic occupation factor is $\frac{1}{e^{\epsilon/k_B T} - 1}$.

$$\text{Internal energy: } U \propto \int_0^\infty \frac{\epsilon g(\epsilon)}{e^{\epsilon/k_B T} - 1} d\epsilon \propto \int_0^\infty \frac{\epsilon^{3/2}}{e^{\epsilon/k_B T} - 1} d\epsilon$$

$$\text{Let } \epsilon = (k_B T)x \text{ then } U \propto (k_B T)^{5/2} \int_0^\infty \frac{x^{3/2}}{e^x - 1} dx \propto T^{5/2}$$

$$\text{The specific heat } C_V = \frac{dU}{dT} \propto T^{3/2}$$

Q69. A binary alloy consists of N_A number of A -type and N_B number of B -type atoms. The atoms sit on the sites of a simple cubic lattice and the nearest neighbours interact with each other. Assume an attractive interaction energy $-J$ ($J > 0$) between two like neighbours (AA or BB pair) and a repulsive interaction energy $+J$ between two unlike neighbours (AB pair). If N is the total number of sites, then the average energy of the system at a very high temperature ($k_B T \gg J$) is

- (a) $-3J \frac{(N_A - N_B)^2}{N}$ (b) $3JN$
(c) $3J \frac{(N_A + N_B)^2}{N}$ (d) $-3J(N_A - N_B)$

Ans.:(a)

Solution.:

1. High Temperature Limit ($k_B T \gg J$)

When the temperature is extremely high, the thermal energy completely dominates over the interaction energies. Consequently, the configurations of the atoms become completely random. The probability of finding an atom of type A or type B at any given lattice site depends solely on their respective concentrations:

Probability of a site being occupied by an A-type atom: $P_A = \frac{N_A}{N}$

Probability of a site being occupied by an B-type atom: $P_B = \frac{N_B}{N}$

2. Finding the total number of nearest-neighbour pairs

Number of nearest neighbours per site for a simple cubic lattice is $z = 6$.

Since each pair connects 2 sites, the total number of nearest-neighbour bonds/pairs (N_{pairs}) in a

lattice of N sites is $N_{pairs} = \frac{zN}{2} = \frac{6N}{2} = 3N$

3. Probabilities and Energies of individual bonds

Because the distributions are completely random in the high-temp. limit, we can compute the probability of occurrence for each pair type:

Pair Type	Interaction Energy (E_i)	Probability of occurrence (P_i)
AA pair	$-J$	$P_A \times P_A = \left(\frac{N_A}{N}\right)^2$
BB pair	$-J$	$P_B \times P_B = \left(\frac{N_B}{N}\right)^2$
AB or BA pair	$+J$	$2 \times P_A \times P_B = 2 \left(\frac{N_A N_B}{N^2}\right)$

4. Calculating the average energy $\langle E \rangle$

The total average energy of the system is the total number of pairs multiplied by the expected energy of a single random pair:

$$\langle E \rangle = N_{pairs} \times \sum P_i E_i = 3N \left[\left(\frac{N_A}{N}\right)^2 (-J) + \left(\frac{N_B}{N}\right)^2 (-J) + 2 \left(\frac{N_A N_B}{N^2}\right) (+J) \right]$$

$$\Rightarrow \langle E \rangle = 3N \left(\frac{-J}{N^2} \right) [N_A^2 + N_B^2 - 2N_A N_B] = -3J \frac{(N_A - N_B)^2}{N}$$



