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CSIR NET-JRF Physical Sciences June-2026

Complete Solution

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CSIR NET-JRF Physical Sciences Paper June-2026
Solution-Mathematical Physics

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PART B

Q36. In a 4-dimensional vector space V_4 , three linearly independent vectors are given by $e_1 = (1, 1, 0, 0)$, $e_2 = (0, 1, 1, 0)$ and $e_3 = (0, 0, 1, 1)$. Which of the following vector e_4 will make $\{e_1, e_2, e_3, e_4\}$ a basis of V_4 ?

- (a) $(1, 1, 1, 1)$ (b) $(1, 1, 1, 0)$
(c) $(1, 0, -1, 0)$ (d) $(0, 1, 1, 0)$

Ans.: (b)

Solution.: To form a basis for V_4 , the determinant of the matrix formed by the four vectors $\{e_1, e_2, e_3, e_4\}$ must be non-zero (linearly independent).

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ a & b & c & d \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ b & c & d \end{vmatrix} - a \cdot \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = b - c + d - a$$

$$(a) \ e_4 = (1, 1, 1, 1): \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = b - c + d - a = 1 - 1 + 1 - 1 = 0. \text{ Not a basis.}$$

$$(b) \ e_4 = (1, 1, 1, 0): \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = b - c + d - a = 1 - 1 + 0 - 1 = -1 \neq 0$$

So $e_4 = (1, 1, 1, 0)$ will make $\{e_1, e_2, e_3, e_4\}$ a basis of V_4 .

$$(c) \ e_4 = (1, 0, -1, 0): \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix} = b - c + d - a = 0 - (-1) + 0 - 1 = 0. \text{ Not a basis.}$$

$$(d) \ e_4 = (0, 1, 1, 0): \begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{vmatrix} = b - c + d - a = 1 - 1 + 0 - 0 = 0. \text{ Not a basis.}$$

Q42. The value of the integral:

$$\oint \frac{d\theta}{5-4\cos\theta}$$

over the circle $|z|=1$ in the anti-clockwise direction can be calculated by substituting $z = e^{i\theta}$.

The value of the integral is:

(a) $\frac{2\pi}{3}$

(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{4\pi}{3}$

Ans.: (a)

Solution.: $z = e^{i\theta} \Rightarrow dz = ie^{i\theta} d\theta \Rightarrow d\theta = \frac{dz}{iz}$, $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} = \frac{z + z^{-1}}{2}$

$$I = \oint_{|z|=1} \frac{d\theta}{5-4\cos\theta} = \oint_{|z|=1} \frac{dz/iz}{5-4\left(\frac{z+z^{-1}}{2}\right)} = \oint_{|z|=1} \frac{dz/iz}{5-2\left(\frac{z^2+1}{z}\right)} = \oint_{|z|=1} \frac{dz}{i(5z-2z^2-2)}$$

$$\Rightarrow I = -\oint_{|z|=1} \frac{dz}{i(2z^2-5z+2)} = -\oint_{|z|=1} \frac{dz}{i(2z-1)(z-2)}$$

Find the poles inside $|z|=1$: $z = \frac{1}{2}$ (inside); $z = 2$ (outside).

$$\text{Residue at } z = \frac{1}{2}: a_{-1} = \lim_{z \rightarrow \frac{1}{2}} \left(z - \frac{1}{2}\right) \frac{1}{i(2z-1)(z-2)} = -\frac{1}{3i}$$

$$\text{Apply Cauchy's Residue Theorem: } I = -(2\pi i \times a_{-1}) = -\left(2\pi i \times \left(-\frac{1}{3i}\right)\right) = \frac{2\pi}{3}$$

Q45. In spherical polar coordinates (r, θ, ϕ) , $\frac{\partial \hat{\theta}}{\partial \phi}$ is:

(a) $\cos\theta \hat{\phi}$

(b) 0

(c) $\sin\theta \hat{\phi}$

(d) $-\cos\theta \hat{r}$

Ans.: (a)

Solution.: In spherical coordinates:

$$\hat{\theta} = \cos\theta \cos\phi \hat{x} + \cos\theta \sin\phi \hat{y} - \sin\theta \hat{z}$$

Differentiate with respect to ϕ :

$$\frac{\partial \hat{\theta}}{\partial \phi} = -\cos\theta \sin\phi \hat{x} + \cos\theta \cos\phi \hat{y} = \cos\theta \hat{\phi}$$

PART C

Q46. Consider a complex analytic function $f(x, y) = u(x, y) + iv(x, y)$. If $u(x, y) = x^3 - 3xy^2$, then $v(x, y)$ is:

- (a) $y^3 - 3xy^2$ (b) $3xy^2 - y^3$
(c) $3x^2y - y^3$ (d) $y^3 + 3xy^2$

Ans.: (c)

Solution.: Use Cauchy-Riemann Condition: $u_x = v_y$, $u_y = -v_x$

$$\because u(x, y) = x^3 - 3xy^2 \text{ thus } u_x = v_y \Rightarrow 3x^2 - 3y^2 = v_y \Rightarrow v = 3x^2y - y^3 + g(x) \Rightarrow v_x = 6xy + g'(x)$$

$$\text{and } u_y = -v_x \Rightarrow -6xy = -v_x \Rightarrow v_x = 6xy. \text{ Hence } g'(x) = 0 \Rightarrow g(x) = C = 0 (\text{Let}).$$

$$\text{Thus } v(x, y) = 3x^2y - y^3$$

Q49. In four dimensions, a rank four tensor is given by T_{abcd} . It is symmetric under interchange of the indices a, b and interchange of the indices c, d . Moreover, the tensor is anti-symmetric under the interchange of pair of indices ab and cd . The number of independent components of the tensor is:

- (a) 100 (b) 45
(c) 36 (d) 21

Ans.: (b)

Solution.:

A symmetric rank-2 pair (a, b) in 4D has $N = \frac{4(4+1)}{2} = 10$ independent pairs.

The tensor $T_{(ab)(cd)}$ behaves as an anti-symmetric pair of these combined indices $A = (a, b)$ and $B = (c, d)$.

Number of independent components for an anti-symmetric matrix of dimension $N = 10$:

$$\frac{N(N-1)}{2} = \frac{10(10-1)}{2} = 45$$

Q57. Newton-Raphson method is used for finding the roots of the polynomial $(x^2 - 1)(x - 2)$. If the initial trial solutions are chosen as 0 and 0.5, the algorithm converges to p and q , respectively. Then, which of the following is correct?

- (a) $p = 1, q = -1$ (b) $p = -1, q = 1$
(c) $p = 2, q = 1$ (d) $p = 1, q = 2$

Ans.: (c)

Solution.:

Polynomial: $f(x) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2$

Derivative: $f'(x) = 3x^2 - 4x - 1$

Newton-Raphson: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Starting at $x_0 = 0$: $f(0) = 2, f'(0) = -1, x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 0 - \frac{2}{(-1)} = 2$

$f(2) = 8 - 8 - 2 + 2 = 0$. so, converges to $p = 2$.

Starting at $x_0 = 0.5$: $f(0.5) = 0.125 - 0.5 - 0.5 + 2 = 1.125, f'(0.5) = 0.75 - 2 - 1 = -2.25$

$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 0.5 - \frac{1.125}{(-2.25)} = 1$

$f(1) = 1 - 2 - 1 + 2 = 0$. so, converges to $q = 1$. So $p = 2, q = 1$

Q69. In an experiment, a student generates 100 random numbers drawn from a uniform distribution in the interval $(0,1)$ and calculates their mean. If the experiment is repeated 100 times, then the standard deviation of the means is:

- (a) $\frac{1}{20\sqrt{3}}$ (b) $\frac{1}{2\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{200\sqrt{3}}$

Ans.: (a)

Solution.: For a uniform distribution $U(0,1)$:

Variance of a single variable $\sigma^2 = \frac{(b-a)^2}{12} = \frac{1}{12} \Rightarrow \sigma = \frac{1}{\sqrt{12}}$

Standard deviation of the mean of $N = 100$ variables: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{1/\sqrt{12}}{\sqrt{100}} = \frac{1}{20\sqrt{3}}$

Q72. A biased random walker confined to the x -axis moves, at each step, with probability 0.6 to the right and with probability 0.4 to the left. If the walker takes 1000 steps starting from the origin, the probability that the walker is found exactly 200 steps to the right of the origin is close to:

- (a) 0.026 (b) 0.125
(c) 0.250 (d) 0.375

Ans.: (a)

Solution.: Total steps $N = 1000$, rightward probability $p = 0.6$, leftward probability $q = 0.4$.

Let n_R be right steps and n_L be left steps: $n_R + n_L = 1000$, $n_R - n_L = 200 \Rightarrow n_R = 600$

Using normal approximation for Binomial distribution $B(1000, 0.6)$:

$$\mu = Np = 1000 \times 0.6 = 600, \sigma = \sqrt{Npq} = \sqrt{1000 \times 0.6 \times 0.4} = \sqrt{240} \approx 15.49$$

Maximum probability density at the mean ($\mu = 600$):

$$P(X = 600) \approx \frac{1}{\sqrt{2\pi\sigma^2}} = \frac{1}{\sqrt{2\pi \times 240}} \approx 0.02575 \approx 0.026$$

Q74. The integral

$$\int_0^4 x^4 dx$$

when compound numerically using Simpson's 1/3rd method and trapezoidal method, each with unit step size, yields values I_s and I_t , respectively. Then which of the following options is correct?

- (a) $I_s = 205.3$, $I_t = 226.0$ (b) $I_s = 205.3$, $I_t = 205.3$
(c) $I_s = 204.8$, $I_t = 226.0$ (d) $I_s = 226.0$, $I_t = 226.0$

Ans.: (a)

Solution.: For $I = \int_0^4 x^4 dx$ with $h = 1$ and points $x \in \{0, 1, 2, 3, 4\}$.

Then function values $y_i = x_i^4$: $y_0 = 0$, $y_1 = 1$, $y_2 = 16$, $y_3 = 81$, $y_4 = 256$

Simpson's 1/3rd Rule: $I_s = \frac{h}{3} [y_0 + y_4 + 4(y_1 + y_3) + 2y_2]$

$$I_s = \frac{1}{3} [0 + 256 + 4(1 + 81) + 2 \times 16] = \frac{616}{3} \approx 205.33$$

Trapezoidal Rule: $I_t = \frac{h}{2} [y_0 + y_4 + 2(y_1 + y_2 + y_3)] \Rightarrow I_t = \frac{1}{2} [0 + 256 + 2(1 + 16 + 81)] = 226$



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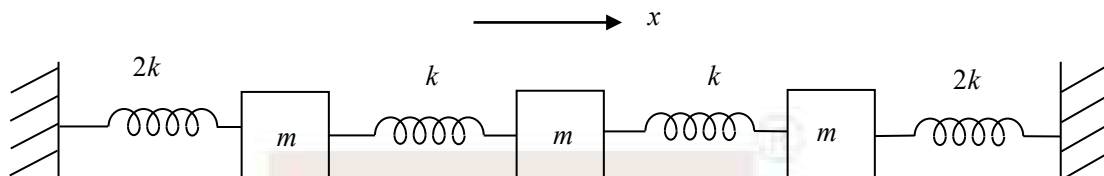
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PART B

Q21. Three equal masses m are free to move along x -axis on a frictionless surface. The masses are coupled together, and to a rigid wall, by springs as shown in the figure. If $\omega_0 = \sqrt{\frac{k}{m}}$, and the angular frequencies of the three normal modes of this system are ω_1, ω_2 and ω_3 then the value of $\omega_1^2 + \omega_2^2 + \omega_3^2$ is:



