



Learn Physics in Right Way

**GATE Physics-2026**  
**Solution-Mathematical Physics**

Learn Physics in Right Way

**Be Part of Disciplined Learning**

## Q.11 – Q.35 Carry ONE mark Each

**Q13.** Consider an operator  $\hat{A}$  which is not Hermitian. Find the possible values of  $c$  and  $d$  such that the operator  $(c\hat{A} - d\hat{A}^\dagger)$  is Hermitian.

- (a)  $c = i$  and  $d = i$  (b)  $c = 1$  and  $d = 1$   
(c)  $c = -1$  and  $d = i$  (d)  $c = i$  and  $d = -i$

**Ans.: (a)**

**Solution.:** Let  $\hat{O} = (c\hat{A} - d\hat{A}^\dagger)$  then for  $\hat{O}$  to be Hermitian  $\hat{O}^\dagger = \hat{O}$ .

$$\Rightarrow (c\hat{A} - d\hat{A}^\dagger)^\dagger = (c\hat{A} - d\hat{A}^\dagger) \Rightarrow (c^* \hat{A}^\dagger - d^* \hat{A}) = (c\hat{A} - d\hat{A}^\dagger) \Rightarrow c^* = -d, -d^* = c \Rightarrow c = -d^*$$

So,  $c = i$  and  $d = i$  will satisfy the condition  $c = -d^*$ .

**Note:**

For any non-Hermitian operator  $\hat{A}$ ,  $i(\hat{A} - \hat{A}^\dagger)$  is anti-Hermitian while  $i(\hat{A} + \hat{A}^\dagger)$  is Hermitian.

**Q21.** Given  $|v_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$  and  $|v_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$ , the tensor product  $|v_1\rangle \otimes |v_2\rangle$  is

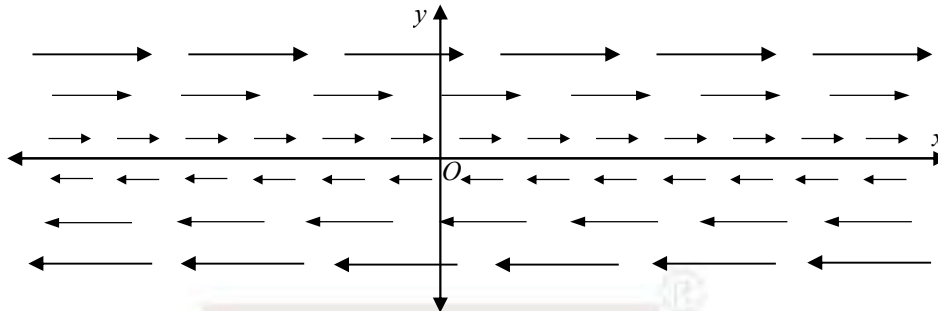
- (a)  $\frac{1}{2} \begin{pmatrix} 1 \\ -i \\ i \\ 1 \end{pmatrix}$  (b)  $\frac{1}{2} \begin{pmatrix} 1 \\ i \\ -i \\ 1 \end{pmatrix}$  (c)  $\frac{1}{2} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}$  (d)  $\frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$

**Ans.: (a)**

**Solution.:** The tensor product of  $a \otimes b = \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix}$

$$|v_1\rangle \otimes |v_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ i \\ 1 \end{pmatrix}$$

**Q22.** Sketch of a two-dimensional vector field  $\vec{V}$  is shown below. Here, length and arrow head of the arrows denote magnitude and direction of the vector field, respectively. Which of the following statements is correct for  $\vec{\nabla} \times \vec{V}$ ?



- (a) It is zero everywhere in the two-dimensional space
- (b) Its magnitude is non-zero and its direction is out of the two-dimensional plane
- (c) Its magnitude is non-zero and its direction is into the two-dimensional plane
- (d) It points in opposite directions above and below the  $x$ -axis

**Ans.: (c)**

**Solution.:** The vector field is directed along  $+\hat{x}$  for  $y > 0$  and along  $-\hat{x}$  for  $y < 0$ :

$$\vec{V} = Cy\hat{x} \quad \text{where } C > 0.$$

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Cy & 0 & 0 \end{vmatrix} = \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}(0-C) \Rightarrow \vec{\nabla} \times \vec{V} = -C\hat{z}$$

Since  $\vec{\nabla} \times \vec{V} = -C\hat{z}$ , its magnitude is non-zero, and its direction is in the  $-\hat{z}$  direction (pointing into the page/plane).

**Q27.** For which of the following functions does the Laplacian vanish?

- (a)  $xe^y - ye^x$
- (b)  $x \cos(y) - y \cos(x)$
- (c)  $e^{x+iy}$
- (d)  $yx^2 - \frac{y^3}{3} - xy$

**Ans.: (c) and (d)**

**Solution.:** The Laplacian is  $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

$$(a) f = xe^y - ye^x \Rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -ye^x + xe^y \neq 0$$

$$(b) f = x \cos(y) - y \cos(x) \Rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = y \cos(x) - x \cos(y) \neq 0$$

$$(c) f = e^{x+iy} \Rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = e^{x+iy} - e^{x+iy} = 0$$

$$(d) f = yx^2 - \frac{y^3}{3} - xy \Rightarrow \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 2y - 2y = 0$$

**Q.36 – Q.55 Carry TWO marks Each**

**Q36.** The function  $f(z)$  of complex variable  $z$  given below,  $f(z) = \frac{z^2 - 5z + 4}{z^3 + 4z - z^2 - 4}$  has singular points at  $z =$

- (a) 1 and  $(2-i)$       (b)  $2i$  and  $-2i$       (c) 1 and  $(2+i)$       (d)  $(2+i)$  only

**Ans.: (b)**

**Solution.:**  $z^2 - 5z + 4 = (z-1)(z-4)$ ;  $z^3 + 4z - z^2 - 4 = z^2(z-1) + 4(z-1) = (z-1)(z^2 + 4)$

$$f(z) = \frac{z^2 - 5z + 4}{z^3 + 4z - z^2 - 4} = \frac{(z-1)(z-4)}{(z-1)(z^2 + 4)} = \frac{(z-4)}{(z^2 + 4)}$$

Thus  $f(z)$  has singular points when  $z^2 + 4 = 0$  and it is at  $2i$  and  $-2i$ .

$z = 1$  has removable singularity.

**Q37.** Consider the Pauli matrices  $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ ,  $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

The value of  $Tr(\sigma_z [\sigma_x, \sigma_y])$  is

- (a)  $2i$       (b)  $i$       (c)  $4i$       (d)  $\frac{i}{2}$

**Ans.: (c)**

**Solution.:**  $[\sigma_x, \sigma_y] = 2i\sigma_z \Rightarrow \sigma_z [\sigma_x, \sigma_y] = 2i\sigma_z^2 = 2iI_2$

$$\Rightarrow Tr(\sigma_z [\sigma_x, \sigma_y]) = Tr(2iI_2) = 2i Tr \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = 2i \times 2 = 4i$$

**Q50.** Which of the following operators is/are self-adjoint?

(a)  $x^2 \frac{d^2}{dx^2} + 3x \frac{d}{dx} + x^2$

(b)  $(1-x^2) \frac{d^2}{dx^2} - 2x \frac{d}{dx} + 3x$

(c)  $(3x-4x^3) \frac{d^2}{dx^2} + (3-12x^2) \frac{d}{dx} + 12$

(d)  $x \frac{d^2}{dx^2} + x^2 \frac{d}{dx} + \frac{5x}{3}$

**Ans.: (b) and (c)**

**Solution.:** For a second-order differential operator

$$L = a_2(x) \frac{d^2}{dx^2} + a_1(x) \frac{d}{dx} + a_0(x)$$

the formal adjoint is  $L^\dagger = \frac{d^2}{dx^2} a_2(x) - \frac{d}{dx} a_1(x) + a_0(x) = \frac{d}{dx} \left( \frac{d}{dx} a_2(x) \right) - \frac{d}{dx} a_1(x) + a_0(x)$

Expanding  $L^\dagger = a_2 \frac{d^2}{dx^2} + (2a_2' - a_1) \frac{d}{dx} + (a_2'' - a_1' + a_0)$

For self-adjointness,  $a_1 = a_2'$ .