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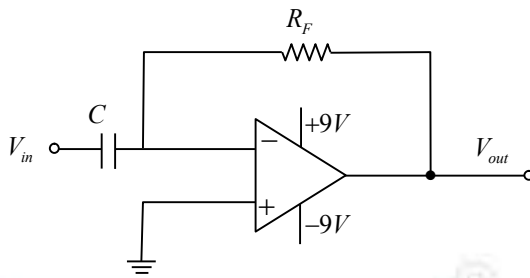
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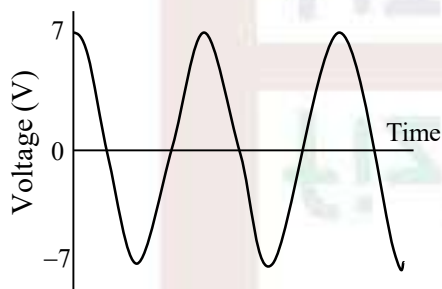
PART B

Q31. In the circuit shown below, the input voltage is $v_{in}(t) = 0.3 \sin 50t$ (Volts) and $C = 100 \mu F$, $R_F = 10 k\Omega$

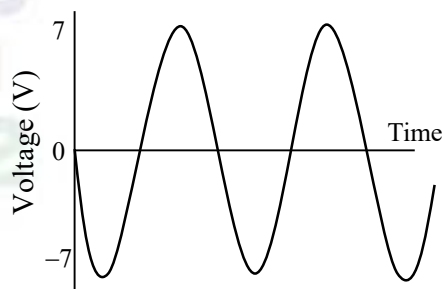


Considering the opamp to be ideal and neglecting the transients, the best representation of the output voltage $V_{out}(t)$ is

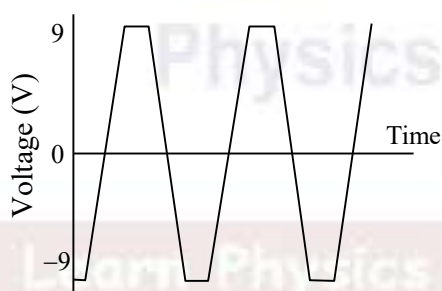
(a)



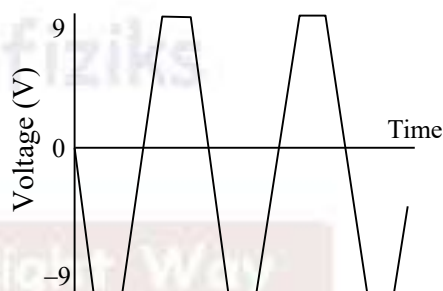
(b)



(c)



(d)



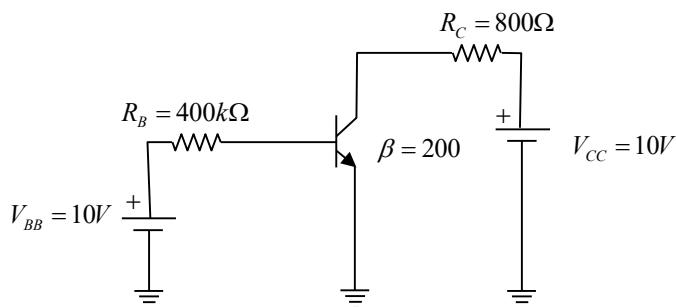
Ans.:(c)

Solution.: $v_o = -R_F C \frac{dv_{in}}{dt} = -(10 \times 10^3)(100 \times 10^{-6}) \frac{d}{dt}(0.3 \sin 50t)$

$$\Rightarrow v_o = -0.3 \times 50 (\cos 50t) = -15 \cos 50t \text{ volts}$$

Since saturation voltage is ± 9 Volts so output voltage cannot exceed this limit so output is option (c).