- (a) $9\omega_0^2$ (b) $6\omega_0^2$
(c) $10\omega_0^2$ (d) $8\omega_0^2$

Ans.: (d)

Solution.: Let x_1, x_2, x_3 be the displacements of the three masses m from their equilibrium positions.

$$\text{Kinetic Energy } T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}m\dot{x}_3^2$$

$$\text{The mass matrix } M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$$

$$\text{Potential Energy } V = \frac{1}{2}(2k)x_1^2 + \frac{1}{2}k(x_2^2 - x_1^2) + \frac{1}{2}k(x_3^2 - x_2^2) + \frac{1}{2}(2k)x_3^2$$

$$\Rightarrow V = \frac{1}{2}(3kx_1^2 + 2kx_2^2 + 3kx_3^2 - 2kx_1x_2 - 2kx_2x_3)$$

$$\text{The potential energy matrix } M = \begin{bmatrix} 3k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 3k \end{bmatrix}$$

The normal mode frequencies $\omega_1, \omega_2, \omega_3$ are the eigenvalues of $M^{-1}K$.

$$M^{-1}K = \frac{k}{m} \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} = \omega_0^2 \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

The sum of the squared normal frequencies equals the trace of the matrix $M^{-1}K$:

$$\omega_1^2 + \omega_2^2 + \omega_3^2 = \text{Tr}(M^{-1}K) = \omega_0^2(3 + 2 + 3) = 8\omega_0^2$$

Q22. A bead of mass M slides along a smooth frictionless and massless wire which is bent in the shape of a parabola $z = cr^2$ in cylindrical coordinates (r, ϕ, z) . Here c is a positive constant. The wire is made to rotate about z -axis at constant angular velocity. Which of the following statements about this system of the wire and the head is correct?

- (a) Energy is conserved but angular momentum is not conserved
- (b) Energy is not conserved but angular momentum is conserved
- (c) Both energy and angular momentum are conserved
- (d) Both energy and angular momentum are not conserved

Ans.: (d)

Solution.:

Angular Momentum:

The wire forces the bead to rotate about the z -axis at a constant angular velocity ω , applying an external torque. Thus, the angular momentum L_z is **not conserved**.

Energy:

Because the constraint depends explicitly on time through the forced rotation ($\dot{\phi} = \omega$), the system is non-conservative in the lab frame. Mechanical energy is **not conserved** (work is constantly done by the external force rotating the wire).

Q43. A particle moves in an orbit $r = k \exp(\alpha\theta)$ under a central force $F(r)$. Here, k and α are constants and $k > 0$. Then $F(r)$ is proportional to:

- (a) $\frac{1}{r}$
- (b) $\frac{1}{r^2}$
- (c) $\frac{1}{r^3}$
- (d) $\frac{1}{r^4}$

Ans.: (c)

Solution.: Using Binet's formula for central force motion: $F(r) = -\frac{L^2 u^2}{m} \left(\frac{d^2 u}{d\theta^2} + u \right)$

$$\because r = k \exp(\alpha\theta) \Rightarrow u = \frac{1}{r} = \frac{1}{k} e^{-\alpha\theta} \Rightarrow \frac{du}{d\theta} = -\frac{\alpha}{k} e^{-\alpha\theta} \Rightarrow \frac{d^2 u}{d\theta^2} = \frac{\alpha^2}{k} e^{-\alpha\theta} = \alpha^2 u$$

$$\Rightarrow F(r) = -\frac{L^2 u^2}{m} (\alpha^2 u + u) = -\frac{L^2 (\alpha^2 + 1)}{m} u^3 \Rightarrow F(r) \propto u^3 \propto \frac{1}{r^3}$$

Q44. The Lagrangian of a system is given by $L = x^2 + x\dot{x} + \frac{\dot{x}^2}{2}$. If p is the generalized momentum, then the corresponding Hamiltonian is:

(a) $\frac{(p^2 - x^2 - 2xp)}{2}$

(b) $\frac{(p^2 - x^2 + 2xp)}{2}$

(c) $\frac{(p^2 + x^2 + 2xp)}{2}$

(d) $\frac{(p^2 + x^2 - 2xp)}{2}$

Ans.: (a)

Solution.: Generalized Momentum $p = \frac{\partial L}{\partial \dot{x}} = x + \dot{x} \Rightarrow \dot{x} = p - x \quad \because L = x^2 + x\dot{x} + \frac{\dot{x}^2}{2}$

$$\text{Hamiltonian } H = p\dot{x} - L = p(p - x) - \left(x^2 + x(p - x) + \frac{(p - x)^2}{2} \right)$$

$$\Rightarrow H = p^2 - px - \left(x^2 + px - x^2 + \frac{p^2 + x^2 - 2px}{2} \right)$$

$$\Rightarrow H = p^2 - px - px - \frac{p^2 + x^2 - 2px}{2} = \frac{p^2 - x^2 + 2px}{2}$$

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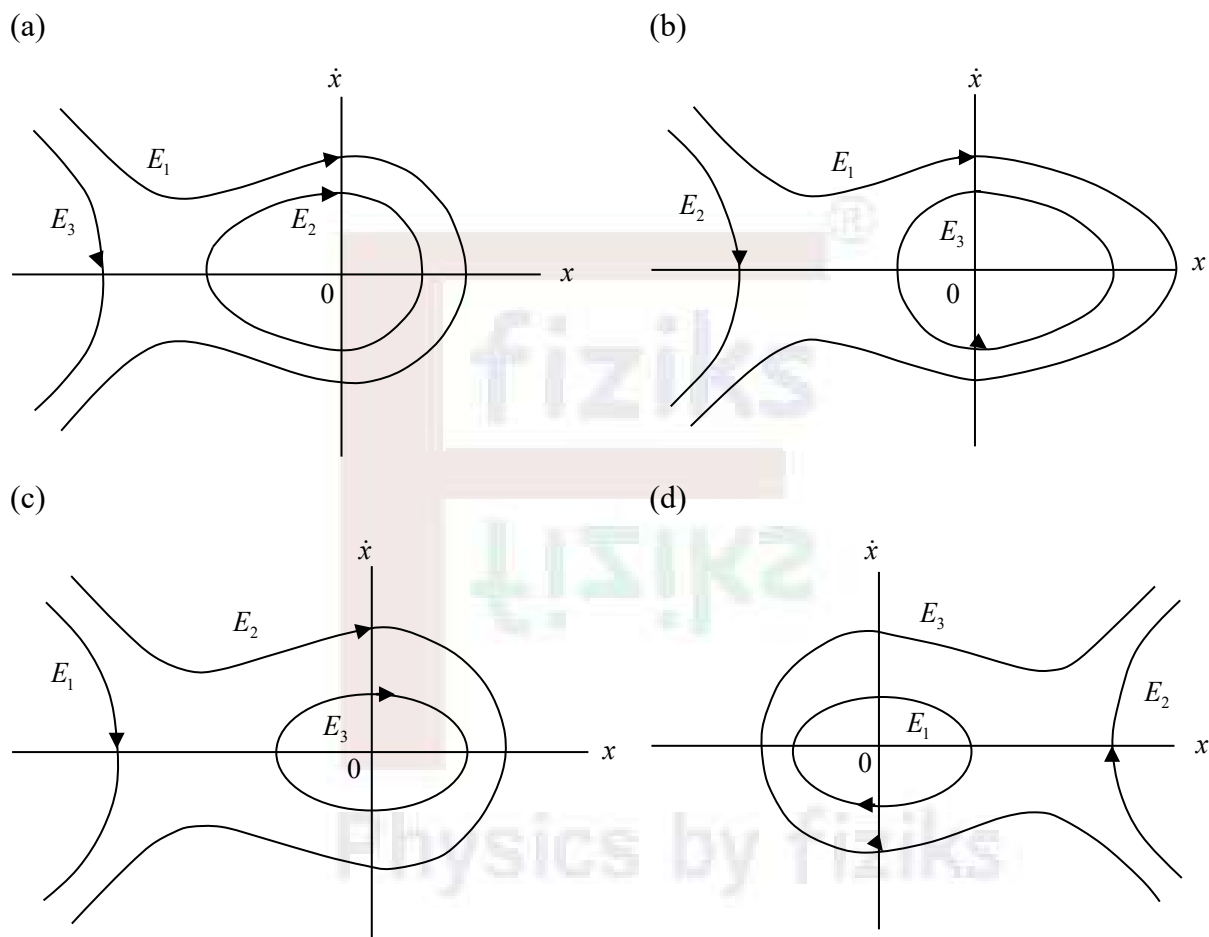
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PART C

Q50. A particle is moving in one dimension in the potential

$$V(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3$$

The schematic phase-space diagrams for the total energies $E_1 > E_2 > E_3$ are best represented by:



Ans.: (a) and (b)

Solution.: $\because V(x) = \frac{1}{2}x^2 + \frac{1}{3}x^3$

For Equilibrium Points: $\frac{dV(x)}{dx} = x + x^2 = 0 \Rightarrow x(1+x) = 0 \Rightarrow x = 0, x = -1$

Local Extremes: $\frac{d^2V(x)}{dx^2} = 1 + 2x$

$\left. \frac{d^2V(x)}{dx^2} \right|_{x=0} = 1 > 0$, **Local Minimum** (stable point, forms closed bounded orbits)

$\left. \frac{d^2V(x)}{dx^2} \right|_{x=-1} = -1 < 0$, **Local Maximum** (unstable point / saddle point)

Behavior of Energies ($E_1 > E_2 > E_3$)

- For low energy (E_3), the motion is trapped in the potential well around the stable equilibrium point $x = 0$, forming a closed, elliptical orbit centered at $x = 0$.
- As energy increases, the trajectories expand towards the left side (unbound region beyond $x = -1$).
- The stable closed loops must be located around the origin ($x = 0$) on the right side of the barrier at $x = -1$.

Identify Correct Trajectory Orientation

In phase space $(x, \dot{x})$:

- When $\dot{x} > 0$ (top half of plot), $\dot{x} = \frac{dx}{dt} > 0$. Thus x increases (motion is to the **right**)
- When $\dot{x} < 0$ (bottom half of plot), $\dot{x} = \frac{dx}{dt} < 0$. Thus x decreases (motion is to the **left**)

Q51. A dynamical system is described by a differential equation $\ddot{x} - \dot{x} + x^2 - 2x = 0$. The nature of the fixed point at the origin is:

- (a) unstable spiral (b) stable node
(c) unstable hyperbola (d) stable spiral

Ans.: (c)

Solution.: Given $\ddot{x} - \dot{x} + x^2 - 2x = 0$.