(a)  $L = x^2 \frac{d^2}{dx^2} + 3x \frac{d}{dx} + x^2$ ;  $a_2 = x^2$ ,  $a_1 = 3x \Rightarrow a_2' = 2x$  thus  $a_1 \neq a_2'$

(b)  $L = (1-x^2) \frac{d^2}{dx^2} - 2x \frac{d}{dx} + 3x$ ;  $a_2 = (1-x^2)$ ,  $a_1 = -2x \Rightarrow a_2' = -2x$  thus  $a_1 = a_2'$

(c)  $L = (3x-4x^3) \frac{d^2}{dx^2} + (3-12x^2) \frac{d}{dx} + 12$ ;

$a_2 = (3x-4x^3)$ ,  $a_1 = (3-12x^2) \Rightarrow a_2' = 3-12x^2$  thus  $a_1 = a_2'$

(d)  $L = x \frac{d^2}{dx^2} + x^2 \frac{d}{dx} + \frac{5x}{3}$ ;  $a_2 = x$ ,  $a_1 = x^2 \Rightarrow a_2' = 1$  thus  $a_1 \neq a_2'$

**Q51.** Consider two operators  $\hat{A}$  and  $\hat{B}$  which are related as  $\hat{A} = \exp(i\theta\hat{B})$ . If  $\theta$  is a non-zero real number, which of the following statements is/are true?

(a) If  $\hat{B}$  is Hermitian, then  $\hat{A}$  is unitary (b) If  $\hat{B}$  is anti-Hermitian, then  $\hat{A}$  is unitary

(c) If  $\hat{B}$  is Hermitian, then  $|\text{Det}(\hat{A})| = 1$  (d) If  $\hat{B}$  is anti-Hermitian, then  $\hat{A}$  is Hermitian

**Ans.: (a), (c) and (d)**

**Solution.:** (a) If  $\hat{B}$  is Hermitian, then  $\hat{A}$  is unitary

$$\hat{B}^\dagger = \hat{B}, \hat{A} = \exp(i\theta\hat{B}) \Rightarrow \hat{A}^\dagger = \exp(-i\theta\hat{B}^\dagger) = \exp(-i\theta\hat{B}) \Rightarrow \hat{A}^\dagger \hat{A} = \exp(-i\theta\hat{B}) \exp(i\theta\hat{B}) = I.$$

So option (a) is correct.

(b) If  $\hat{B}$  is anti-Hermitian, then  $\hat{A}$  is unitary

$$\hat{B}^\dagger = -\hat{B}, \hat{A} = \exp(i\theta\hat{B}) \Rightarrow \hat{A}^\dagger = \exp(-i\theta\hat{B}^\dagger) = \exp(i\theta\hat{B}) \Rightarrow \hat{A}^\dagger \hat{A} = \exp(i\theta\hat{B}) \exp(i\theta\hat{B}) \neq I.$$

So option (b) is incorrect.

(c) If  $\hat{B}$  is Hermitian, then  $|\text{Det}(\hat{A})| = 1$

If  $\hat{B}$  is Hermitian then its eigenvalues  $b_n$  are real then eigenvalues of  $\hat{A} = \exp(i\theta\hat{B})$  are  $\exp(i\theta b_n)$ .

$$|\text{Det}(\hat{A})| = \left| \prod_n \exp(i\theta b_n) \right| = 1. \text{ So option (c) is correct.}$$

(d) If  $\hat{B}$  is anti-Hermitian, then  $\hat{A}$  is Hermitian

$$\hat{B}^\dagger = -\hat{B}, \hat{A} = \exp(i\theta\hat{B}) \Rightarrow \hat{A}^\dagger = \exp(-i\theta\hat{B}^\dagger) = \exp(i\theta\hat{B}) = \hat{A} \text{ then } \hat{A} \text{ is Hermitian.}$$

So option (d) is correct.

**Extra (Buffer) Questions: Carry ONE mark only**

**Q1A.** Consider the matrix  $M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ . The exponential of this matrix  $e^M$  is given by

- (a)  $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$       (b)  $\begin{pmatrix} 0 & e \\ 0 & 0 \end{pmatrix}$       (c)  $\begin{pmatrix} 1 & e \\ 0 & 1 \end{pmatrix}$       (d)  $\begin{pmatrix} 1 & e \\ 1 & 1 \end{pmatrix}$

**Ans.: (a)**

$$\text{Solution.: } M = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, M^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, M^3 = M^2 M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \dots$$

$$e^M = I + M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

**Extra (Buffer) Questions: Carry TWO mark only**

**Q2A.** The two dimensional integral,  $\iint_S d\vec{r} [a \cos(\theta) + r] \delta^{(2)}(\vec{r} - \vec{r}_0)$  over the entire plane, where

$(r, \theta)$  are plane polar coordinates,  $\delta^{(2)}(\vec{r} - \vec{r}_0)$  is the Dirac delta function with  $\vec{r}_0 = \frac{\hat{x} + \hat{y}}{\sqrt{2}}$ , is

- (a)  $1 + \frac{a}{\sqrt{2}}$       (b) 0  
(c)  $1 - \frac{a}{\sqrt{2}}$       (d) 1

**Ans.: (a)**

**Solution.:**  $I = \iint_S d\vec{r} [a \cos(\theta) + r] \delta^{(2)}(\vec{r} - \vec{r}_0) = [a \cos(\theta_0) + r_0]$

$$\because \vec{r}_0 = \frac{\hat{x} + \hat{y}}{\sqrt{2}} \Rightarrow r_0 = \sqrt{\frac{1}{2} + \frac{1}{2}} = 1, \theta_0 = \frac{\pi}{4} \Rightarrow I = a \cos \frac{\pi}{4} + 1 = \frac{a}{\sqrt{2}} + 1$$

**Extra (Buffer) Questions: Carry TWO mark only**

**Q2D.** If two indistinguishable particles are to be placed randomly in three boxes colored red, blue and green, then the probability of finding one or both particles in the blue box is

- (a) 25% (b) 33%  
(c) 47% (d) 50%

**Ans.: (d)**

**Solution.:**

Number of particles =  $N$ , Number of boxes =  $g$  (Red, Blue, Green).

Total ways to distribute  $N$  indistinguishable particles in  $g$  boxes =  ${}^{N+g-1}C_N$

Total ways to distribute 2 indistinguishable particles in 3 boxes =  ${}^{2+3-1}C_2 = {}^4C_2 = \frac{4!}{2!2!} = 6$

The 6 possible configurations are:

1. (2, 0, 0) - Both in Red
2. (0, 2, 0) - Both in Blue
3. (0, 0, 2) - Both in Green
4. (1, 1, 0) - One in Red, One in Blue
5. (1, 0, 1) - One in Red, One in Green
6. (0, 1, 1) - One in Blue, One in Green

Configurations with **one or both particles in the blue box:**

- (0, 2, 0), (1, 1, 0), (0, 1, 1) → **3 configurations**

Probability =  $\frac{3}{6} = 0.5$  or 50%



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**Solution-Classical Mechanics**

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## Q.11 – Q.35 Carry ONE mark Each

**Q16.** On a horizontal plane, a projectile of mass  $m$  is launched from the ground with speed  $v_0$  at an angle  $\theta_0$  with the horizontal. In addition to the gravitational force ( $mg$ ), it also experiences a drag force  $\vec{F}_{drag} = -\gamma\vec{v}$ , where  $\vec{v}$  is its velocity and  $\gamma$  is a constant. It hits the ground at a distance  $R$  from the point of launch with its velocity making an angle  $\theta$  with the horizontal, as shown schematically in the figure. Then which of the following options is correct?



(a)  $R = \frac{v_0^2 \sin 2\theta}{g}$      $\theta < \theta_0$

(b)  $R < \frac{v_0^2 \sin 2\theta_0}{g}$      $\theta < \theta_0$

(c)  $R < \frac{v_0^2 \sin 2\theta_0}{g}$      $\theta > \theta_0$

(d)  $R = \frac{v_0^2 \sin 2\theta}{2}$      $\theta > \theta_0$

**Ans.: (c)**

**Solution.:**

**Range Comparison:**

In the absence of air drag, the range of a projectile is given by  $R_{vacuum} = \frac{v_0^2 \sin 2\theta_0}{g}$ .

The presence of a drag force  $\vec{F}_{drag} = -\gamma\vec{v}$  always reduces the horizontal distance traveled by the projectile compared to the ideal case. Therefore,  $R < \frac{v_0^2 \sin 2\theta_0}{g}$ .

**Angle Comparison:**

The drag force opposes the motion. The horizontal component of velocity  $v_x$  decreases continuously. The vertical component  $v_y$  goes from positive to negative. At the point of impact, the projectile has lost more horizontal speed relative to its vertical speed (in terms of the ratio  $v_y / v_x$ ) compared to the no-drag case. This results in a steeper trajectory at impact, meaning the angle of velocity with the horizontal  $\theta$  is greater than the launch angle  $\theta_0$ . Thus,  $\theta > \theta_0$ .