Q36. In the transistor circuit given below the voltage V_{CC} fluctuates by 5%. Then the fluctuation in V_{CE} would be closest to (take $V_{BE} = 0.7V$)



- (a) 8% (b) 7% (c) 6% (d) 5%

Ans.:(a)

Solution.: $I_B = \frac{V_{BB} - V_{BE}}{R_B} = \frac{10V - 0.7V}{400 \times 10^3} = 23.25 \mu A \Rightarrow I_C = \beta I_B = 200 \times 23.25 \mu A = 4.65 mA$

$$V_{CE} = V_{CC} - I_C R_C = 10V - (4.65 \times 10^{-3})(800) = 10V - 3.72V = 6.28V$$

Since V_{BB} is independent of V_{CC} then I_B, I_C remains unchanged. Thus

$$\Delta V_{CE} = \Delta V_{CC} = 10V \times \frac{5}{100} = 0.5V$$

Then the fluctuation in V_{CE} would be $= \frac{\Delta V_{CE}}{V_{CE}} \times 100 = \frac{0.5}{6.28} \times 100 \approx 8\%$

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PART C

Q54. A thermistor measures an object's temperature T , by measuring its resistance R according to $R = AT^{-n}$, where A and n are positive constants. The observed resistances for different values of temperature (including environmental and instrumental sources of error) are

$T(K)$	$R(\Omega)$
250	140
300	110
350	90

The estimated value of the exponent n , from the above data, is closest to

- (a) 2.0 (b) 0.8
(c) 1.3 (d) 2.7

Ans.:(c)

Solution.: $\because R = AT^{-n} \Rightarrow \ln R = \ln A - n \ln T$.

It's a straight-line relation between $\ln R$ and $\ln T$ whose slope is $-n$.

Using the first and third data points: $(T_1, R_1) = (250, 140)$ and $(T_3, R_3) = (350, 90)$

$$\text{So } -n = \frac{\ln R_3 - \ln R_1}{\ln T_3 - \ln T_1} = \frac{\ln 90 - \ln 140}{\ln 350 - \ln 250} = -\frac{0.442}{0.336} \Rightarrow n \approx 1.32$$

Q71. In the circuit shown below, the input impedance of voltmeters V_1 and V_2 are $10 M\Omega$. If

$R_L = 10 M\Omega$ and $V_{in} = 1.5V$. The measured

voltages by V_1 and V_2 are closest to

- (a) $0.5V$ and $1.0V$, respectively
(b) $0V$ and $1.5V$, respectively
(c) $1.5V$ and $0V$, respectively
(d) $1.0V$ and $0.5V$, respectively

Ans.:(a)

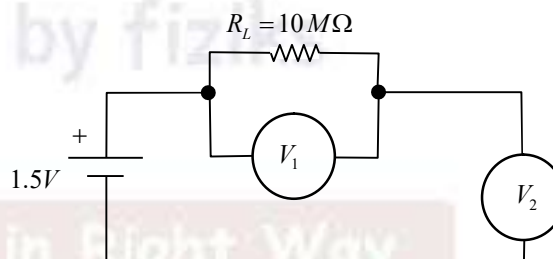
Solution.: The input impedance of voltmeters V_1 and V_2 are $R_{V_1} = R_{V_2} = 10 M\Omega$.

$$\text{Let } R_{eq} = R_L \parallel R_{V_1} = \frac{10 \times 10}{10 + 10} = 5 M\Omega \text{ then } I = \frac{1.5V}{R_{eq} + R_{V_2}} = \frac{1.5V}{5 M\Omega + 10 M\Omega} = 0.1 \mu A$$

Voltmeters V_1 measures the voltage across the parallel branch so

$$V_1 = IR_{eq} = (0.1 \mu A)(5 M\Omega) = 0.5V$$

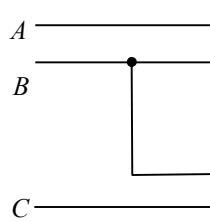
$$\text{and } V_2 = IR_{V_2} = (0.1 \mu A)(10 M\Omega) = 1.0V$$



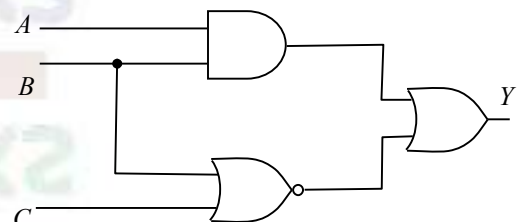
Q73. The digital logic circuit that would give the following truth table