Let $x_1 = x$ and $x_2 = \dot{x}$. The non-linear system is: $\dot{x}_1 = x_2$, $\dot{x}_2 = \dot{x} - x^2 + 2x = x_2 - x_1^2 + 2x_1$

Linearize Around the Origin $(0,0)$:

Evaluating the Jacobian matrix J at $(x_1, x_2) = (0, 0)$

$$J = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix}_{(0,0)} = \begin{bmatrix} 0 & 1 \\ 2-2x_1 & 1 \end{bmatrix}_{(0,0)} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}_{(0,0)}$$

For Eigenvalues $|J - \lambda I| = 0 \Rightarrow \begin{bmatrix} -\lambda & 1 \\ 2 & 1 - \lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 - \lambda - 2 = 0 \Rightarrow \lambda_1 = 2, \lambda_2 = -1$

Since the eigenvalues are real, non-zero, and have **opposite signs**, the origin is a **Saddle Point** (unstable hyperbolic point)

Q52. For a one-dimensional system the Hamiltonian is given by $H = \frac{p^2}{2} - \frac{1}{2q^2}$. If a constant of motion of the system is $\left(apq + bp^2t + \frac{ct}{q^2}\right)$, where a, b and c are constants and t denotes the time, then which of the following options is correct:

- (a) $a = b = c$ (b) $a = -b = c$
(c) $a = -b = -c$ (d) $a = b = -c$

Ans.: (b)

Solution.: $H = \frac{p^2}{2} - \frac{1}{2q^2}$ and $I = apq + bp^2t + \frac{ct}{q^2}$

For I to be a constant of motion, its total time derivative must vanish

$$\frac{dI}{dt} = \frac{\partial I}{\partial t} + \{I, H\} = 0.$$

Now, $\frac{\partial I}{\partial t} = bp^2 + \frac{c}{q^2}$ and Hamilton's Equations: $\dot{q} = \frac{\partial H}{\partial p} = p$, $\dot{p} = -\frac{\partial H}{\partial q} = -\frac{1}{q^3}$

Thus $\frac{dI}{dt} = \left(bp^2 + \frac{c}{q^2}\right) + \frac{\partial I}{\partial q}\dot{q} + \frac{\partial I}{\partial p}\dot{p} = 0$

$$\Rightarrow \left(bp^2 + \frac{c}{q^2}\right) + \left(ap - \frac{2ct}{q^3}\right)p + (aq + 2bpt)\left(-\frac{1}{q^3}\right) = 0$$

$$\Rightarrow p^2(b+a) + \frac{1}{q^2}(c-a) + \frac{pt}{q^3}(-2c-2b) = 0 \Rightarrow b = -a, c = a, c = -b \Rightarrow a = -b = c$$

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Solution-Electromagnetic Theory

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PART B

Q24. A particle with charge q , which is confined to move in the xy -plane, is subjected to a magnetic field with magnitude B pointing in the $+z$ direction. The particle moves in a circle (C) of radius R . If $\vec{p}$ is the canonical momentum then the following integral $\oint_C \vec{p} \cdot d\vec{l}$ has the magnitude: ($d\vec{l}$ is in the direction of motion)

(a) $qB\pi R^2$

(b) $2qB\pi R^2$

(c) $3qB\pi R^2$

(d) $4qB\pi R^2$

Ans.: (a)

Solution.: Canonical momentum is $\vec{p} = \vec{p}_{mech} + q\vec{A}$ thus $\oint_C \vec{p} \cdot d\vec{l} = \oint_C \vec{p}_{mech} \cdot d\vec{l} + q\oint_C \vec{A} \cdot d\vec{l}$

$$\Rightarrow \oint_C \vec{p} \cdot d\vec{l} = m\oint_C \vec{v} \cdot d\vec{l} + q\oint_C \vec{A} \cdot d\vec{l}$$

Since the magnetic force acts as a centripetal force pointing towards the center, the velocity $\vec{v}$ must circulate clockwise when viewed from above ($+z$) to experience an inward force $\vec{F} = q(\vec{v} \times B\hat{z})$.

The magnetic force provides the required centripetal force: $\frac{mv^2}{R} = -qvB \Rightarrow mv = -qBR$

$$\Rightarrow m\oint_C \vec{v} \cdot d\vec{l} = mv\oint_C dl = (-qBR)(2\pi R) = -2\pi qBR^2$$

$$\text{and } q\oint_C \vec{A} \cdot d\vec{l} = q\int_s (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} = q\int_s \vec{B} \cdot d\vec{a} = qB \times \pi R^2 = \pi qBR^2$$

$$\Rightarrow \oint_C \vec{p} \cdot d\vec{l} = m\oint_C \vec{v} \cdot d\vec{l} + q\oint_C \vec{A} \cdot d\vec{l} = -2\pi qBR^2 + \pi qBR^2 = -\pi qBR^2 \Rightarrow \left| \oint_C \vec{p} \cdot d\vec{l} \right| = \pi qBR^2$$

Q26. Consider the Poisson's equation $\nabla^2 V = -\frac{q}{\epsilon_0} \delta^3(\vec{r} - \vec{a})$ in half-space $z > 0$ with boundary

condition $V = 0$ in the $z = 0$ plane. Here, $\vec{a} = a_0 \hat{k}$ with $a_0 > 0$. If $\frac{q}{4\pi\epsilon_0 a_0} = 24$ volt, then

which of the following is correct?

(a) At $\vec{r} = (0, 0, 2a_0)$, $V = 16$ Volt

(b) At $\vec{r} = (0, 0, 3a_0)$, $V = 8$ Volt

(c) At $\vec{r} = (0, 0, 4a_0)$, $V = 4$ Volt

(d) At $\vec{r} = (0, 0, a_0/2)$, $V = 32$ Volt

Ans.: (a) and (d)

Solution.: This is the method of images. The boundary condition $V = 0$ in the $z = 0$ plane is satisfied by placing an image charge $-q$ at $z = a_0$. The potential at a point $(0, 0, z)$ on the z -axis is:

$$\text{is: } V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{|z - a_0|} - \frac{1}{|z + a_0|} \right]$$

$$(a) \text{ At } \vec{r} = (0, 0, 2a_0) \Rightarrow V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{a_0} - \frac{1}{3a_0} \right] = 24 \times \frac{2}{3} = 16 \text{ Volts}$$

$$(b) \text{ At } \vec{r} = (0, 0, 3a_0) \Rightarrow V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{2a_0} - \frac{1}{4a_0} \right] = 24 \times \frac{1}{4} = 6 \text{ Volts}$$

$$(c) \text{ At } \vec{r} = (0, 0, 4a_0) \Rightarrow V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{3a_0} - \frac{1}{5a_0} \right] = 24 \times \frac{2}{15} = 3.2 \text{ Volts}$$

$$(d) \text{ At } \vec{r} = (0, 0, a_0/2) \Rightarrow V(z) = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{a_0/2} - \frac{1}{3a_0/2} \right] = 24 \times \frac{4}{3} = 32 \text{ Volts}$$

Q39. A linearly polarized electromagnetic wave is incident normally on a material of refractive index n_R for right circular polarization and n_L for left circular polarization, where $n_R \neq n_L$. The reflected wave's polarization is:

- (a) Linear
- (b) Elliptical
- (c) Circular
- (d) Random

Ans.: (b)

Solution.: A linearly polarized wave can be represented as the superposition of equal-amplitude right- and left-circularly polarized waves:

$$\text{Linear} = \text{RCP} + \text{LCP}.$$

Because $n_R \neq n_L$ the reflection coefficients for the two circular components are different.

Therefore, after reflection, the two circular components have **different amplitudes**.

When two oppositely rotating circular components have unequal amplitudes, their superposition is generally **elliptically polarized**.

PART C

Q58. Transverse electric modes TE_{mn} propagate in a hollow straight infinite metallic waveguide having rectangular cross section of sides 3 cm and 2 cm . The frequency (in GHz) of the lowest degenerate mode is:

(Given: speed of light = $3 \times 10^8 \text{ m/s}$)

- (a) 5.0 (b) 7.5
(c) 9.0 (d) 15.0

Ans.: (d)

Solution.:

The cutoff frequency for TE_{mn} is $f_{mn} = \frac{c}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$

Degenerate modes occur when two distinct mode sets (m, n) gives the exact same cutoff frequency. For rectangular waveguides, degeneracy happens when:

$$\frac{m_1^2}{a^2} + \frac{n_1^2}{b^2} = \frac{m_2^2}{a^2} + \frac{n_2^2}{b^2} \Rightarrow \frac{m_1^2}{9} + \frac{n_1^2}{4} = \frac{m_2^2}{9} + \frac{n_2^2}{4} \text{ since } a = 3\text{ cm and } b = 2\text{ cm}$$
$$\Rightarrow 4m_1^2 + 9n_1^2 = 4m_2^2 + 9n_2^2$$

Lowest Degenerate Combination:

The lowest integer combination satisfying this is $(m_1, n_1) = (3, 0)$ and $(m_2, n_2) = (0, 2)$

$$\Rightarrow 4(3)^2 + 9(0)^2 = 4(0)^2 + 9(2)^2 = 36$$

For TE modes, TE_{30} and TE_{02} are both valid modes.

$$f = \frac{(3 \times 10^8)}{2} \sqrt{\frac{(3)^2}{(0.03)^2} + 0} = \frac{(3 \times 10^8)}{2} \times 100 = 15.0 \text{ GHz}$$

Q60. An inertial observer S finds the electric field due to a large rectangular parallel plate capacitor to be directed along the y -axis. For another observer, moving at a relativistic speed with respect to S along the positive x -axis, which of the following statements is correct for the electric field:

- (a) Direction remains same but magnitude increases
(b) Direction remains same but magnitude decreases
(c) Direction and magnitude both remain the same
(d) Direction tilts towards x -axis but the magnitude remains same

Ans.: (a)

Solution.: In rest frame S , the electric field is perpendicular to motion (x -axis):

$$E_x = 0, E_y = E_0, E_z = 0$$

Lorentz Transformation for Electric Field:

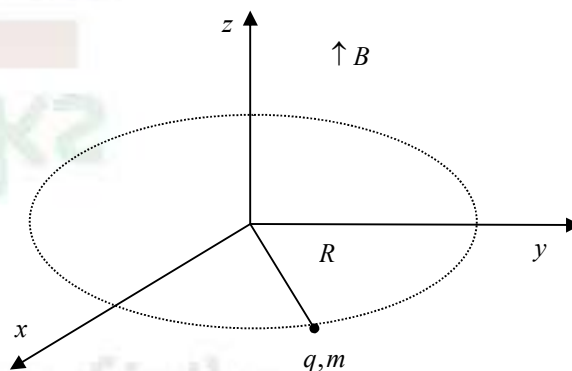
Moving along $+x$ with velocity v : (where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$)

$$E'_x = E_x = 0, \quad E'_y = \gamma(E_y - vB_z) = \gamma E_0, \quad E'_z = \gamma(E_z + vB_y) = 0$$

The transverse electric field component scales by $\gamma > 1$. Therefore, the direction remains along the y -axis, but its magnitude increases to γE_0 .

Q64. A particle of mass m and charge q is fixed at one end of a rigid insulating rod of length R . The other end of the rod is pivoted such that it can rotate about the z -axis in the xy -plane as shown in the figure. A time dependent magnetic field $\vec{B} = \beta t \hat{z}$ is turned on at $t = 0$. Here β is a constant. Then the force in the rod at time t will be:

- (a) Compressive and of magnitude $q^2 \beta^2 t^2 R/m$
- (b) Zero
- (c) Compressive and of magnitude $q^2 \beta^2 t^2 R/(4m)$
- (d) Tensile and of magnitude $q^2 \beta^2 t^2 R/(4m)$



Ans.: (c)

Solution.: We first find the induced electric field using Faraday's law.

$$\text{For a circular path of radius } R: \oint \vec{E} d\vec{l} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{E} d\vec{l} = -\frac{d\phi_B}{dt} \Rightarrow E(2\pi R) = -\frac{d}{dt}[\beta t \times \pi R^2] \Rightarrow |E| = \frac{\beta R}{2}$$

$$\text{The tangential force on the charge is } F_t = qE = \frac{q\beta R}{2}.$$

$$\text{Thus, the tangential acceleration is } a_t = \frac{F_t}{m} = \frac{q\beta R}{2m}$$

$$\text{So, the angular acceleration is } \alpha = \frac{a_t}{R} = \frac{q\beta}{2m}$$

$$\text{Starting from rest, } \omega(t) = \alpha t = \frac{q\beta}{2m} t \Rightarrow v = R\omega = \frac{q\beta R}{2m} t$$

The magnetic force is radial: $F_B = qvB = q\left(\frac{q\beta R}{2m}t\right)\beta t \Rightarrow F_B = \frac{q^2\beta^2 t^2 R}{2m}$

But the required centripetal force is $F_C = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{q\beta R}{2m}t\right)^2 = \frac{q^2\beta^2 t^2 R}{4m}$

The magnetic force is twice the required centripetal force: $F_B = 2F_C$

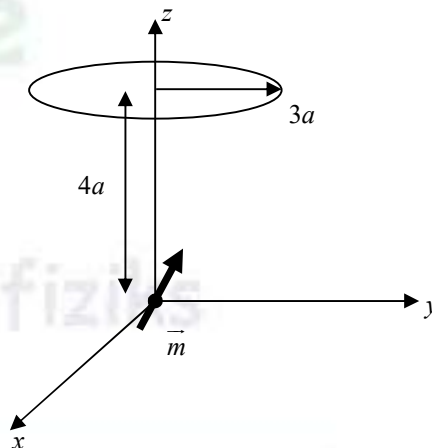
Therefore, the rod must provide an outward force of magnitude

$$F_{rod} = F_B - F_C = 2F_C - F_C = F_C \Rightarrow F_{rod} = \frac{q^2\beta^2 t^2 R}{4m}$$

The rod has to **pull** the particle outward, so the rod is under **tension**.