**Q28.** A projectile of mass  $m$  is launched from the ground with the initial speed  $v_0$  at an angle  $30^\circ$  from the horizontal. Take the ground to be horizontal. Ignoring the drag, the magnitude of Hamilton's action  $\int L dt$  for the particle from the beginning till it hits the ground is  $f \times \left( \frac{mv_0^3}{g} \right)$ .

The value of  $f$  (rounded off to two decimal places) is \_\_\_\_\_

**Ans.: 0.33**

**Solution.:**

For a projectile without drag under uniform gravity:

$$x(t) = (v_0 \cos 30^\circ)t = \frac{\sqrt{3}}{2} v_0 t \Rightarrow \dot{x} = \frac{\sqrt{3}}{2} v_0$$

$$y(t) = (v_0 \sin 30^\circ)t - \frac{1}{2} g t^2 = \frac{v_0}{2} t - \frac{1}{2} g t^2 \Rightarrow \dot{y} = \frac{v_0}{2} - g t$$

$$\text{Total time of flight: } T_0 = \frac{2v_0 \sin 30^\circ}{g} = \frac{v_0}{g}$$

The Lagrangian for a projectile (ignoring drag) is  $L = T - V = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) - mgy$

$$L = \frac{1}{2} m \left( \frac{3}{4} v_0^2 + \left( \frac{v_0}{2} - g t \right)^2 \right) - mg \left( \frac{v_0}{2} t - \frac{1}{2} g t^2 \right) = \frac{1}{2} m \left( \frac{3}{4} v_0^2 + \frac{v_0^2}{4} + g^2 t^2 - v_0 g t \right) - mg \left( \frac{v_0}{2} t - \frac{1}{2} g t^2 \right)$$

$$\Rightarrow L = \frac{1}{2} m v_0^2 + m g^2 t^2 - m g v_0 t$$

$$\text{The action } S = \int L dt = \int_0^{T_0=v_0/g} \left( \frac{1}{2} m v_0^2 + m g^2 t^2 - m g v_0 t \right) dt = \frac{m v_0^3}{g} \left( \frac{1}{2} - \frac{1}{2} + \frac{1}{3} \right) = \frac{1}{3} \frac{m v_0^3}{g} \approx 0.33 \left( \frac{m v_0^3}{g} \right)$$

**Q35.** A rocket of length  $18.0 \text{ m}$  is moving at speed  $0.9c$  (where  $c$  is the speed of light) parallel to its own length, relative to the earth. The length of the rocket measured in meters by an observer on earth (rounded off to two decimal places) is \_\_\_\_\_

**Ans.: 7.6 to 8.0**

**Solution.:**

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 18 \sqrt{1 - \frac{(0.9c)^2}{c^2}} = 8 \sqrt{0.19} \approx 7.85 \text{ m}$$

Q.36 – Q.55 Carry TWO marks Each

Q45. The Lagrangian

$$L_0 = \frac{1}{2} m \dot{q}^2 - \frac{1}{2} m \omega^2 q^2$$

with the generalized coordinate  $q$  is transformed to  $L = L_0 + \alpha \frac{df(q)}{dt}$ . Consider the following statements:

(i) Expression for the canonical momentum does not change

(ii) The equation of the motion does not change

Which of the following options is correct for the above statements?

(a) Both (i) and (ii) are correct

(b) Both (i) and (ii) are not correct

(c) (i) is correct and (ii) is not correct

(d) (i) is not correct and (ii) is correct

**Ans.: (d)**

**Solution.:** The transformation is  $L = L_0 + \alpha \frac{df(q)}{dt} = L_0 + \alpha f'(q) \dot{q} \quad \therefore \frac{df(q)}{dt} = f'(q) \dot{q}$

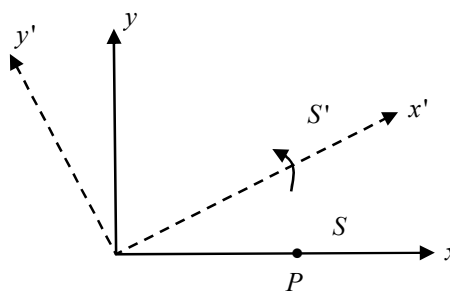
Canonical momentum becomes  $p = \frac{\partial L}{\partial \dot{q}} = \frac{\partial L_0}{\partial \dot{q}} + \alpha f'(q) = m \dot{q} + \alpha f'(q)$

Therefore, the **canonical momentum changes**.

However, adding a total time derivative to the Lagrangian does **not** change the Euler-Lagrange equation. Hence the equation of motion remains unchanged.

So: (i) False and (ii) True

**Q49.** Two frames  $S$  (solid lines) and  $S'$  (dashed lines) with common origin are shown in the figure below. Frame  $S$  is inertial while  $S'$  is rotating about the common  $z$ -axis. There is a point mass fixed at  $P$  on the  $x$ -axis of the  $S$  frame. The magnitude of the centrifugal force and the Coriolis force experienced by the mass in the  $S'$  frame is  $F_{cen}$  and  $F_{cor}$ , respectively. Which of the following options is correct for these forces?

(a)  $F_{cen} = 0$  and  $F_{cor} = 0$ (b)  $F_{cen} \neq 0$  and  $F_{cor} \neq 0$  and  $F_{cen} = \frac{F_{cor}}{2}$ (c)  $F_{cen} \neq 0$  and  $F_{cor} \neq 0$  and  $F_{cen} = 2F_{cor}$ (d)  $F_{cen} \neq 0$  and  $F_{cor} \neq 0$  and  $F_{cen} = F_{cor}$ **Ans.: (b)**

**Solution.:** The particle is fixed in the inertial frame  $S$ , but appears to move in the rotating frame  $S'$ . Its velocity in  $S'$  is  $\vec{v}' = -\vec{\omega} \times \vec{r}$ . Therefore,  $|\vec{v}'| = \omega r$ .

Centrifugal force:  $F_{cen} = m\omega^2 r$ ; Coriolis force:  $F_{cor} = 2m\omega v' = 2m\omega^2 r \Rightarrow F_{cen} = \frac{F_{cor}}{2}$

**Q55.** A symmetric rigid body has moment of inertia  $I_1, I_2, I_3$  about its principal axes 1, 2 and 3, respectively, with  $I_1 = I_3 = I_\perp$  and  $I_2 \neq I_\perp$ . It is rotating in space with no torque on it so that its angular momentum  $\vec{L}$  is constant. Let  $\omega_1, \omega_2, \omega_3$  be the components of its angular velocity along the principal axes 1, 2 and 3, respectively. Which of the following quantities is/are constant during the motion of this rigid body?

- (a)  $\omega_1 + \omega_3$  (b)  $\omega_1^2 + \omega_3^2$   
(c) Angle between axis 2 and  $\vec{L}$  (d)  $\omega_2$

**Ans.: (b), (c) and (d)**

**Solution.:**

- For a symmetric top with  $I_1 = I_3 = I_\perp$  and  $I_2 \neq I_\perp$ , the angular momentum  $\vec{L}$  is constant.
- The equations of motion are Euler's equations. For torque-free motion:
  - $\omega_2$  is constant (since  $I_1 = I_3$ ).
  - The component of  $\vec{L}$  along the symmetry axis (axis 2) is constant, so the angle between axis 2 and  $\vec{L}$  is constant.
  - The magnitude of the angular velocity in the 1-3 plane,  $\omega_\perp = \sqrt{\omega_1^2 + \omega_3^2}$ , is constant.