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

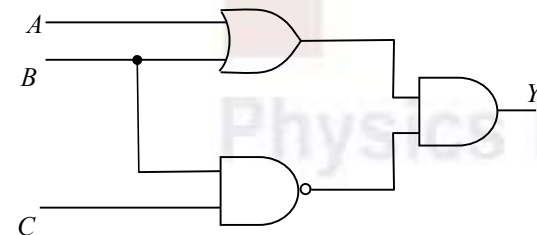
(a)



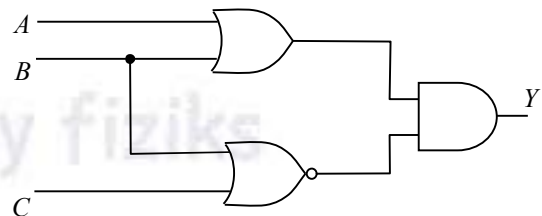
(b)



(c)



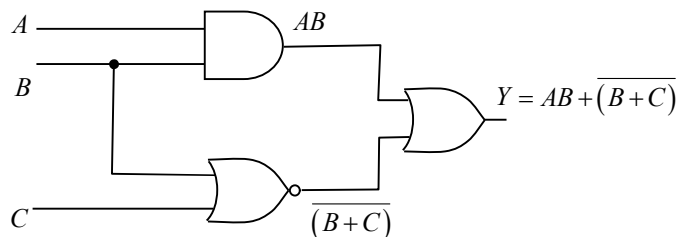
(d)



Ans.:(b)

Solution.: From truth table $Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC = \bar{B}\bar{C}(\bar{A} + A) + AB(\bar{C} + C)$

$$\Rightarrow Y = \bar{B}\bar{C} + AB = AB + \overline{(B+C)}$$





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PART B

Q43. An isolated two-electron quantum state is described by the orbital angular momentum quantum number l and the total spin S . An allowed value of l and S is

- (a) $S = 1, l = 0$ (b) $S = 0, l = 1$
(c) $S = 1, l = 1$ (d) $S = 1, l = 2$

Ans.:(c)

Solution.:

For two electrons, total spin $S = 1$ is a triplet, whose spin part is symmetric under exchange. By Pauli exclusion, the spatial/orbital part must then be antisymmetric. For total orbital angular momentum l , exchange symmetry is $(-1)^l$, so l must be odd. Thus $S = 1, l = 1$ is allowed, while $S = 1, l = 0$ is not.

PART C

Q46. Consider an emission line of wavelength $\lambda = 550\text{nm}$ of Argon ($A = 40, Z = 18$) at a temperature 400K . The full Doppler width of the emission line will be closest to

- (a) 10^{-2}nm (b) 10^{-1}nm
(c) 10^{-3}nm (d) 10^{-5}nm

Ans.:(c)

Solution.:

The full width at half maximum (FWHM) Doppler broadening is:

$$\Delta\lambda_D = \lambda_0 \sqrt{\frac{8k_B T \ln 2}{mc^2}} = 550 \times 10^{-9} \sqrt{\frac{8 \ln 2 (1.38 \times 10^{-23}) \times 400}{(40 \times 1.66 \times 10^{-27})(3 \times 10^8)^2}}$$

$$\Rightarrow \Delta\lambda_D = 550 \times 10^{-9} \sqrt{\frac{8 \ln 2 \times 5.52 \times 10^{-21}}{5.98 \times 10^{-9}}} \approx 1.24 \times 10^{-12} \text{m} = 1.24 \times 10^{-3} \text{nm} \approx 10^{-3} \text{nm}$$

Q57. An optical cavity of a laser, formed by two plane mirrors, is filled up with an active medium. The medium emits radiation at wavelengths 450nm , 600nm and 750nm . If the medium is continuously pumped, at which cavity length among the following, will all three wavelengths be amplified?