Q67. A magnetic dipole $\vec{m}$ is kept at the origin with its direction along $+z$ -axis. At time $t = 0$ it starts spinning about x -axis with angular speed ω . A wire loop of radius $3a$ is kept with its plane parallel to xy -plane and centered at $z = 4a$ as shown in the figure. The emf E induced in the loop for $t > 0$ is: (Note that the magnetic flux through the given loop will be the same as through an appropriate section of a spherical shell of radius $5a$ with its center at the origin.)

- (a) $\left(\frac{9\mu_0 m\omega}{500a}\right)\sin\omega t$
- (b) $\left(\frac{9\mu_0 m\omega}{500a}\right)\cos\omega t$
- (c) $\left(\frac{9\mu_0 m\omega}{250a}\right)\sin\omega t$
- (d) $\left(\frac{9\mu_0 m\omega}{250a}\right)\cos\omega t$



Ans.: (c)

Solution.:

Dipole Rotation:

The magnetic dipole starts at $+z$ and rotates in the yz -plane about the x -axis with angular frequency ω : $\vec{m}(t) = (m \sin \omega t) \hat{y} + (m \cos \omega t) \hat{z} = m_y \hat{y} + m_z \hat{z}$

Magnetic Flux calculation using Equivalent Spherical Cap:

The flux through the planar circular loop ($r = 3a, z = 4a$) equals the flux through a spherical cap of radius $R = \sqrt{(3a)^2 + (4a)^2} = 5a$ centered at origin. So, the loop corresponds to a circular section of a sphere of radius $5a$.

For a magnetic dipole at the origin field is:

$$\vec{B} = \frac{\mu_0}{4\pi r^3} [3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}]$$

On the spherical surface, the radial component is

$$B_r = \vec{B} \cdot \hat{r} = \frac{\mu_0}{4\pi r^3} [3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}] \cdot \hat{r} = \frac{\mu_0}{4\pi r^3} [2(\vec{m} \cdot \hat{r})]$$

$$2(\vec{m} \cdot \hat{r}) = 2(m_y \hat{y} \cdot \hat{r} + m_z \hat{z} \cdot \hat{r}) = 2m_y \sin \theta \sin \phi + 2m_z \cos \theta$$

$$\Rightarrow B_r = \frac{\mu_0}{4\pi r^3} [2m_y \sin \theta \sin \phi + 2m_z \cos \theta]$$

Integrating over the cap: polar angle $\theta = 0$ to $\theta = \theta_0$ where $\sin \theta_0 = \frac{3}{5}$, $\cos \theta_0 = \frac{4}{5}$.

(i) The y -component contribution integrates to zero due to azimuthal symmetry over $[0, 2\pi]$.

(ii) Only the z -component $m_z = m \cos \omega t$ contributes to the flux:

$$\phi(t) = \int_0^{2\pi} \int_0^{\theta_0} \left(\frac{\mu_0}{4\pi} \frac{2m_z \cos \theta}{r^3} \right) (r^2 \sin \theta d\theta d\phi) = \int_0^{2\pi} \int_0^{\theta_0} \left(\frac{\mu_0}{4\pi} \frac{2m \cos(\omega t) \cos \theta}{5a} \right) (\sin \theta d\theta d\phi)$$

$$\Rightarrow \phi(t) = \frac{\mu_0 m \cos(\omega t)}{5a} \frac{\sin^2 \theta_0}{2} = \frac{\mu_0 m \cos(\omega t)}{10a} \sin^2 \theta_0 = \frac{\mu_0 m \cos(\omega t)}{10a} \left(\frac{3}{5} \right)^2 \quad \because \sin \theta_0 = \frac{3}{5}$$

$$\Rightarrow \phi(t) = \frac{9\mu_0 m \cos(\omega t)}{250a}$$

$$\text{Induced EMF: } \varepsilon = -\frac{d\phi(t)}{dt} = -\frac{d}{dt} \left[\frac{9\mu_0 m \cos(\omega t)}{250a} \right] = \left(\frac{9\mu_0 \omega m}{250a} \right) \sin(\omega t)$$

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PART B

Q25. A particle is in the ground state of a one-dimensional infinite well potential with walls at $x = 0$ and at $x = a$. The wall at $x = a$ is moved to $x = 2a$ in two separate, sudden and adiabatic, protocols so that $\langle E \rangle_s$ and $\langle E \rangle_a$ are the respective expectation values of the energy in the final states. Then the ratio $\frac{\langle E \rangle_s}{\langle E \rangle_a}$ is:

- (a) 4:1 (b) 2:1 (c) 1:1 (d) 1:4

Ans.: (a)

Solution.:

Sudden Expansion: The wavefunction remains instantaneously unchanged, so the expectation value of energy is equal to its initial ground state energy: $\langle E \rangle_s = \frac{\pi^2 \hbar^2}{2ma^2}$

Adiabatic Expansion:

The particle remains in the ground state ($n = 1$) of the expanded box ($2a$):

$$\langle E \rangle_a = \frac{\pi^2 \hbar^2}{2m(2a)^2} = \frac{1}{4} \frac{\pi^2 \hbar^2}{2ma^2}. \quad \text{Thus } \frac{\langle E \rangle_s}{\langle E \rangle_a} = 4.$$

Q27. Consider the spin matrix S_r projected along the radial direction in spherical polar

coordinates (r, θ, ϕ) . The probability that a measurement of S_r on the state $|\beta\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}$

will yield the value $\frac{\hbar}{2}$ is:

- (a) $\sin^2 \theta$ (b) $\cos^2 \theta$ (c) $\frac{1}{2}$ (d) $\cos^2 \frac{\theta}{2}$

Ans.: (a)

Solution.: Given spin state $|\psi\rangle = |\beta\rangle = \begin{pmatrix} \sin \frac{\theta}{2} \\ e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix}$

Eigenstate of $\hat{S}_r$ for $+\frac{\hbar}{2}$: For a radial direction $\hat{r} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, the spin operator along radial direction is

$$\hat{S}_r = +\frac{\hbar}{2} \sigma \cdot \hat{r}$$

Its normalized eigenstate corresponding to $+\frac{\hbar}{2}$ is

$$|+\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}$$

So, the required probability is

$$P = |\langle + | \beta \rangle|^2 = \left| \cos \frac{\theta}{2} \sin \frac{\theta}{2} + \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right|^2 = \left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)^2 = \sin^2 \theta$$

Q29. A three-state system is described by a certain Hamiltonian having eigen energies $E_0, 2E_0$

and $3E_0$ with the corresponding eigen states $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, respectively. If the state of the

system at time $t = 0$ is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, then what is the probability of finding the system in the same state

at time $t = \frac{\pi\hbar}{2E_0}$?

- (a) 1 (b) $\frac{1}{3}$ (c) $\frac{5}{18}$ (d) $\frac{2}{9}$

Ans.: (c)

Solution.:

Normalized eigenstates: $|1\rangle = \frac{1}{\sqrt{3}}(1,1,1)^T$, $|2\rangle = \frac{1}{\sqrt{2}}(1,0,-1)^T$, $|3\rangle = \frac{1}{\sqrt{6}}(1,-2,1)^T$

Initial state: $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = c_1|1\rangle + c_2|2\rangle + c_3|3\rangle$

$$c_1 = \langle 1 | \psi(0) \rangle = \frac{1}{\sqrt{3}}, \quad c_2 = \langle 2 | \psi(0) \rangle = \frac{1}{\sqrt{2}}, \quad c_3 = \langle 3 | \psi(0) \rangle = \frac{1}{\sqrt{6}}$$

Time evolution: $|\psi(t)\rangle = c_1 e^{-iE_0 t/\hbar} |1\rangle + c_2 e^{-i(2E_0)t/\hbar} |2\rangle + c_3 e^{-i(3E_0)t/\hbar} |3\rangle$

At time $t = \frac{\pi\hbar}{2E_0}$: $|\psi(t)\rangle = c_1 e^{-i\pi/2} |1\rangle + c_2 e^{-i\pi} |2\rangle + c_3 e^{-i3\pi/2} |3\rangle$

$$\Rightarrow |\psi(t)\rangle = c_1 (-i) |1\rangle + c_2 (-1) |2\rangle + c_3 (i) |3\rangle$$

Probability to be in $|\psi(0)\rangle$ is $|\langle\psi(0)|\psi(t)\rangle|^2$.

$$\langle\psi(0)|\psi(t)\rangle = c_1^*(-i)c_1 + c_2^*(-1)c_2 + c_3^*(i)c_3 = -i|c_1|^2 - |c_2|^2 + i|c_3|^2$$

$$\Rightarrow \langle\psi(0)|\psi(t)\rangle = -i\frac{1}{3} - \frac{1}{2} + i\frac{1}{6} = -\frac{1}{2} - \frac{1}{6}i \Rightarrow |\langle\psi(0)|\psi(t)\rangle|^2 = \frac{1}{4} + \frac{1}{36} = \frac{5}{18}$$

Q30. Consider two operators $\hat{R}$ and $\hat{S}$ where $\hat{R}$ commutes with $[\hat{R}, \hat{S}]$. If α is a constant, then

$[e^{\alpha\hat{R}}, \hat{S}]$ is:

- (a) $[\hat{R}^\alpha, \hat{S}]$ (b) $\alpha e^{\alpha\hat{R}}[\hat{R}, \hat{S}]$ (c) 0 (d) $\alpha e^{\hat{R}}[e^\alpha \hat{R}, \hat{S}]$

Ans.: (b)

Solution.: Given $[\hat{R}, [\hat{R}, \hat{S}]] = 0$ and $e^{\alpha\hat{R}} = 1 + \alpha\hat{R} + \frac{\alpha^2\hat{R}^2}{2!} + \frac{\alpha^3\hat{R}^3}{3!} + \dots$

$$[e^{\alpha\hat{R}}, \hat{S}] = \alpha[\hat{R}, \hat{S}] + \frac{\alpha^2}{2!}[\hat{R}^2, \hat{S}] + \frac{\alpha^3}{3!}[\hat{R}^3, \hat{S}] + \dots$$

$$[\hat{R}^2, \hat{S}] = \hat{R}[\hat{R}, \hat{S}] + [\hat{R}, \hat{S}]\hat{R} = 2\hat{R}[\hat{R}, \hat{S}] \quad \because [\hat{R}, [\hat{R}, \hat{S}]] = 0$$

$$[\hat{R}^3, \hat{S}] = 3\hat{R}^2[\hat{R}, \hat{S}], \dots \dots [\hat{R}^n, \hat{S}] = n\hat{R}^{n-1}[\hat{R}, \hat{S}]$$

$$[e^{\alpha\hat{R}}, \hat{S}] = \left(\alpha + \alpha^2\hat{R} + \frac{\alpha^3}{2!}\hat{R}^2 + \dots \right) [\hat{R}, \hat{S}] = \alpha e^{\alpha\hat{R}}[\hat{R}, \hat{S}]$$

Note: Operator identity $[f(\hat{R}), \hat{S}] = f'(\hat{R})[\hat{R}, \hat{S}]$ if $[\hat{R}, [\hat{R}, \hat{S}]] = 0$

Q31. You are given a time-independent observable O for a system with time independent Hamiltonian H and a general time dependent state $|\phi(t)\rangle$. If the expectation value $\langle O(t) \rangle_\phi = \langle \phi(t) | O | \phi(t) \rangle$, then consider the following statements (A) to (D) and choose the correct option.