**Q56.** An  $\alpha$  particle moves towards a fixed nucleus carrying charge  $Ze$ , with initial speed  $v_0$  and impact parameter  $b$ . Starting from a large distance from the nucleus, its distance of closest approach is  $r_m$  and its speed there is  $v_m$ . Then which of the following options is/are correct?

$$\left( k = \frac{1}{4\pi\epsilon_0} \text{ and } r_0 = k \frac{Ze^2}{mv_0^2} \right)$$

- (a)  $v_0 b = v_m r_m$   
(b)  $v_0 b = 2v_m r_0$   
(c) For  $\frac{b}{r_0} \ll 1, r_m = 4r_0 + \frac{b^2}{2r_0}$  ignoring higher order corrections in  $\frac{b}{r_0}$   
(d) For  $\frac{b}{r_0} \ll 1, r_m = 4r_0 + \frac{b^2}{8r_0}$  ignoring higher order corrections in  $\frac{b}{r_0}$

**Ans.: (a) and (d)**

**Solution.:** Given  $r_0 = k \frac{Ze^2}{mv_0^2}$ .

For an alpha-particle, charge is  $2e$ , so the Coulomb potential is  $V(r) = k \frac{2Ze^2}{r}$

Angular momentum conservation

At infinity,  $L = mv_0 b$ ; At closest approach,  $L = mv_m r_m \Rightarrow v_0 b = v_m r_m$

So, **option (a) is correct.**

Now using energy conservation,

$$\frac{1}{2} mv_0^2 = \frac{1}{2} mv_m^2 + k \frac{2Ze^2}{r_m} \Rightarrow \frac{1}{2} mv_0^2 = \frac{1}{2} mv_m^2 + \frac{2mv_0^2 r_0}{r_m} \quad \because mv_0^2 r_0 = kZe^2$$

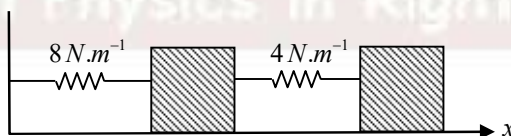
$$\Rightarrow v_m^2 = v_0^2 \left( 1 - \frac{4r_0}{r_m} \right) \Rightarrow \left( \frac{v_0 b}{r_m} \right)^2 = v_0^2 \left( 1 - \frac{4r_0}{r_m} \right) \Rightarrow \frac{b^2}{r_m^2} = 1 - \frac{4r_0}{r_m} \Rightarrow b^2 = r_m^2 - 4r_0 r_m$$

$$\Rightarrow r_m^2 - 4r_0 r_m - b^2 = 0 \Rightarrow r_m = 2r_0 + \sqrt{4r_0^2 + b^2}$$

$$\frac{b}{r_0} \ll 1: \Rightarrow r_m = 2r_0 + 2r_0 \sqrt{1 + \frac{b^2}{4r_0^2}} = 4r_0 + \frac{b^2}{8r_0} + \dots$$

So, **option (d) is correct.**

**Q64.** Two  $1\text{ kg}$  blocks are connected to two massless springs of spring constants  $8\text{ N.m}^{-1}$  and  $4\text{ N.m}^{-1}$ . The system is kept on a frictionless horizontal floor with one end of a spring attached to a wall (see figure below). They are performing oscillatory motion along the  $x$ -axis with the normal mode frequencies  $\omega_H$  and  $\omega_L$  ( $\omega_H > \omega_L$ ). The ratio  $\frac{\omega_H}{\omega_L}$  (rounded off to two decimal places) is \_\_\_\_\_



**Ans.: 2.35 to 2.45**

**Solution.:**  $m_1 = m_2 = 1\text{ kg}$ ;  $k_1 = 8\text{ N.m}^{-1}$ ,  $k_2 = 4\text{ N.m}^{-1}$

Equations of motion:  $m\ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1)$ ;  $m\ddot{x}_2 = -k_2 (x_2 - x_1)$

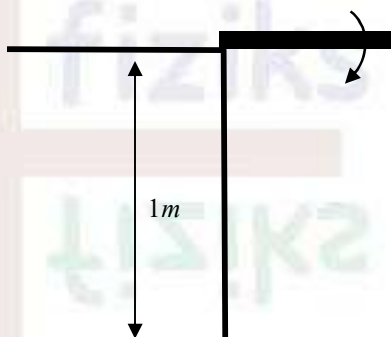
$$\ddot{x}_1 = -8x_1 + 4(x_2 - x_1); \quad \ddot{x}_2 = -4(x_2 - x_1)$$

$$\ddot{x}_1 = -12x_1 + 4x_2; \quad \ddot{x}_2 = 4x_1 - 4x_2$$

For normal modes,  $x_j = x_j e^{i\omega t}$  so,  $\begin{vmatrix} 12 - \omega^2 & -4 \\ -4 & 4 - \omega^2 \end{vmatrix} = 0 \Rightarrow (12 - \omega^2)(4 - \omega^2) - 16 = 0$

$$\Rightarrow \omega^4 - 16\omega^2 + 32 = 0 \Rightarrow \omega_{H,L}^2 = 8 \pm 4\sqrt{2}. \text{ Thus } \frac{\omega_H}{\omega_L} = \sqrt{\frac{8+4\sqrt{2}}{8-4\sqrt{2}}} = 1 + \sqrt{2} = 2.41$$

**Q65.** A  $15\text{ cm}$  long scale is held horizontally with one of its ends on the edge of a  $1\text{ m}$  high table and the other end resting on one's index finger. As the finger is removed (see figure below), the scale starts rotating about its end on the table. After  $0.1\text{ s}$ , during which it has rotated by a negligibly small angle but has gained a rotational speed as it leaves the table and falls vertically towards the ground. When its center of mass has fallen by  $0.5\text{ m}$ , it has rotated by an angle  $\theta$ . The value of  $\theta$  in degrees (rounded off to one decimal place) is \_\_\_\_\_



**Ans.: 179.0 to 180.0**

**Solution.:** Moment of inertia about the end:  $I = \frac{1}{3}mL^2$ . Torque due to gravity:  $\tau = mg \frac{L}{2}$

Therefore, angular acceleration is  $\alpha = \frac{\tau}{I} = \frac{mgL/2}{mL^2/3} \Rightarrow \alpha = \frac{3g}{2L} = \frac{3 \times 9.8}{2 \times 0.15} = 98 \text{ rad/sec}$

During the initial  $0.1\text{ sec}$ ;  $\omega = \alpha t = 98 \times 0.1 = 9.8 \text{ rad}$

After leaving the table, gravity acts through the center of mass, so there is no torque about the CM. Thus, angular velocity remains approximately constant.

The CM falls by  $0.5\text{ m}$ :  $0.5 = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{1}{g}} \approx 0.319 \text{ sec}$

Hence  $\theta = \omega t = 9.8(0.19) \approx 3.13 \text{ rad} = 3.13 \times \frac{180}{\pi} \text{ degree} \approx 179.4^\circ$

**Extra (Buffer) Questions: Carry ONE mark only**

**Q1E.** A particle has energy  $E$  and its performing periodic motion on the  $x$ -axis with its potential energy being  $V(x) = F|x|$ . If its action per period  $J = \int \vec{p} \cdot d\vec{x} \propto E^\beta$  then the value of  $\beta$  (rounded off to one decimal place) is \_\_\_\_\_

**Ans.: 1.5**

**Solution.:** For a potential power-law  $V(x) \propto |x|^k$  for  $k = 1$ .

By dimensional analysis/scaling:  $x \propto E^{1/k}$ ,  $p \propto \sqrt{mE} \propto E^{1/2}$

Action per period  $J = \int p dx \propto E^{1/2} E^{1/k} \propto E^{\frac{1}{2} + \frac{1}{k}} \propto E^{\frac{3}{2}} \propto E^\beta$  for  $k = 1$ .

$\Rightarrow \beta = 1.5$



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**GATE Physics-2026**  
**Solution-Electromagnetic Theory**

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**Q.11 – Q.35 Carry ONE mark Each**

**Q11.** In free space, an electromagnetic wave is travelling whose wavevector is  $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y})m^{-1}$ . The electric field component of this electromagnetic wave is given by  $\vec{E}(\vec{r}, t) = \hat{z} 600 \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-1}$ . The speed of light in free space is  $c = 3.0 \times 10^8 m \cdot s^{-1}$ . The corresponding magnetic field  $\vec{B}(\vec{r}, t)$  is

- (a)  $\vec{B}(\vec{r}, t) = 2 \times 10^{-6} (\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-2} \cdot s$   
 (b)  $\vec{B}(\vec{r}, t) = 10^{-6} (\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-2} \cdot s$   
 (c)  $\vec{B}(\vec{r}, t) = 2 \times 10^{-5} (-\sqrt{3}\hat{x} + \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-2} \cdot s$   
 (d)  $\vec{B}(\vec{r}, t) = 10^{-5} (\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-2} \cdot s$

**Ans.: (b)**

**Solution.:**  $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y})m^{-1}$  and  $\vec{E}(\vec{r}, t) = \hat{z} 600 \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-1}$

$$\hat{k} = \frac{\vec{k}}{|\vec{k}|} = \frac{10(\hat{x} + \sqrt{3}\hat{y})}{10\sqrt{1+3}} m^{-1} = \frac{(\hat{x} + \sqrt{3}\hat{y})}{2}$$

$$\vec{B} = \frac{1}{c} (\hat{k} \times \vec{E}) = \frac{1}{2c} (\hat{x} + \sqrt{3}\hat{y}) \times \hat{z} 600 \cos(\vec{k} \cdot \vec{r} - \omega t) = \frac{600}{6.0 \times 10^8} (-\hat{y} + \sqrt{3}\hat{x}) \cos(\vec{k} \cdot \vec{r} - \omega t)$$

$$\Rightarrow \vec{B}(\vec{r}, t) = 10^{-6} (\sqrt{3}\hat{x} - \hat{y}) \cos(\vec{k} \cdot \vec{r} - \omega t) V \cdot m^{-2} \cdot s$$