- (a) $750\mu\text{m}$ (b) $1500\mu\text{m}$
(c) $600\mu\text{m}$ (d) $450\mu\text{m}$

Ans.:(d)

Solution.:

For a plane-mirror cavity of length L , amplification occurs when

$$L = m \frac{\lambda}{2} \quad \text{or} \quad \frac{2L}{\lambda} = \text{integer}$$

Check $L = 450 \mu\text{m}$: $2L = 900 \mu\text{m} = 900000 \text{ nm}$

For $\lambda = 450 \text{ nm}$, $\frac{2L}{\lambda} = \frac{900000}{450} = 2000$

For $\lambda = 600 \text{ nm}$, $\frac{2L}{\lambda} = \frac{900000}{600} = 1500$

For $\lambda = 750 \text{ nm}$, $\frac{2L}{\lambda} = \frac{900000}{750} = 1200$

All are integers. Hence all three wavelengths are amplified.

Q59. A hydrogen atom is in a weak external magnetic field $\vec{B}$. Consider an electron of this atom with $\left(l=1, s=\frac{1}{2}\right)$ and total $j=\frac{3}{2}$. There are multiple energy levels for this electron due to the magnetic field. The energy spacing between any two adjacent levels (in units of $\mu_B B$) is

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{3}{4}$

(d) $\frac{4}{3}$

Ans.:(d)

Solution.:

The energy shift in a weak magnetic field (Zeeman effect) is given by:

$$\Delta E = g_J \mu_B B m_J$$

Where $g_J = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$ with $\left(l=1, s=\frac{1}{2}\right)$ and total $j=\frac{3}{2}$.

$$\Rightarrow g_J = 1 + \frac{\frac{3}{2} \times \frac{5}{2} + \frac{1}{2} \times \frac{3}{2} - 1 \times 2}{2 \times \frac{3}{2} \times \frac{5}{2}} = 1 + \frac{\frac{18}{4} - 2}{\frac{15}{2}} = 1 + \frac{10}{4} \times \frac{2}{15} = 1 + \frac{1}{3} \Rightarrow g_J = \frac{4}{3}$$

The spacing between any two adjacent m_J levels ($\Delta m_J = 1$) in units of $\mu_B B$ is equal to $g_J = \frac{4}{3}$.

Q72. The bond dissociation energy of OH molecule is 4.18 eV with rotational constant 18.8 cm^{-1} . For rotational induced dissociation, the minimum value of rotational quantum number is closest to

- (a) 114 (b) 454
(c) 45 (d) 90

Ans.:(c)

Solution.: Rotational energy: $E_J = BJ(J+1)$ where $B = 18.8\text{ cm}^{-1}$

The bond dissociation energy

$$D = 4.18\text{ eV} = 4.18 \times 8065.54\text{ cm}^{-1} = 33714\text{ cm}^{-1} \quad \because 1\text{ eV} = 8065.54\text{ cm}^{-1}$$

$$\text{Now } BJ(J+1) = D \Rightarrow 18.8J(J+1) = 33714 \Rightarrow J(J+1) \approx 1793.3$$

$$\Rightarrow J^2 + J - 1793.3 = 0 \Rightarrow J = 41.85 \approx 42.$$

So best option is (c).



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PART C

Q53. In a one-dimensional chain of atoms, the phonon energy dispersion is given by $E = A|\sin ka|$. Here, A is a constant, k is a vector in the reciprocal space and a is lattice spacing. The density of states is proportional to

- (a) $\frac{1}{\sqrt{A^2 - E^2}}$ (b) $\frac{1}{\sqrt{A^2 + E^2}}$ (c) $\frac{1}{\sqrt{A - E}}$ (d) $\frac{1}{\sqrt{A + E}}$

Ans.:(a)