- (A) If O commutes with H then $\langle O(t) \rangle_\phi$ is independent of time
 (B) If $|\phi(t)\rangle$ is an eigen state of H then $\langle O(t) \rangle_\phi$ is independent of time
 (C) If $|\phi(t)\rangle$ is an eigen state of O then $\langle O(t) \rangle_\phi$ is independent of time
 (D) If $|\phi(t)\rangle$ is an eigen state of $[O, H]$ then $\langle O(t) \rangle_\phi$ is independent of time
 (a) Only (A) and (B) are correct (b) Only (A) is correct
 (c) Only (B), (C) and (D) are correct (d) Only (A), (B) and (C) are correct

Ans.: (d)

Solution.: For an operator O with no explicit time dependence, Ehrenfest's theorem gives

$$\frac{d\langle O \rangle}{dt} = \frac{i}{\hbar} \langle [H, O] \rangle \quad \because \frac{\partial \langle O \rangle}{\partial t} = 0$$

Therefore, if $[H, O] = 0$ then $\frac{d\langle O \rangle}{dt} = 0$. So the expectation value is conserved.

(A) If O commutes with H then $\langle O(t) \rangle_\phi$ is independent of time

$$\because [O, H] = 0 \Rightarrow [H, O] = 0 \quad \because [H, O] = -[O, H]$$

$$\Rightarrow \frac{d\langle O \rangle}{dt} = 0 \Rightarrow \langle O \rangle = \text{constant}. \text{ Therefore, (A) is CORRECT.}$$

(B) If $|\phi(t)\rangle$ is an eigen state of H then $\langle O(t) \rangle_\phi$ is independent of time

$$\text{Suppose } H|\phi\rangle = E|\phi\rangle$$

Since H is time-independent, an energy eigenstate evolves only by an overall phase:

$$|\phi(t)\rangle = e^{-iEt/\hbar} |\phi(0)\rangle.$$

$$\text{Thus } \langle O(t) \rangle_\phi = \langle \phi(t) | O | \phi(t) \rangle = e^{+iEt/\hbar} \langle \phi(0) | O | e^{-iEt/\hbar} \phi(0) \rangle = \langle \phi(0) | O | \phi(0) \rangle$$

Thus, it does not depend on time. Therefore, **(B) is CORRECT.**

(C) If $|\phi(t)\rangle$ is an eigen state of O then $\langle O(t) \rangle_\phi$ is independent of time

$$\text{Suppose } O|\phi\rangle = o|\phi\rangle; \text{ then } \langle O(t) \rangle_\phi = \langle \phi(t) | O | \phi(t) \rangle = o \langle \phi(t) | \phi(t) \rangle = o \quad \because \langle \phi(t) | \phi(t) \rangle = 1$$

The eigenvalue o is constant, so the expectation value is time-independent.

Therefore, **(C) is CORRECT.**

(D) If $|\phi(t)\rangle$ is an eigen state of $[O, H]$ then $\langle O(t) \rangle_\phi$ is independent of time

$$\text{Suppose } [O, H]|\phi\rangle = c|\phi\rangle \text{ then } \langle [O, H] \rangle = \langle \phi(t) | c | \phi(t) \rangle = c \langle \phi(t) | \phi(t) \rangle = c \Rightarrow \langle [H, O] \rangle = -c$$

$$\because \frac{d\langle O \rangle}{dt} = \frac{i}{\hbar} \langle [H, O] \rangle \Rightarrow \frac{d\langle O \rangle}{dt} = -\frac{i}{\hbar} c$$

There is no reason for c to be zero merely because the state is an eigenstate of $[O, H]$.

Hence $\langle O(t) \rangle_\phi$ need not be constant. Therefore, **(D) is INCORRECT.**

Q33. The eigen energies and the normalized eigenfunctions in the n^{th} state of a particle, confined in a one-dimensional infinite potential well, are denoted by E_n and ψ_n , respectively. If $E_1 = 1.2 \text{ eV}$ then the expectation value of energy in the state $\psi = 3\psi_1 + 2\psi_2 + \sqrt{3}\psi_4$ is close to:

- (a) 5.5 eV (b) 5.0 eV
(c) 4.5 eV (d) 6.0 eV

Ans.: (a)

Solution.:

Energy levels: $E_n = n^2 E_1$; $E_1 = 1.2 \text{ eV}$, $E_2 = 4 \times 1.2 \text{ eV} = 4.8 \text{ eV}$, $E_3 = 16 \times 1.2 \text{ eV} = 19.2 \text{ eV}$

Given state: $\psi = 3\psi_1 + 2\psi_2 + \sqrt{3}\psi_4$; Norm squared: $N = 3^2 + 2^2 + (\sqrt{3})^2 = 16$

Expectation Value: $\langle E \rangle = \frac{9}{16} E_1 + \frac{4}{16} E_2 + \frac{3}{16} E_4 = \frac{9}{16} (1.2) + \frac{4}{16} (4.8) + \frac{3}{16} (19.2)$

$$\Rightarrow \langle E \rangle = \frac{10.8 + 19.2 + 57.6}{16} = \frac{87.6}{16} = 5.475 \text{ eV} \approx 5.5 \text{ eV}$$

PART C

Q48. The Hamiltonian of a particle of mass m , moving in a circle of a fixed radius R , is given

by $H_0 = \frac{L_z^2}{2mR^2}$, where $L_z = -i\hbar \frac{d}{d\phi}$ and ϕ is the azimuthal angle. This particle is perturbed by

$H' = V \cos(2\phi)$ with $V \ll \frac{\hbar^2}{2mR^2}$. Within the first-order degenerate perturbation theory, the magnitude of the energy splitting of the first excited state is:

- (a) $\frac{V}{2}$ (b) V
(c) $2V$ (d) $\frac{V}{4}$

Ans.: (b)

Solution.: The unperturbed Hamiltonian is $H_0 = \frac{L_z^2}{2mR^2}$ where $L_z = -i\hbar \frac{d}{d\phi}$.

The eigen state of L_z are $\psi(\phi) = \frac{1}{\sqrt{2\pi}} e^{im_l\phi}$ where $m_l = 0, \pm 1, \pm 2, \dots$

$$L_z \psi(\phi) = m_l \hbar \psi(\phi): E_0^{(0)} = \frac{m_l^2 \hbar^2}{2mR^2} = 0 \text{ (for ground state } m_l = 0)$$

First excited state corresponds to $m_l = \pm 1$: $E_{+1}^{(0)} = E_{-1}^{(0)} = \frac{\hbar^2}{2mR^2}$ which are degenerate.

(i) Now use degenerate perturbation theory in two-dimensional space $\{|+1\rangle, |-1\rangle\}$ and construct perturbation matrix

$$H' = V \cos(2\phi) = \frac{V}{2} (e^{i2\phi} + e^{-i2\phi})$$

(ii) Diagonal elements

$\langle +1 | H' | +1 \rangle = 0$ because H' changes m_l by ± 2 , so it can not connect back to $m_l = +1$.

Similarly, $\langle -1 | H' | -1 \rangle = 0$.

(iii) Off diagonal elements

$$e^{i2\phi} |-1\rangle \rightarrow |+1\rangle \text{ and } e^{-i2\phi} |+1\rangle \rightarrow |-1\rangle$$

$$\langle +1 | H' | -1 \rangle = \frac{V}{2}, \langle -1 | H' | +1 \rangle = \frac{V}{2}$$

(iv) Perturbation Matrix $H' = \begin{bmatrix} 0 & V/2 \\ V/2 & 0 \end{bmatrix}$

(v) Energy corrections: $\begin{vmatrix} -\lambda & V/2 \\ V/2 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{V^2}{4} = 0 \Rightarrow \lambda_{\pm} = \pm \frac{V}{2}$

So, two first excited state energies become $E_{\pm} = \frac{\hbar^2}{2mR^2} + \pm \frac{V}{2}$

Hence Energy Splitting: $\Delta E = E_+ - E_- = \frac{V}{2} - \left(-\frac{V}{2}\right) = V$

Q54. The Hamiltonian of a quantum mechanical system is given by $\hat{H} = \alpha \hat{L}^2 + \beta \hbar \hat{L}_z$. Here, α and β are positive constants and $\hat{L}$ represents the orbital angular momentum operator. If l and m are the azimuthal and magnetic quantum numbers then the lowest energy state for this system is:

- (a) $l=1, m=-1$ for $\alpha = \beta$ (b) $l=0, m=0$ for $\alpha < \frac{\beta}{2}$
 (c) $l=1, m=-1$ for $\alpha < \frac{\beta}{2}$ (d) $l=1, m=+1$ for $\alpha < \frac{\beta}{2}$

Ans.: (c)

Solution.: $\hat{H}|l, m\rangle = [\alpha \hat{L}^2 + \beta \hbar \hat{L}_z]|l, m\rangle = [\alpha \hbar^2 l(l+1) + (\beta \hbar) m_l \hbar]|l, m\rangle$

$\Rightarrow \hat{H}|l, m\rangle = \hbar^2 [\alpha l(l+1) + \beta m_l]|l, m\rangle$

Hence the energy is $E_{l, m_l} = \hbar^2 [\alpha l(l+1) + \beta m_l]$

Energy can be minimized if m_l is having lowest value.