**Q12.** An infinitely large non-conducting thin sheet in the  $xy$  plane ( $z=0$ ) carries a uniform surface charge density  $\sigma = 17.70 \times 10^{-12} C \cdot m^{-2}$ . The electric field in the region  $z < 0$  is  $\vec{E}_2 = \hat{x} + 2\hat{y} + 3\hat{z}$ . Then, the electric field  $\vec{E}_1$  in the region  $z > 0$  will be ( $\epsilon_0 = 8.85 \times 10^{-12} C^2 \cdot N^{-1} \cdot m^{-2}$ )

- (a)  $\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$  (b)  $\vec{E}_1 = \hat{x} + 2\hat{y} + 4\hat{z}$   
 (c)  $\vec{E}_1 = \hat{x} + 2\hat{y} + 3\hat{z}$  (d)  $\vec{E}_1 = \hat{x} + 4\hat{y} + \hat{z}$

**Ans.: (a)**

**Solution.:** Since  $xy$  plane is the interface between two medium so  $\vec{E}_2^{\parallel} = \hat{x} + 2\hat{y} = \vec{E}_1^{\parallel}$  and  $\vec{E}_2^{\perp} = 3\hat{z}$ .

$$\therefore \vec{E}_1^{\perp} - \vec{E}_2^{\perp} = \frac{\sigma}{\epsilon_0} \hat{z} \Rightarrow \vec{E}_1^{\perp} = \vec{E}_2^{\perp} + \frac{\sigma}{\epsilon_0} \hat{z} = 3\hat{z} + \frac{17.70 \times 10^{-12}}{8.85 \times 10^{-12}} \hat{z} = 3\hat{z} + 2\hat{z} = 5\hat{z} \Rightarrow \vec{E}_1^{\perp} = 5\hat{z}$$

$$\Rightarrow \vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$$

**Q14.** For a scalar field  $\psi(\vec{r})$  and a vector field  $\vec{A}(\vec{r})$ ,  $\vec{\nabla} \times (\vec{A}\psi)$  is equivalent to the expression

- (a)  $\psi(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}\psi)$  (b)  $\psi(\vec{\nabla} \times \vec{A}) + \vec{A} \times (\vec{\nabla}\psi)$   
(c) Null vector (d)  $\vec{A} \times (\vec{\nabla}\psi) - \psi(\vec{\nabla} \times \vec{A})$

**Ans.: (a)**

**Solution.:**  $\vec{\nabla} \times (\vec{A}\psi) = \psi(\vec{\nabla} \times \vec{A}) + (\vec{\nabla}\psi) \times \vec{A} = \psi(\vec{\nabla} \times \vec{A}) - \vec{A} \times (\vec{\nabla}\psi)$

**Q15.** Which of the following options is correct for transformation of electric field  $\vec{E}$  and magnetic field  $\vec{B}$  under time reversal, i.e.,  $t \rightarrow -t$ ?

- (a)  $\vec{E} \rightarrow \vec{E}$  and  $\vec{B} \rightarrow \vec{B}$  (b)  $\vec{E} \rightarrow -\vec{E}$  and  $\vec{B} \rightarrow \vec{B}$   
(c)  $\vec{E} \rightarrow \vec{E}$  and  $\vec{B} \rightarrow -\vec{B}$  (d)  $\vec{E} \rightarrow -\vec{E}$  and  $\vec{B} \rightarrow -\vec{B}$

**Ans.: (c)**

**Solution.:** Consider the Lorentz force:  $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Under time reversal,  $\vec{v} \rightarrow -\vec{v}$  so to preserve the form of the electromagnetic equations  $\vec{F}$ ,

$$\vec{E} \rightarrow \vec{E} \text{ and } \vec{B} \rightarrow -\vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q(\vec{E} + (-\vec{v}) \times (-\vec{B})).$$

**Q19.** For the electric field of an electromagnetic wave given below, which of the following statements is correct?  $\vec{E} = \hat{x}E_0 \cos(\omega t) + \hat{y}2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$

- (a) The electric field is linearly polarised with slope 2  
(b) The electric field is circularly polarised with radius  $E_0$   
(c) The electric field is elliptically polarised with a ratio of major to minor axis being 2  
(d) The electric field is unpolarised with the two components being phase shifted by  $\pi/2$

**Ans.: (c)**

**Solution.:**  $\vec{E} = \hat{x}E_0 \cos(\omega t) + \hat{y}2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$

$$\Rightarrow E_x = E_0 \cos(\omega t), E_y = 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -2E_0 \sin(\omega t)$$

$$\text{Thus } \frac{E_x^2}{E_0^2} + \frac{E_y^2}{4E_0^2} = 1 \Rightarrow \frac{E_x^2}{E_0^2} + \frac{E_y^2}{(2E_0)^2} = 1$$

Thus, electric field is elliptically polarised with a ratio of major to minor axis being 2.

**Q30.** A dielectric sphere carries a uniform polarization  $P = 26 \mu C.cm^{-2}$ . The magnitude of the electric field at the center of the sphere is  $E \times 10^9 N.C^{-1}$ . The value of  $E$  (rounded off to one decimal place) is \_\_\_\_\_

$$(\epsilon_0 = 8.85 \times 10^{-12} C^2.N^{-1}.m^{-2})$$

**Ans.: 9.0 to 10.0**

**Solution.:** For a uniformly polarized sphere, the depolarization field inside is  $\vec{E}_1 = -\frac{\vec{P}}{3\epsilon_0}$ .

Therefore, the magnitude is  $E_1 = \frac{P}{3\epsilon_0}$ .

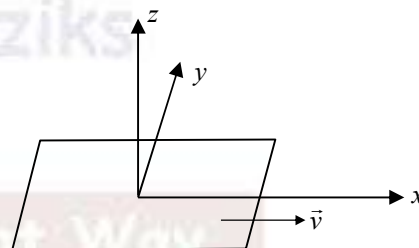
$$P = 26 \mu C.cm^{-2} = \frac{26 \times 10^{-6}}{10^{-4}} C.m^{-2} = 0.26 C.m^{-2}$$

$$\Rightarrow E_1 = \frac{0.26}{3 \times 8.85 \times 10^{-12}} N.C^{-1} = 9.8 \times 10^9 N.C^{-1} = E \times 10^9 N.C^{-1} \Rightarrow E = 9.8$$

**Q.36 – Q.55 Carry TWO marks Each**

**Q47.** An infinitely large thin sheet in the  $xy$ -plane carries uniform positive charge density and is moving with constant velocity  $\vec{v}$  in the  $+x$  direction (see figure below). The direction of the corresponding Poynting vector is

- (a)  $+x$  for both  $z < 0$  and  $z > 0$
- (b)  $+x$  for  $z < 0$  and  $-x$  for  $z > 0$
- (c)  $-x$  for  $z < 0$  and  $+x$  for  $z > 0$
- (d)  $-x$  for both  $z < 0$  and  $z > 0$



**Ans.: (a)**

**Solution.:** Thin sheet in the  $xy$ -plane carries uniform positive charge density and is moving with constant velocity  $\vec{v} = v \hat{x}$  then surface current density  $\vec{K} \rightarrow \hat{x}$ .

$$z > 0: \vec{E} \rightarrow \hat{z}, \vec{B} \rightarrow -\hat{y} \text{ so } \vec{S} \propto (\vec{E} \times \vec{B}) \rightarrow \hat{z} \times (-\hat{y}) \rightarrow +\hat{x}$$

$$z < 0: \vec{E} \rightarrow -\hat{z}, \vec{B} \rightarrow +\hat{y} \text{ so } \vec{S} \propto (\vec{E} \times \vec{B}) \rightarrow (-\hat{z}) \times \hat{y} \rightarrow +\hat{x}$$

**Q48.** A positive point charge is fixed at the origin. At some distance from it on the  $x$  axis, a point dipole is kept pointing in the  $+y$  direction. The force on the dipole is

- (a) 0 (b) in the  $+y$  direction  
(c) in the  $-y$  direction (d) in the  $+x$  direction

**Ans.: (b)**

**Solution.:** Let the positive charge be  $q$  and let the dipole be at  $(x, 0, 0)$

Its dipole moment is  $\vec{p} = p\hat{y}$ .