Solution.: The density of states $g(E)$ in one dimension is proportional to the inverse of the group velocity magnitude: $g(E) = \frac{1}{\pi} \left| \frac{dk}{dE} \right|$

Differentiating E with respect to k for $ka \in [0, \pi]$:

$$\frac{dE}{dk} = Aa \cos ka = Aa \sqrt{1 - \sin^2 ka} = a \sqrt{A^2 - E^2} \quad \because E = A|\sin ka|$$

$$\Rightarrow g(E) \propto \left| \frac{dk}{dE} \right| \propto \frac{1}{\sqrt{A^2 - E^2}}$$

Q68. Consider a one-dimensional chain of atoms with lattice constant a . The energy of an electron with wave-vector k is $\epsilon(k) = \mu - 2\gamma \cos ka$, where μ and γ are constants. If an electric field $\vec{E}$ is applied along the chain, the time dependent velocity of the electron is proportional to (assume initial wave vector $k = k_0$ at $t = 0$)

- (a) $\sin^2 \left(k_0 a - \frac{eEa}{\hbar} t \right)$ (b) $\cos \left(k_0 a - \frac{eEa}{\hbar} t \right)$
(c) $\sin \left(k_0 a - \frac{eEa}{\hbar} t \right)$ (d) $\cos^2 \left(k_0 a - \frac{eEa}{\hbar} t \right)$

Ans.:(c)

Solution.: The energy of an electron with wave-vector k is $\epsilon(k) = \mu - 2\gamma \cos ka$, where μ and γ are constants.

$$\text{Velocity: } v(t) = \frac{1}{\hbar} \frac{d\epsilon}{dk} = \frac{2\gamma a}{\hbar} \sin ka$$

Under an electric field, the semiclassical equation is $\hbar \frac{dk}{dt} = -eE \Rightarrow k(t) = k_0 - \frac{eE}{\hbar} t$

$$\Rightarrow v(t) \propto \sin \left(k_0 a - \frac{eEa}{\hbar} t \right)$$

Q48. A cubic sample of edge length L is maintained at a temperature of $4K$. The speed of sound in the material of the sample is $5 \times 10^3 \text{ m/s}$. The minimum value of L required to excite the lowest frequency phonon mode is closest to

- (a) 10 nm (b) 30 nm (c) 20 nm (d) 5 nm

Ans.:(b)

Solution.: Lowest phonon mode in a cubic sample of edge L has maximum wavelength

$$\lambda_{\max} = 2L$$

So its minimum frequency is $\nu_{\min} = \frac{v}{2L}$.

Its energy is $E_{\min} = h\nu_{\min} = \frac{h\nu}{2L}$.

For the mode to be excited at temperature T , $E_{\min} \approx k_B T = \frac{h\nu}{2L} \Rightarrow L = \frac{h\nu}{2k_B T}$

$$\Rightarrow L = \frac{(6.6 \times 10^{-34})(5 \times 10^3)}{2(1.38 \times 10^{-23}) \times 4} \approx 3 \times 10^{-8} \text{ m} = 30 \text{ nm}$$

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PART C

Q62. Consider the cross-sections

$$\sigma_1 = \sigma(p + n \rightarrow \Delta^+ + n) \text{ and } \sigma_2 = \sigma(p + n \rightarrow \Delta^0 + p)$$

where the (Δ^+, Δ^0) are part of the baryon decuplet. Then

- (a) one of the $\sigma_{1,2}$ vanishes identically (b) $\sigma_1 \gg \sigma_2$, with both being non-zero
(c) $\sigma_1 \ll \sigma_2$, with both being non-zero (d) $\sigma_1 \approx \sigma_2$

Ans.:(d)

Solution.: $\sigma_1 = \sigma(p + n \rightarrow \Delta^+ + n)$ and $\sigma_2 = \sigma(p + n \rightarrow \Delta^0 + p)$

Both are nucleons with isospin $I = \frac{1}{2}$. Baryon decuplet and have isospin $I = \frac{3}{2}$

If isospin is conserved, the initial and final states must have a common total isospin.