(i) $l=0, m_l=0 \rightarrow E_{0,0} = 0$

(ii) $l=1, m_l=0, \pm 1$, so minimum energy state will have $m_l = -1$

$E_{1,-1} = \hbar^2 [2\alpha - \beta]$

$\therefore E_{1,-1} < E_{0,0} \Rightarrow \hbar^2 [2\alpha - \beta] < 0 \Rightarrow \alpha < \frac{\beta}{2}$

Q59. A particle of mass m with momentum $\hbar \vec{k}$ scatters elastically to momentum $\hbar(\vec{k} + \vec{q})$ from a spherical potential

$$V(\vec{r}) = \begin{cases} V_0 & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases}$$

Here, $q^2 = 4k^2 \sin^2 \frac{\theta}{2}$ with θ being the scattering angle. Within the first-order Born approximation and for $qR \ll 1$, the differential scattering cross-section to the leading order in R is proportional to:

- (a) R^2 (b) R^4
(c) R^6 (d) R^8

Ans.: (c)

Solution.: Born Approximation Amplitude: $f(\theta) \propto \int V(r) e^{iqr} d^3r$

Low momentum limit ($qR \ll 1$): $f(\theta) \propto V_0 \int 4\pi r^2 dr \propto V_0 R^3$

Differential Cross-Section: $\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \propto (R^3)^2 \propto R^6$

Q68. Consider a particle of mass m in an infinite potential well with walls at $x=0$ and $x=2L$. The particle is perturbed by a weak perturbation $H' = V_0 L \delta(x - 3L/2)$. The first-order correction to its first excited state energy is:

- (a) $2V_0$ (b) V_0
(c) $\frac{V_0}{2}$ (d) 0

Ans.: (b)

Solution.: First excited state ($n=2$): $\psi_2(x) = \sqrt{\frac{2}{2L}} \sin\left(\frac{2\pi x}{2L}\right) = \frac{1}{\sqrt{L}} \sin\left(\frac{2\pi x}{2L}\right)$

First-order correction to first excited state energy:

$$E_2^{(1)} = \langle \psi_2 | H' | \psi_2 \rangle = \int_0^{2L} |\psi_2(x)|^2 H' dx = \int_0^{2L} |\psi_2(x)|^2 V_0 L \delta(x - 3L/2) dx$$

$$\Rightarrow E_2^{(1)} = V_0 L \int_0^{2L} \frac{1}{L} \sin^2\left(\frac{2\pi x}{2L}\right) \delta(x - 3L/2) dx = V_0 \sin^2\left(\frac{3\pi}{2}\right) = V_0$$



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PART B

Q28. A micro-canonical ensemble consists of six non-interacting and distinguishable spin 1 particles in a uniform magnetic field. Each particle can thus be in energy states $-E_0, 0$ and E_0 .

The number of microstates in the state with total energy zero is:

- (a) 51 (b) 191
(c) 141 (d) 55

Ans.: (c)

Solution.: Let n_1, n_2, n_3 be the number of particles in states $-E_0, 0$ and E_0 , respectively.

We have two conditions:

$$(i) \quad n_1 + n_2 + n_3 = 6 \quad (ii) \quad n_1(-E_0) + n_2(0) + n_3(E_0) = 0 \Rightarrow n_1 = n_3$$

$$\Rightarrow 2n_1 + n_2 = 6 \Rightarrow n_2 = 6 - 2n_1$$

Since $n_1 \geq 0$ and $n_2 \geq 0$, the valid tuples (n_1, n_2, n_3) are:

- For $n_1 = 0: (0, 6, 0) \Rightarrow \frac{6!}{0!6!0!} = 1$
- For $n_1 = 1: (1, 4, 1) \Rightarrow \frac{6!}{1!4!1!} = 30$
- For $n_1 = 2: (2, 2, 2) \Rightarrow \frac{6!}{2!2!2!} = 90$
- For $n_1 = 3: (3, 0, 3) \Rightarrow \frac{6!}{3!0!3!} = 20$

Total number of microstates $\Omega = 1 + 30 + 90 + 20 = 141$

Q32. Internal energy of one mole of a non-ideal gas is given by $U = \frac{3}{2}RT - \frac{a}{V}$, where V is volume of the gas at temperature T and a is a positive constant. An insulated container of volume V_2 contains this gas confined in a volume V_1 at temperature T_1 . The remaining volume $V_2 - V_1$ of this container, which is in vacuum, is separated by a wall. The wall is suddenly removed which makes the gas volume V_2 and temperature T_2 . The temperature T_2 is:

(R is the molar gas constant)

- (a) $T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} + \frac{1}{V_1} \right)$ (b) $T_1 - \frac{2a}{3R} \left(\frac{1}{V_2} + \frac{1}{V_1} \right)$
(c) $T_1 + \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$ (d) $T_1 - \frac{2a}{3R} \left(\frac{1}{V_2} - \frac{1}{V_1} \right)$

Ans.: (c)

Solution.: Free expansion into a vacuum ($\Delta Q = 0$, $\Delta W = 0 \Rightarrow \Delta U = 0$).

$$\because \Delta U = 0 \Rightarrow U_1 = U_2 \Rightarrow \frac{3}{2}RT_1 - \frac{a}{V_1} = \frac{3}{2}RT_2 - \frac{a}{V_2} \Rightarrow \frac{3}{2}R(T_2 - T_1) = a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$\Rightarrow T_2 - T_1 = \frac{2a}{3R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right) \Rightarrow T_2 = T_1 + \frac{2a}{3R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Q35. Two particles occupy ten degenerate energy levels. There are three possible scenarios where the two particles are either identical bosons or identical fermions or distinguishable particles, with the associated number of accessible microstates denoted as Ω_B, Ω_F and Ω_C , respectively. Then, which of the following options is correct?

- (a) $\Omega_B = 55, \Omega_F = 45, \Omega_C = 100$ (b) $\Omega_B = 65, \Omega_F = 55, \Omega_C = 100$
(c) $\Omega_B = 45, \Omega_F = 25, \Omega_C = 55$ (d) $\Omega_B = 55, \Omega_F = 45, \Omega_C = 75$

Ans.: (a)

Solution.: Given: $N = 2$ particles occupying $g = 10$ degenerate energy levels.

$$\text{Bosons } \Omega_B = {}^{g+N-1}C_N = {}^{11}C_2 = \frac{11!}{2!9!} = \frac{11 \times 10}{2} = 55$$

$$\text{Fermions } \Omega_F = {}^gC_N = {}^{10}C_2 = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45$$

$$\text{Classical / Distinguishable } \Omega_C = g^N = 10^2 = 100$$

Q40. Water, having density 1 gm/cm^3 , freezes into ice at 273 K temperature and 1 atm pressure. The latent heat of melting of ice is 334 J/gm and the density of ice is 0.917 gm/cm^3 . Then for the melting temperature (T_m) which of the following options is correct?

- (a) At 140 atm pressure, $T_m \approx 274 \text{ K}$ (b) At 420 atm pressure, $T_m \approx 274 \text{ K}$
(c) At 140 atm pressure, $T_m \approx 272 \text{ K}$ (d) At 420 atm pressure, $T_m \approx 272 \text{ K}$

Ans.: (c)

Solution.: Given: $T_1 = 273 \text{ K}$, $P_1 = 1 \text{ atm}$, $L = 334 \text{ J/gm} = 334 \times 10^3 \text{ J/kg}$

$$\text{Density of water } \rho_w = 1 \text{ gm/cm}^3 = 10^3 \text{ kg/m}^3 \Rightarrow v_w = 10^{-3} \text{ m}^3/\text{kg}$$

Density of water

$$\rho_i = 0.917 \text{ gm/cm}^3 = 0.917 \times 10^3 \text{ kg/m}^3 \Rightarrow v_i = \frac{10^{-3}}{0.917} \text{ m}^3/\text{kg} = 1.0905 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$\Delta v = v_w - v_i = (1 - 1.0905) \times 10^{-3} \text{ m}^3/\text{kg} = -0.0905 \times 10^{-3} \text{ m}^3/\text{kg}$$

Use Clausius–Clapeyron equation:

$$\frac{dP}{dT} = \frac{L}{T\Delta v} \Rightarrow \Delta P = \frac{L}{T\Delta v} \Delta T \Rightarrow \Delta T = \frac{T\Delta v}{L} \Delta P$$

$$\Rightarrow \Delta T = \frac{273 \times (-0.0905 \times 10^{-3})}{334 \times 10^3} (139 \times 10^5) \quad \because \Delta P = 139 \times 10^5 \text{ Pa at } 140 \text{ atm.}$$

$$\Rightarrow \Delta T = -1.04 \text{ K} \approx -1 \text{ K} \Rightarrow T_m - T_1 = -1 \text{ K} \Rightarrow T_m = T_1 - 1 \text{ K} = 273 \text{ K} - 1 \text{ K} = 272 \text{ K} \Rightarrow T_m = 272 \text{ K}$$

PART C

Q62. Resistivity of a monovalent metal, having atomic density $4 \times 10^{22} \text{ atoms/cm}^3$, is found to be $1 \mu\Omega - \text{cm}$. The approximate value of the mean free path of electrons in this metal is:

(Given: $\hbar = 10^{-34} \text{ J-s}$ and $e = 1.6 \times 10^{-19} \text{ C}$)

(a) 1 nm

(b) 100 nm

(c) 10^4 nm

(d) 10^6 nm

Ans.: (b)

Solution.: According to the free electron model:

$$\rho = \frac{p_F}{ne^2\lambda} = \frac{\hbar k_F}{ne^2\lambda} \quad \text{where Fermi wavenumber } k_F = (3\pi^2 n)^{1/3}.$$

Given that

$$n = 4 \times 10^{22} \text{ atoms/cm}^3 = 4 \times 10^{28} \text{ atoms/m}^3, \quad \rho = 10^{-8} \Omega \text{m}, \quad \hbar = 10^{-34} \text{ J-s}, \quad e = 1.6 \times 10^{-19} \text{ C}$$

$$\Rightarrow k_F = (3\pi^2 n)^{1/3} = (3 \times 10 \times 4 \times 10^{28})^{1/3} \approx 1.05 \times 10^{10} \text{ m}^{-1}$$

$$\lambda = \frac{(10^{-34})(1.05 \times 10^{10})}{(4 \times 10^{28})(1.6 \times 10^{-19})^2 (10^{-8})} \approx 102.5 \text{ nm} \approx 100 \text{ nm}$$

Q56. There is a ferromagnetic interaction J between two Ising spins, $S_1, S_2 = \pm 1$, and they are under the influence of an external magnetic field h . The Hamiltonian of this system is,

$$H = -JS_1S_2 - hS_1 - hS_2$$

The spins are in equilibrium at temperature T and $\beta = 1/k_B T$ with k_B as Boltzmann constant.

The average value of either spin, for small h , is:

- (a) $\frac{4h \exp(\beta J)}{\cosh(\beta J)}$ (b) $\frac{h \exp(2\beta J)}{\cos(\beta J)}$
 (c) $\frac{8h \exp(4\beta J)}{\sinh(\beta J)}$ (d) $\frac{2h \exp(2\beta J)}{\sin(\beta J)}$

DROP

Solution.: Hamiltonian $H = -JS_1S_2 - hS_1 - hS_2 = -JS_1S_2 - h(S_1 + S_2)$ with $S_1, S_2 = \pm 1$.