The force on a dipole in a nonuniform electric field is  $\vec{F} = (\vec{p} \cdot \nabla) \vec{E}$

where  $\vec{E} = \frac{q}{4\pi\epsilon_0 r^3} (x\hat{x} + y\hat{y} + z\hat{z})$

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E} = \left( p \frac{\partial}{\partial y} E_y \right) \hat{y} \quad \text{where} \quad p \frac{\partial E_y}{\partial y} = p \frac{\partial}{\partial y} \frac{qy}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \bigg|_{y=z=0} = \frac{pq}{4\pi\epsilon_0 x^3}$$

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E} = \frac{pq}{4\pi\epsilon_0 x^3} \hat{y}$$

**Q57.** Consider a particle of mass  $m = 9.0 \times 10^{-5} \text{ kg}$  and charge  $q = 3.0 \times 10^{-4} \text{ C}$  in a uniform electromagnetic field  $\vec{E} = 2\hat{x} \text{ V.m}^{-1}$ ,  $\vec{B} = 3\hat{z} \text{ V.m}^{-2} \cdot \text{s}$ . The particle is released from the coordinates  $(0, 5m, 0)$  at time  $t = 0$ . Starting from initial speed zero, it comes back to the  $y$ -axis for the first time at time  $t$ . The value of  $t$  in seconds (rounded off to two decimal places) is

**Ans.: 0.60 to 0.70**

**Solution.:** Cyclotron Frequency:  $\omega = \frac{qB}{m} = \frac{(3.0 \times 10^{-4})(3)}{9.0 \times 10^{-5}} = 10 \text{ rad/sec}$

The motion in the  $x$ - $y$  plane is cycloidal. The  $x$ -coordinate as a function of time is:

$$x(t) = \frac{E}{B} \left( t - \frac{\sin \omega t}{\omega} \right)$$

Starting from initial speed zero, it comes back to the  $y$ -axis for the first time at time  $t$ ,  $x(t) = 0$

$$\Rightarrow \omega t = \sin \omega t \Rightarrow \omega t = 2\pi \Rightarrow t = \frac{2\pi}{\omega} = \frac{2\pi}{10} \approx 0.63 \text{ sec}$$

**Q61.** A capacitor is made of two circular metal plates of radius  $1\text{ m}$  separated by a distance of  $d = 1\text{ mm}$ . The space between them is filled by a dielectric with permittivity  $\epsilon_r = 5$ . The capacitor is connected to a voltage  $V = 10 \sin(2\pi \times 10^6 \times t)$  volts, where  $t$  is in seconds. The maximum value of the magnetic field in between the plates at a radial distance  $r = 0.5\text{ m}$  from the center of the capacitor is  $B \times 10^{-6}\text{ T}$ . The value of  $B$  (rounded off to two decimal places) is \_\_\_\_\_

(Speed of light in vacuum  $c = 3 \times 10^8\text{ m.s}^{-1}$ )

**Ans.: 0.80 to 0.90**

**Solution.:** The electric field between the plates is approximately  $E = \frac{V}{d} \Rightarrow \frac{\partial E}{\partial t} = \frac{1}{d} \frac{\partial V}{\partial t}$

The displacement current density is  $J_d = \epsilon \frac{\partial E}{\partial t} = \frac{\epsilon_0 \epsilon_r}{d} \frac{\partial V}{\partial t}$

Only the area of radius  $r$  is enclosed by the Amperian loop:  $I_d = J_d \times \pi r^2 = \frac{\epsilon_0 \epsilon_r}{d} \frac{\partial V}{\partial t} \pi r^2$

From Ampere's Law:  $B \times 2\pi r = \mu_0 \left( \frac{\epsilon_0 \epsilon_r}{d} \frac{\partial V}{\partial t} \pi r^2 \right) \Rightarrow B = \mu_0 \frac{\epsilon_0 \epsilon_r}{2d} r \frac{\partial V}{\partial t}$

$$\Rightarrow B_{\max} = \mu_0 \frac{\epsilon_0 \epsilon_r}{2d} r \left. \frac{\partial V}{\partial t} \right|_{\max} = \mu_0 \frac{\epsilon_0 \epsilon_r}{2d} r V_0 \omega \Rightarrow B_{\max} = \mu_0 \frac{\epsilon_0 (5)}{2d} r V_0 \omega = \frac{5r V_0 \omega}{2c^2 d}$$

$$\Rightarrow B_{\max} = \frac{5(0.5)(10)(2\pi \times 10^6)}{2(3 \times 10^8)^2 (10^{-3})} = 0.873 \times 10^{-6}\text{ T} = B \times 10^{-6}\text{ T} \Rightarrow B = 0.87$$

**Extra (Buffer) Questions: Carry ONE mark only**

**Q1C.** If the intensity of solar radiation is  $1200\text{ W.m}^{-2}$ , the pressure (in micro-Pascals) exerted by sunlight incident on a perfectly reflecting mirror in the normal direction is (speed of light in vacuum is  $c = 3 \times 10^8\text{ m.s}^{-1}$ )

- (a) 8 (b) 4 (c) 16 (d) 0

**Ans.: (a)**

**Solution.:** Radiation Pressure for Perfect Reflection:

$$P = \frac{2I}{c} = \frac{2 \times 1200}{3 \times 10^8} = 8 \times 10^{-6}\text{ Pa} = 8\text{ }\mu\text{Pa}$$



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**GATE Physics-2026**  
**Solution-Quantum Mechanics**

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**Q.11 – Q.35 Carry ONE mark Each**

**Q26.** If the perturbation  $V = \lambda x^3$  is added to the Hamiltonian of a one-dimensional harmonic oscillator, the matrix element  $\langle m|V|0\rangle$  is/are non-zero for which of the following states? Here, the eigenstates of the harmonic oscillator are denoted by  $|n\rangle$ .

- (a)  $|m=3\rangle$  (b)  $|m=1\rangle$   
(c)  $|m=2\rangle$  (d)  $|m=5\rangle$

**Ans.: (a) and (b)**

**Solution.:** For a 1D harmonic oscillator, the position operator  $x$  in terms of ladder operators

$$a \text{ and } a^\dagger \text{ is: } x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger) \Rightarrow x^3 \propto (a + a^\dagger)^3$$

$$\because a|0\rangle = 0 \text{ and } a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$(a + a^\dagger)|0\rangle = |1\rangle \Rightarrow (a + a^\dagger)^2|0\rangle = (a + a^\dagger)|1\rangle = |0\rangle + \sqrt{2}|2\rangle$$

$$\Rightarrow (a + a^\dagger)^3|0\rangle = (a + a^\dagger)|0\rangle + \sqrt{2}|2\rangle = |1\rangle + |3\rangle$$

Therefore,  $\langle m|V|0\rangle \neq 0$  only for  $m=1$  and  $m=3$ .

**Q29.** Consider an electron in the energy eigenstate  $\psi_{211}(\vec{r})$  of the hydrogen atom. Given that the radial probability distribution of the electron in such a state takes its maximum value at  $r = n_0 a$ , where  $a$  is the Bohr radius, and  $n_0$  is an integer. The value of  $n_0$  (in integer) is \_\_\_\_\_

The radial part of the wavefunction  $\psi_{211}(\vec{r})$  is given by  $R_{21}(r) = \frac{1}{\sqrt{24}a^5} r e^{-r/2a}$ .