Calculate the total isospin of the initial state $(p + n)$

allowed total values are $I = I_1 + I_2, I_1 + I_2 - 1, \dots, |I_1 - I_2|$

So, the initial state can have $I = 0$ or $I = 1$.

The third component of isospin is $I_3 = 0$, because the proton has $I_3 = +\frac{1}{2}$ and the neutron has $I_3 = -\frac{1}{2}$.

Calculate the total isospin of the final state $(\Delta^+ + n)$ and $(\Delta^0 + p)$

The final particles have isospins $\frac{3}{2}$ and $\frac{1}{2}$. The possible total isospins are $I = 2$ or $I = 1$

Because the initial state allows $I = 0, 1$, while the final state allows $I = 1, 2$, the only common value is $I = 1$.

Therefore, the reactions can occur through the $I = 1$ channel.

Both final states have total third component of isospin $I_3 = 0$.

The two final states, , have equal-magnitude isospin Clebsch–Gordan coefficients for the $I = 1$, $I_3 = 0$ state. Therefore, assuming isospin conservation and neglecting isospin-breaking effects, their cross-sections are equal: $\sigma_1 \approx \sigma_2$

NOTE: When comparing strong-interaction reactions involving members of the same isospin multiplets, use isospin conservation and Clebsch–Gordan coefficients to compare transition amplitudes.

Q67. Suppose that the volume and the surface terms are the most dominant ones in the semi-empirical formula for the binding energy of a nucleus. Let C_s and C_v be the coefficients of the surface and volume terms. Which of the following is a criterion for stability of the nucleus?

- (a) $A > \left(\frac{C_s}{C_v}\right)^3$ (b) $A < \left(\frac{C_s}{C_v}\right)^3$
(c) $A > \left(\frac{2C_s}{3C_v}\right)^3$ (d) $A < \left(\frac{2C_s}{3C_v}\right)^3$

Ans.:(a)

Solution.: For a nucleus to be bound (stable against complete disintegration into individual nucleons), its total binding energy must be positive ($B > 0$):

$$B = C_v A - C_s A^{2/3} > 0$$

$$\Rightarrow C_v A > C_s A^{2/3} \Rightarrow C_v A^{1/3} > C_s \Rightarrow A > \left(\frac{C_s}{C_v}\right)^3$$

Q70. For a spherical nucleus, consider the interior charge distribution to be

$$\rho(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]}$$

where ρ_0, R and a are constants of appropriate dimensions. In the limit $a \rightarrow 0^+$, the number of protons (charge e) inside a sphere of radius $2R$ is given by

- (a) $\frac{2\rho_0}{e} \left(\frac{4}{3}\pi R^3\right)$ (b) $\frac{\rho_0}{e} \left(\frac{4}{3}\pi R^3\right)$
(c) $\frac{8\rho_0}{e} \left(\frac{4}{3}\pi R^3\right)$ (d) $\frac{4\rho_0}{e} \left(\frac{4}{3}\pi R^3\right)$

Ans.:(b)

Solution.: Given $\rho(r) = \frac{\rho_0}{1 + \exp[(r - R)/a]}$. In the limit $a \rightarrow 0^+$: $\rho(r) = \begin{cases} \rho_0, & r < R \\ 0, & r > R \end{cases}$

Total charge inside a sphere of radius $2R$ is $Q = \int_0^{2R} \rho(r) \times 4\pi r^2 dr = \int_0^R \rho_0 \times 4\pi r^2 dr + 0$

$$\Rightarrow Q = \rho_0 \left(\frac{4}{3}\pi R^3\right)$$

The number of protons is the total charge Q divided by the charge of a single proton (e):

$$N_p = \frac{Q}{e} = \frac{\rho_0}{e} \left(\frac{4}{3}\pi R^3\right)$$