Let us List all 4 possible spin configurations (S_1, S_2) :

(S_1, S_2)	H	S_1	$e^{-\beta H}$
$(+1, +1)$	$-J - 2h$	$+1$	$e^{\beta J + 2\beta h}$
$(+1, -1)$	$+J$	$+1$	$e^{-\beta J}$
$(-1, +1)$	$+J$	-1	$e^{-\beta J}$
$(-1, -1)$	$-J + 2h$	-1	$e^{\beta J - 2\beta h}$

$$\text{Partition Function } Z = e^{\beta J + 2\beta h} + 2e^{-\beta J} + e^{\beta J - 2\beta h} = 2e^{\beta J} \cosh(2\beta h) + 2e^{-\beta J}$$

$$\text{Average Spin } \langle S_1 \rangle = \frac{1}{Z} \left[(+1)e^{\beta J + 2\beta h} + (+1)e^{-\beta J} + (-1)e^{-\beta J} + (-1)e^{\beta J - 2\beta h} \right]$$

$$\Rightarrow \langle S_1 \rangle = \frac{2e^{\beta J} \sinh(2\beta h)}{2e^{\beta J} \cosh(2\beta h) + 2e^{-\beta J}} = \frac{2e^{\beta J} (2\beta h)}{2e^{\beta J} + 2e^{-\beta J}} = \frac{2e^{\beta J} (2\beta h)}{4 \cosh(\beta J)} = \frac{\beta h e^{\beta J}}{\cosh(\beta J)}$$

For small h , $\sinh(2\beta h) \approx 2\beta h$, $\cosh(2\beta h) \approx 1$



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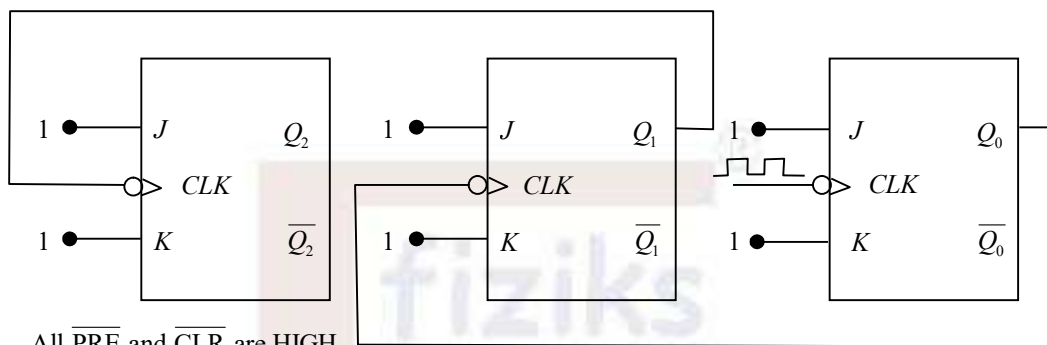
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PART B

Q34. Three J-K flip-flops are connected together to make a counter as shown in the figure. The J and K inputs are tied to logical 1 and a clock pulse is applied only to CLK input of the flip flop Q_0 . If the counter is initially in the state $Q_2Q_1Q_0 = 100$, then the state of the counter after 9 clock cycles is:

Figure:



All $\overline{\text{PRE}}$ and $\overline{\text{CLR}}$ are HIGH

(a) $Q_2Q_1Q_0 = 001$

(b) $Q_2Q_1Q_0 = 110$

(c) $Q_2Q_1Q_0 = 101$

(d) $Q_2Q_1Q_0 = 111$

Ans.: (c)

Solution.:

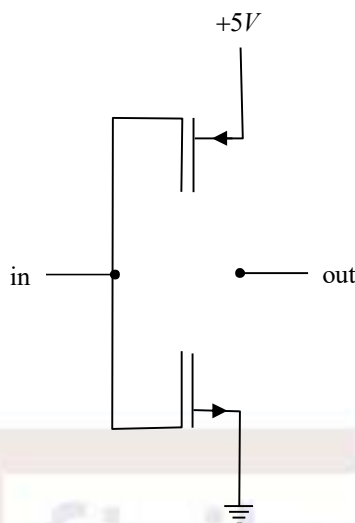
Clock	Q_2	Q_1	Q_0
Initially	1	0	0
1	1	0	1
2	1	1	0
3	1	1	1
4	0	0	0
5	0	0	1
6	0	1	0
7	0	1	1
8	1	0	0
9	1	0	1

Q_0 will toggle at each negative edge of clock.

Q_1 will toggle when Q_0 change from $1 \rightarrow 0$.

Q_2 will toggle when Q_1 change from $1 \rightarrow 0$.

Q37. The circuit shown below could be used as a:



(a) TTL NOT Gate

(b) TTL Buffer

(c) CMOS NOT Gate

(d) CMOS Buffer

Ans.: (c)

Solution.: The given circuit uses a pair of complementary MOSFETs - one p -channel MOSFET on top connected to +5V and one n -channel MOSFET on the bottom connected to ground, with gates tied together as the input. It is the standard configuration of a CMOS Inverter (NOT Gate).

Q41. An analog-to-digital converter monitors the dc voltage output from a battery. The output of the converter is a three-digit binary number ABC, corresponding to battery voltage in steps of $0.5V$, with A as the most significant bit (MSB). The output of the converter is fed into a logic circuit which produces a HIGH output when the battery voltage exceeds $2V$. The expression for this logic circuit is:

(a) $AB + AC$

(b) $AB + BC + AC$

(c) ABC

(d) AC

Ans.: (a)

Solution.: The ADC converts battery voltage into a 3-bit binary number ABC in steps of $0.5V$:
 $(000)_2 \rightarrow 0.0V$; $(001)_2 \rightarrow 0.5V$; $(010)_2 \rightarrow 1.0V$; $(011)_2 \rightarrow 1.5V$; $(100)_2 \rightarrow 2.0V$

For voltage exceeding $2.0V$, the binary output must correspond to values 5, 6, 7

$$(101)_2 \rightarrow 2.5V; (110)_2 \rightarrow 3.0V; (111)_2 \rightarrow 3.5V.$$

So in truth table $Y = 1$ when $ABC > 0$: $(101)_2 \rightarrow A\bar{B}\bar{C}$; $(110)_2 \rightarrow AB\bar{C}$; $(111)_2 \rightarrow ABC$

$$Y = A\bar{B}\bar{C} + AB\bar{C} + ABC = A\bar{B}\bar{C} + AB(\bar{C} + C) = A\bar{B}\bar{C} + AB = A(B + \bar{B}\bar{C}) = A(B + C)$$

$$\Rightarrow Y = AB + AC$$

PART C

Q55. Given is a set of 5 data points of the form (x_i, y_i, σ_i) with σ_i as the estimated uncertainty in y_i . This data is fitted to the form $y = ax + b$ and the minimized χ^2 (chi-square) is found to be 5.1. The reduced χ^2 for this fitting is:

- (a) 1.0 (b) 1.2
(c) 1.5 (d) 1.7

Ans.: (d)

Solution.: Number of data points N : 5

Number of fitted parameters (k): 2 (for the linear equation $y = ax + b$)

Degrees of freedom (ν): $\nu = N - k = 5 - 3 = 2$

Reduced χ^2 is $\chi_{red}^2 = \frac{\chi^2}{\nu} = \frac{5.1}{3} = 1.7$

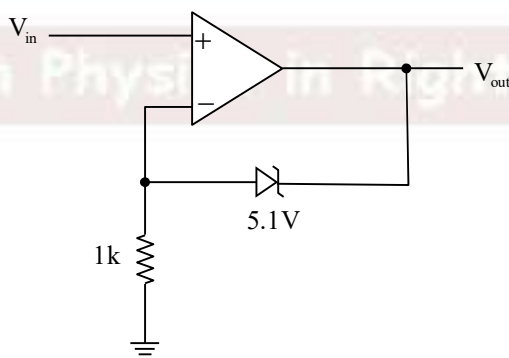
Q63. Modulation techniques are used to shift a signal to higher frequencies to

- (a) remove the effects of white noise (b) reduce the effects of $1/f$ -noise and drift
(c) reduce shot noise and Johnson noise (d) smoothen the signal and amplify it

Ans.: (b)

Solution.: Low-frequency electronic signals are typically dominated by flicker noise (also known as $1/f$ -noise) and low-frequency baseline drift. By using modulation techniques to shift the signal spectrum to a much higher carrier frequency, the signal is moved out of the high $1/f$ -noise region into a region where thermal/white noise dominates at much lower power spectral densities.

Q75. In the circuit shown below, if $V_{in} = 2.0V$ then V_{out} is



- (a) $2.0V$ (b) $3.1V$
(c) $5.1V$ (d) $7.1V$

Ans.: (d)

Solution.:

If output were 7.1 V:

The Zener is in breakdown, current flows from output through Zener into the $1k\Omega$ to ground, developing 2.0 V at inverting input. The op-amp is balanced.

If output were 2.0 V:

Then voltage across Zener = 0, no current - impossible, since inverting node needs 2 mA to ground.

If output were 3.1 V:

Zener voltage = 1.1 V, not enough for 5.1 V Zener to break down; it wouldn't conduct in reverse. In reverse below 5.1 V, it's like open circuit $\rightarrow$ no feedback $\rightarrow$ op-amp saturates. So not possible.

If output 5.1 V:

Zener voltage = 3.1 V, still below breakdown $\rightarrow$ no conduction, no feedback.

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PART B

Q23. Vanadium has electron configuration $[Ar]3d^3 4s^2$. In LS coupling, its ground state configuration is

(a) ${}^4F_{\frac{9}{2}}$

(b) ${}^2P_{\frac{1}{2}}$

(c) ${}^4F_{\frac{3}{2}}$

(d) ${}^4S_{\frac{3}{2}}$

Ans.: (c)

Solution.: Find total orbital angular momentum (L):

Electronic configuration for valence electrons: $3d^3$ (unfilled subshell; $4s^2$ is fully filled with $L=0, S=0$).

For a d -shell ($l=0$): $m_l = +2, +1, 0, -1, -2$

Maximizing L using Hund's first rule for 3 electrons:

$$m_l = +2, +1, 0 \Rightarrow L = \sum m_l = +2 + 1 + 0 = 3 \text{ (F State)}$$

Find total spin angular momentum (S):

Maximizing S : 3 unpaired electrons with parallel spins $\Rightarrow S = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

Spin multiplicity: $2S + 1 = 2 \times \frac{3}{2} + 1 = 4$

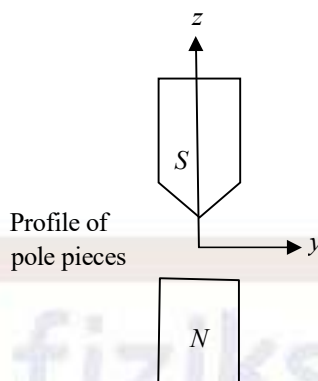
Find total angular momentum (J):

Since the $3d$ shell is **less than half-filled** (3 electrons out of 10 maximum), the ground state has

the minimum J value: $J = |L - S| = \left| 3 - \frac{3}{2} \right| = \frac{3}{2}$

Term Symbol: ${}^{2S+1}L_J = {}^4F_{\frac{3}{2}}$

Q38. A beam of copper atoms ($\text{Cu} : [\text{Ar}]4s^1 3d^{10}$) is passed through a Stern-Gerlach setup (see figure). If the gradient of the z -component of the magnetic field in the z -direction is 10^3 Tesla/m, then the magnitude of the z -component of the force on an atom is: (μ_B is the Bohr magneton)



(a) $5 \times 10^2 \mu_B$

(b) $10^3 \mu_B$

(c) 0

(d) $2\pi \times 10^3 \mu_B$

Ans.: (b)

Solution.:

Magnetic Moment of Copper Atom:

Outer configuration: $4s^1 3d^{10}$;

Complete shell $3d^{10}$ contributes zero angular momentum.