**Ans.: 4**

**Solution.:** Radial Probability Distribution function

$$P(r) = r^2 |R_{21}(r)|^2 = \frac{1}{24a^5} r^4 e^{-r/a}$$

Finding the maximum of  $P(r)$ : take the derivative with respect to  $r$  and set it to 0

$$\frac{dP}{dr} = \frac{1}{24a^5} \left[ 4r^3 e^{-r/a} + r^4 e^{-r/a} \left( -\frac{1}{a} \right) \right] = 0 \Rightarrow 4r^3 e^{-r/a} = \frac{r^4}{a} e^{-r/a} \Rightarrow r = 4a = n_0 a$$

$$\Rightarrow n_0 = 4$$

**Q.36 – Q.55 Carry TWO marks Each**

**Q39.** The Hamiltonian for a quantum particle of mass  $m$  is given below, where  $\omega < \Omega$ . The Schrodinger equation for this system can be solved exactly using the orthogonal transformations:

$$x = \frac{x_1 - x_2}{\sqrt{2}} \text{ and } y = \frac{x_1 + x_2}{\sqrt{2}}.$$

$$H = -\frac{\hbar^2}{2m} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] + \frac{1}{2} m \Omega^2 (x^2 + y^2) + m \omega^2 xy$$

The ground state energy of this system is

- (a)  $\frac{\hbar}{2} \left[ \sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2} \right]$  (b)  $\hbar \left[ \sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2} \right]$   
 (c)  $\frac{\hbar}{2} \left[ \sqrt{\Omega^2 + \Omega\omega} + \sqrt{\Omega^2 - \Omega\omega} \right]$  (d)  $\hbar \left[ \sqrt{\Omega^2 + \Omega\omega} - \sqrt{\Omega^2 - \Omega\omega} \right]$

**Ans.: (a)**

**Solution.:**

$$\because x = \frac{x_1 - x_2}{\sqrt{2}} \text{ and } y = \frac{x_1 + x_2}{\sqrt{2}}$$

$$\Rightarrow x^2 + y^2 = \frac{(x_1 - x_2)^2 + (x_1 + x_2)^2}{2} = x_1^2 + x_2^2 \text{ and } xy = \frac{(x_1 - x_2)(x_1 + x_2)}{2} = \frac{x_1^2 - x_2^2}{2}$$

Therefore, the potential becomes

$$V = \frac{1}{2} m \Omega^2 (x^2 + y^2) + m \omega^2 xy = \frac{1}{2} m \Omega^2 (x_1^2 + x_2^2) + m \omega^2 \frac{(x_1^2 - x_2^2)}{2}$$

$$\Rightarrow V = \frac{1}{2} m (\Omega^2 + \omega^2) x_1^2 + \frac{1}{2} m (\Omega^2 - \omega^2) x_2^2$$

Thus, we have two independent harmonic oscillators with frequencies

$$\omega_1 = \sqrt{\Omega^2 + \omega^2} \text{ and } \omega_2 = \sqrt{\Omega^2 - \omega^2}$$

The ground-state energy of each oscillator is

$$E_0 = \frac{1}{2} \hbar \omega_1 + \frac{1}{2} \hbar \omega_2 = \frac{\hbar}{2} \left[ \sqrt{\Omega^2 + \omega^2} + \sqrt{\Omega^2 - \omega^2} \right]$$

**Q40.** Consider two particles with angular momenta  $j_1 = 2\hbar$  and  $j_2 = \hbar/2$ . If the expression

$$|j = 5/2, m = 3/2\rangle = \begin{cases} c_1 |j_1 = 2, m_1 = 1\rangle |j_2 = 1/2, m_2 = 1/2\rangle + \\ c_2 |j_1 = 2, m_1 = 2\rangle |j_2 = 1/2, m_2 = -1/2\rangle \end{cases}$$

gives an eigenstate of the total angular momentum of the two particles, using standard notation.

Which of the following is true?

(Hint:  $\hat{J}_{\pm} |j, m\rangle = \sqrt{j(j+1) - m(m \pm 1)} |j, m \pm 1\rangle$ )

(a)  $c_1 = \frac{2}{\sqrt{5}}, c_2 = \frac{1}{\sqrt{5}}$

(b)  $c_1 = \frac{1}{\sqrt{5}}, c_2 = \frac{2}{\sqrt{5}}$

(c)  $c_1 = \frac{1}{\sqrt{2}}, c_2 = \frac{1}{\sqrt{2}}$

(d)  $c_1 = 0, c_2 = 1$

**Ans.: (a)**

**Solution.:**  $\hat{J}_{\pm} |j, m\rangle = \sqrt{j(j+1) - m(m \pm 1)} |j, m \pm 1\rangle$

Apply the raising operator  $\hat{J}_+ = \hat{J}_{1+} + \hat{J}_{2+}$  to state  $|j = 5/2, m = 3/2\rangle$

$$\hat{J}_+ |j = 5/2, m = 3/2\rangle = \sqrt{\frac{5}{2} \cdot \frac{7}{2} - \frac{3}{2} \cdot \frac{5}{2}} \left| \frac{5}{2}, \frac{5}{2} \right\rangle = \sqrt{5} \left| \frac{5}{2}, \frac{5}{2} \right\rangle$$

$$\hat{J}_+ (c_1 |2, 1\rangle |1/2, 1/2\rangle) = c_1 (\sqrt{2(3) - 1(2)} |2, 2\rangle |1/2, 1/2\rangle + 0) = 2c_1 |2, 2\rangle |1/2, 1/2\rangle$$

$$\hat{J}_+ (c_2 |2, 2\rangle |1/2, -1/2\rangle) = c_2 \left( 0 + \sqrt{\frac{1}{2} \left( \frac{3}{2} \right) - \left( -\frac{1}{2} \right) \left( \frac{1}{2} \right)} |2, 2\rangle |1/2, 1/2\rangle \right) = c_2 |2, 2\rangle |1/2, 1/2\rangle$$

$$\text{Equating both sides: } \sqrt{5} \left| \frac{5}{2}, \frac{5}{2} \right\rangle = (2c_1 + c_2) |2, 2\rangle |1/2, 1/2\rangle$$

$$\text{We know that: } \left| \frac{5}{2}, \frac{5}{2} \right\rangle = |2, 2\rangle |1/2, 1/2\rangle \Rightarrow 2c_1 + c_2 = \sqrt{5} \text{ also } c_1^2 + c_2^2 = 1$$

$$\text{Solving : } c_1 = \frac{2}{\sqrt{5}}, c_2 = \frac{1}{\sqrt{5}}$$

**Q41.** The energy  $E$  and degeneracy  $d$  of the second excited state of a three-dimensional, isotropic quantum harmonic oscillator with angular frequency  $\omega$  are

(a)  $E = \frac{7}{2} \hbar \omega, d = 6$

(b)  $E = \frac{7}{2} \hbar \omega, d = 3$

(c)  $E = \frac{5}{2} \hbar \omega, d = 3$

(d)  $E = \frac{5}{2} \hbar \omega, d = 6$

**Ans.: (a)**

**Solution.:** For a three-dimensional isotropic harmonic oscillator,

$$E = \left( N + \frac{3}{2} \right) \hbar \omega \text{ where } N = n_x + n_y + n_z$$

For second excited state:  $N = 2$  and  $E = \frac{7}{2} \hbar \omega$

We need the number of non-negative integer solutions of  $n_x + n_y + n_z = 2$ .

The possibilities are: (2,0,0), (0,2,0), (0,0,2) and (1,1,0), (1,0,1), (0,1,1).

Thus,  $d = 6$ .

**Q52.** Consider operators  $\hat{A}, \hat{B}$  and  $\hat{C}$  for three observables of a quantum system satisfying  $[\hat{A}, \hat{B}] = 0, [\hat{B}, \hat{C}] = 0$  and  $[\hat{A}, \hat{C}] \neq 0$ , with uncertainties  $\Delta A, \Delta B, \Delta C$ , respectively. From the options given below, which is/are implied by the commutation relations among  $\hat{A}, \hat{B}$  and  $\hat{C}$ ?

- (a)  $\Delta A \Delta B > 0$
- (b)  $\Delta A \Delta C > 0$
- (c)  $\hat{A}, \hat{B}$  can be simultaneously diagonalized
- (d)  $\hat{A}, \hat{B}, \hat{C}$  can be simultaneously diagonalized

**Ans.: (b) and (c)**

**Solution.:**  $[\hat{A}, \hat{B}] = 0, [\hat{B}, \hat{C}] = 0$  and  $[\hat{A}, \hat{C}] \neq 0$

Two commuting operators share a complete set of simultaneous eigenfunctions. Thus,  $\hat{A}$  and  $\hat{B}$  can be simultaneously diagonalized.

(a) The uncertainty relation gives  $\Delta A \Delta B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle| = 0$ . Since  $[\hat{A}, \hat{B}] = 0$

This does **not** imply that the product must be strictly greater than zero.

It can be zero, for example in a simultaneous eigenstate.

$\Delta A \Delta B > 0$  is not true

(b) The uncertainty relation gives  $\Delta A \Delta C \geq \frac{1}{2} |\langle [\hat{A}, \hat{C}] \rangle| > 0$ . Since  $[\hat{A}, \hat{C}] \neq 0$

$\Delta A \Delta C > 0$  is true.

(c)  $\hat{A}, \hat{B}$  can be simultaneously diagonalized - True, since they commute.

(d)  $\hat{A}, \hat{B}, \hat{C}$  can be simultaneously diagonalized- False, since  $A$  and  $C$  don't commute.