Single $4s^1$ electron has $l=0$ and $s=\frac{1}{2}$, so the total magnetic moment along z is due purely to

spin: $\mu_z = g_s \mu_B S_z = \mp \mu_B$

Force in Stern-Gerlach Setup:

The force acting on the magnetic dipole in an inhomogeneous magnetic field is:

$$F_z = \left| \frac{\partial B_z}{\partial z} \mu_z \right| = 10^3 \mu_B$$

PART C

Q61. A container has a gas at temperature T with its N_1 molecules in the ground state and N_2 molecules in the excited state of energy $h\nu$. Both the levels are non-degenerate and A and B are the Einstein coefficients for spontaneous and stimulated emissions, respectively, between the two levels. Then the spectral density $\rho(\nu, T)$ of radiation inside the container is proportional to:

(a) $\frac{A}{B} \left(\frac{N_2}{N_1 - N_2} \right)$

(b) $\frac{A}{B} \left(\frac{N_2}{N_1 + N_2} \right)$

(c) $\frac{N_2}{\left(\frac{A}{B} \right) N_1 - N_2}$

(d) $\frac{N_2}{\left(\frac{A}{B} \right) N_1 + N_2}$

Ans.: (a)

Solution.:

Detailed Balance Condition: Under thermal equilibrium,

Rate of Absorption = Rate of Spontaneous Emission + Rate of Stimulated Emission:

$$N_1 B_{12} \rho(\nu, T) = N_2 A_{21} + N_2 B_{21} \rho(\nu, T)$$

Einstein Relations for Non-Degenerate Levels: $B_{12} = B_{21} = B$, $A_{21} = A$

Solve for Radiation Spectral Density $\rho(\nu, T)$:

$$(N_1 - N_2) B \rho(\nu, T) = N_2 A \Rightarrow \rho(\nu, T) = \frac{A}{B} \left(\frac{N_2}{N_1 - N_2} \right)$$



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PART C

Q47. In a superconductor, the current density $\vec{J}$ and magnetic field $\vec{B}$ are related by the equation:

(Here, n is the number density of electrons, e is electron charge, m_e is electron mass and μ_0 is permeability of vacuum.)

(a) $\frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{J} - \frac{ne^2}{m_e} \vec{B} \right) = 0$

(b) $\left(\vec{\nabla} \times \vec{J} + \frac{ne^2}{m_e} \vec{B} \right) = 0$

(c) $\left(\nabla^2 \vec{B} - \mu_0 \vec{\nabla} \times \vec{J} \right) = 0$

(d) $\left(\nabla^2 \vec{J} - \mu_0 \vec{\nabla} \times \vec{B} \right) = 0$

Ans.: (b)

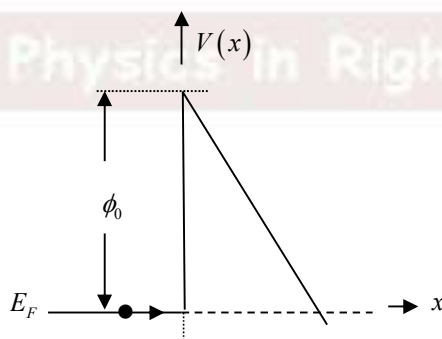
Solution.: The second London equation relates the magnetic field $\vec{B}$ and the supercurrent density $\vec{J}$ inside a superconductor:

$$\vec{\nabla} \times \vec{J} = -\frac{ne^2}{m_e} \vec{B} \Rightarrow \left(\vec{\nabla} \times \vec{J} + \frac{ne^2}{m_e} \vec{B} \right) = 0.$$

Q71. The surface of a metal of work function ϕ_0 is subjected to an electric field ε . As a result, the electric potential profile just outside the surface is given by $V(x) = \phi_0 - x\varepsilon$ as shown in the figure. For electric field ε , the probability $p(\varepsilon)$ of tunneling of an electron moving at the Fermi energy (E_F) can be found using the WKB approximation. If $\lambda = \sqrt{\frac{2me\phi_0^3}{\hbar^2}}$, then the value of the

ratio $\frac{p(\varepsilon = 2\lambda)}{p(\varepsilon = \lambda)}$ will be:

(Here, e and m are electron's charge and mass, respectively)



(a) 1.36

(b) 2.73

(c) 1.94

(d) 5.13

Ans.: (c)

Solution.: $p(\varepsilon) \propto \exp\left(-\frac{4}{3} \frac{\lambda}{\varepsilon}\right)$

$$\frac{p(\varepsilon = 2\lambda)}{p(\varepsilon = \lambda)} = \frac{\exp\left(-\frac{4}{3} \frac{1}{2}\right)}{\exp\left(-\frac{4}{3} \frac{1}{1}\right)} = \exp\left(\frac{2}{3}\right) \approx e^{0.667} \approx 1.94$$

Q73. Potassium chloride (KCl) crystallizes in NaCl structure. The atomic form factors of K^+ and Cl^- are the same. Then the first four X-ray diffraction peaks for KCl are:

- (a) (100),(110),(111),(200) (b) (110),(200),(222),(310)
(c) (111),(200),(220),(311) (d) (200),(220),(222),(400)

Ans.: (d)

Solution.:

KCl has the NaCl structure, which can be regarded as an FCC lattice with a two-atom basis.

Diffraction peaks occur **only when h, k, l are all even**. The first four allowed planes in increasing order of $h^2 + k^2 + l^2$ are (200),(220),(222),(400).

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PART C

Q53. The table shows the g -factors of proton (p) and neutron (n) for orbital (g_l) and spin (g_s) angular momentum.

	g_l	g_s
p	1	5.58
n	0	-3.82

The magnetic moment of $^{17}_8\text{O}$ nucleus, according to the single particle shell model, in units of nuclear magneton (μ_N) is:

- (a) 0.55 (b) 4.79
(c) -1.91 (d) -3.82

Ans.: (c)

Solution.:

Unpaired Nucleon Identification for $^{17}_8\text{O}$:

Proton number: $Z = 8$ (Even); Neutron number: $N = 17 - 8 = 9$ (Odd).

The properties are determined by the single unpaired neutron ($N = 9$).

Nuclear Shell Model Configuration for $N = 9$: $1s_{1/2}^2, 1p_{3/2}^4, 1p_{1/2}^2, 1d_{5/2}^1$

The unpaired neutron occupies the $1d_{5/2}$ state, so $l = 2, j = \frac{5}{2} = l + \frac{1}{2}$

Magnetic Moment Formula for $j = l + \frac{1}{2}$: $\mu = g_l l + \frac{1}{2} g_s$

$$\mu = 0 \times 2 + \frac{1}{2}(-3.82) = -1.91 \mu_N$$

Q65. Consider the following decay processes of ρ^0

- (A) $\rho^0 \rightarrow \pi^+ + \pi^-$ (B) $\rho^0 \rightarrow \pi^0 + \pi^0$

For the conservation of angular momentum and parity, which of the following options is correct?

- (a) Both processes (A) and (B) are allowed
(b) Both processes (A) and (B) are forbidden
(c) Only process (A) is allowed
(d) Only process (B) is allowed

Ans.: (c)

Solution.:

Quantum numbers for ρ^0 meson

Spin $J = 1$, Parity $P = -1$, Charge conjugation $C = -1$

Quantum numbers for (π^+, π^0, π^-) meson

$J^P = 0^-$. Spin 0, intrinsic parity negative.

(A) $\rho^0 \rightarrow \pi^+ + \pi^-$

Angular momentum: Initial: $J = 1$. Final: two spin-0 particles $\rightarrow$ total spin $S = 0$.

Orbital angular momentum L between them must satisfy:

$J = L + S = L + 0 = 1 \Rightarrow L = 1$. So $L = 1$ is allowed (P -wave).

Angular momentum conserved.

Parity: Initial parity $P_i = -1$.

Parity of a two-particle system: $P_f = P_\pi P_\pi (-1)^L$. Intrinsic parity of each pion $= -1$

So, $P_f = (-1)(-1)(-1)^1 = -1$. Thus $P_f = P_i = -1$. Hence Parity conserved.

π^+ and π^- are not identical, so no symmetrization restriction.

Thus **(A) is allowed**.

(B) $\rho^0 \rightarrow \pi^0 + \pi^0$

Angular momentum: Same as before: $J = 1$ requires $L = 1$.

But now the two final particles are **identical bosons** (π^0).

For two identical bosons, the total wavefunction must be symmetric under exchange.

$$\psi_{Total}^S = \psi_{Space} \psi_{Spin}.$$

Since ψ_{Spin} will be symmetric under exchange symmetry for two bosons of zero spin so ψ_{Space}

will also be symmetric i.e. $(-1)^L = +ve$. So L must be **even**: $L = 0, 2, 4, \dots$

But angular momentum conservation requires $L = 1$, which is **odd**.

So parity is not conserved. So **(B) is forbidden**.

Q66. Assume a spherically uniform charge distribution in a nucleus having mass number A and radius $R = R_0 A^{1/3}$ with $R_0 \approx 1.5 \text{ fm}$. If the difference in electrostatic energy between two mirror nuclei, ${}^A_Z X$ and ${}^A_{Z-1} Y$, is 3.53 MeV, then the value of atomic number Z is closest to:

(Given: $\frac{e^2}{4\pi\epsilon_0} \approx 1.44 \text{ MeV}\cdot\text{fm}$ and $1 \text{ fm} = 10^{-15} \text{ m}$)

- (a) 6 (b) 8
(c) 10 (d) 12

Ans.: (b)

Solution.: For a uniformly charged sphere of total charge Ze and radius R :

$$E_C = \frac{3}{5} \frac{(Ze)^2}{4\pi\epsilon_0 R} = \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R}$$

Difference between mirror nuclei

For ${}^A_Z X$ charge is Ze and radius is $R = R_0 A^{1/3}$

For ${}^A_{Z-1} Y$ charge is $(Z-1)e$ same radius R and same A .

$$E_C(Z) = \frac{3}{5} \frac{Z^2 e^2}{4\pi\epsilon_0 R}, \quad E_C(Z-1) = \frac{3}{5} \frac{(Z-1)^2 e^2}{4\pi\epsilon_0 R}$$

$$\text{Difference: } \Delta E = E_C(Z) - E_C(Z-1) = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R} [Z^2 - (Z-1)^2] \Rightarrow \Delta E = \frac{3}{5} \frac{e^2}{4\pi\epsilon_0 R} (2Z-1)$$

$$\Rightarrow \Delta E = \frac{3}{5} \frac{1.44}{1.5 A^{1/3}} (2Z-1) \text{ MeV} \quad \because \frac{e^2}{4\pi\epsilon_0} \approx 1.44 \text{ MeV}\cdot\text{fm} \text{ and } R = R_0 A^{1/3} \text{ with } R_0 \approx 1.5 \text{ fm}$$

$$\Rightarrow 3.53 = \frac{0.576}{A^{1/3}} (2Z-1) \Rightarrow 2Z-1 = \frac{3.53}{0.576} A^{1/3} \approx 6.128 A^{1/3}$$

For mirror nuclei ${}^A_Z X$ and ${}^A_{Z-1} Y$ we know $A = 2Z-1$, so

$$\Rightarrow (2Z-1) = 6.128 (2Z-1)^{1/3} \Rightarrow (2Z-1)^{2/3} = 6.128 \Rightarrow (2Z-1) = (6.128)^{3/2} \approx 15.17$$

$$\Rightarrow 2Z \approx 16.17 \Rightarrow Z \approx 8.08$$

Q70. A kaon (K^0) decays into two neutral pions (π^0). Given that the mass of the K^0 is $495 \text{ MeV}/c^2$ and the mass of the π^0 is $135 \text{ MeV}/c^2$, the final momentum of the π^0 (in MeV/c) in the center of mass frame is approximately:

- (a) 207 (b) 359
(c) 157 (d) 103

Ans.: (a)

Solution.: $K^0 \rightarrow \pi^0 + \pi^0$

In the center-of-mass frame of K^0 the total rest mass energy splits equally between the two identical π^0 pions:

$$E_{\pi^0} = \frac{m_K c^2}{2} = \frac{495}{2} \text{ MeV} = 247.5 \text{ MeV}$$

Calculating Momentum (p):

$$E^2 = (pc)^2 + (m_{\pi^0} c^2)^2 \Rightarrow (pc)^2 = E^2 - (m_{\pi^0} c^2)^2 = (247.5)^2 - (135)^2$$

$$\Rightarrow (pc)^2 = 61256.25 - 18225 = 43031.25 \Rightarrow pc = 207.44 \text{ MeV} \Rightarrow p \approx 207 \text{ MeV} / c$$



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