**Q59.** The Hamiltonian of two interacting spin-1/2 particles is  $H = \frac{A}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2$ , where  $\vec{S}_1$  and  $\vec{S}_2$  are the spin angular momenta of particles 1 and 2, respectively. Here,  $A = 10.56 \text{ eV}$ . The energy in  $\text{eV}$  required to induce an excitation from the ground state to the excited state (rounded off to two decimal places) is \_\_\_\_\_

**Ans.: 10.56**

**Solution.:**  $H = \frac{A}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2$  and  $A = 10.56 \text{ eV}$

$$\text{Use } \vec{J} = \vec{S}_1 + \vec{S}_2 \Rightarrow J^2 = S_1^2 + S_2^2 + 2\vec{S}_1 \cdot \vec{S}_2 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}(J^2 - S_1^2 - S_2^2)$$

$$\text{For two spin-1/2 particles: } S_1^2 = S_2^2 = \frac{3}{4} \hbar^2.$$

There are two possible total spins.

$$\text{Singlet state: } S = 0 \Rightarrow J^2 = S(S+1)\hbar^2 = 0 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} \left( 0 - \frac{3}{4} \hbar^2 - \frac{3}{4} \hbar^2 \right) = -\frac{3}{4} \hbar^2$$

$$E_{\text{singlet}} = \frac{A}{\hbar^2} \left( -\frac{3}{4} \hbar^2 \right) = -\frac{3A}{4}$$

$$\text{Triplet state: } S = 1 \Rightarrow J^2 = S(S+1)\hbar^2 = 2\hbar^2 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2} \left( 2\hbar^2 - \frac{3}{4} \hbar^2 - \frac{3}{4} \hbar^2 \right) = \frac{1}{4} \hbar^2$$

$$E_{\text{triplet}} = \frac{A}{\hbar^2} \left( \frac{1}{4} \hbar^2 \right) = \frac{A}{4}$$

$$\text{The excitation energy is } \Delta E = E_{\text{triplet}} - E_{\text{singlet}} = \frac{A}{4} - \left( -\frac{3A}{4} \right) = A \Rightarrow \Delta E = 10.56 \text{ eV}$$

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**Extra (Buffer) Questions: Carry ONE mark only**

**Q1B.** Consider a spin  $\frac{1}{2}$  quantum particle initially in the eigenstate of spin along  $x$ -direction with eigenvalue  $\frac{\hbar}{2}$ . If a constant uniform magnetic field of magnitude  $B_0$  along the  $y$ -direction is applied for a time  $t$ , and after this a measurement is made of the spin along the  $z$ -direction. The probability,  $P(t)$ , of finding the measurement result  $+\frac{\hbar}{2}$  satisfies (with  $\omega = \frac{\gamma B_0 \hbar}{2}$ ,  $\gamma$  is the gyromagnetic ratio of the particle):

Possibly Useful Information – The Pauli matrices are  $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ ,  $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(a)  $P\left(t = \frac{\pi}{4\omega}\right) = \frac{1}{2}$

(b)  $P\left(t = \frac{\pi}{2\omega}\right) = 1$

(c)  $P\left(t = \frac{3\pi}{4\omega}\right) = \frac{1}{3}$

(d)  $P\left(t = \frac{\pi}{\omega}\right) = \frac{1}{2}$

**Ans.: (d)**

**Solution.:** The particle initially has spin along the positive  $x$ -direction.

A magnetic field along the  $y$ -direction causes the spin to rotate about the  $y$ -axis.

The probability of measuring spin  $+\frac{\hbar}{2}$  along the  $z$ -direction is  $P_{+z}(t) = \frac{1}{2}(1 + \sin 2\omega t)$ .

Check option (d): for  $t = \frac{\pi}{\omega} \Rightarrow P\left(t = \frac{\pi}{\omega}\right) = \frac{1}{2}(1 + \sin 2\pi t) = \frac{1}{2}$

**Extra (Buffer) Questions: Carry TWO mark only**

**Q2B.** A particle of mass  $m$  is travelling from far left to right along the  $x$ -axis with momentum  $\hbar k$  in the presence of a localized potential  $V(x)$ . The integral representation of the time-independent Schrodinger equation for this system is given by

$$\psi(x) = e^{ikx} - \frac{im}{\hbar^2 k} \int_{-\infty}^{\infty} e^{ik|x-x'|} V(x') \psi(x') dx'$$

Within the first-order Born approximation, the reflection amplitude for the potential  $V(x) = V_0 \delta(x)$  is

(a)  $\frac{-imV_0}{\hbar^2 k}$

(b) 0

(c)  $\frac{-2imV_0}{\hbar^2 k}$

(d)  $\frac{-imV_0}{2\hbar^2 k}$

**Ans.: (a)**



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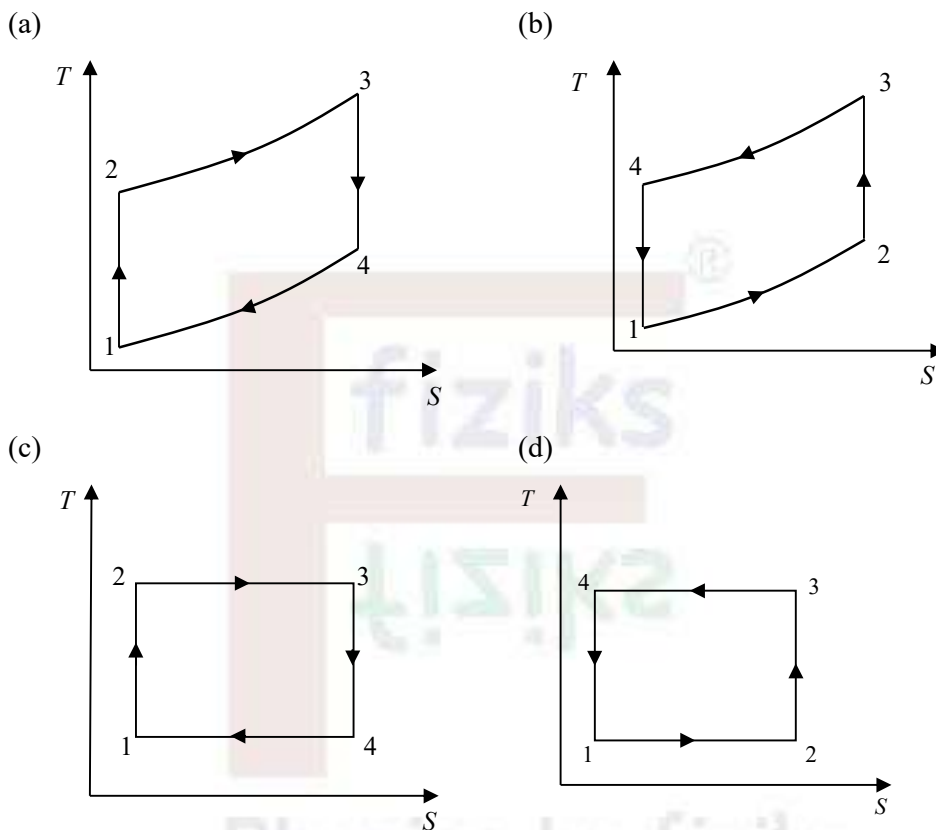
Solution-Thermodynamics and Statistical Mechanics

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Q.11 – Q.35 Carry ONE mark Each

Q17. Consider the Otto cycle for an ideal gas engine consisting of two quasistatic adiabatic and two quasistatic isochoric processes. The correct temperature-entropy ( $T$ - $S$ ) phase diagram for the cycle is



Ans.: (a)

**Solution.:** Adiabatic processes are vertical lines in a  $T$ - $S$  diagram.

For an isochoric process,  $dQ = nC_V dT \Rightarrow dS = \frac{nC_V dT}{T}$

Therefore, entropy changes while temperature changes, so the isochoric processes are **curved lines** in the  $T$ - $S$  plane.

During heat addition at constant volume ( $2 \rightarrow 3$ ), entropy  $S$  increases as temperature  $T$  increases.

During adiabatic expansion ( $3 \rightarrow 4$ ), entropy remains constant while temperature decreases.

During heat rejection at constant volume ( $4 \rightarrow 1$ ), entropy and temperature both decrease.

During adiabatic compression ( $1 \rightarrow 2$ ), entropy remains constant while temperature increases.

$1 \rightarrow 2$ : Adiabatic compression;       $2 \rightarrow 3$ : Isochoric heat addition

$3 \rightarrow 4$ : Adiabatic expansion;       $4 \rightarrow 1$ : Isochoric heat rejection

**Q18.** The formula for energy  $E$  of a photon gas at temperature  $T$  in a two-dimensional box at equilibrium with  $g_{2d}(\nu)$  denoting the density of states of photons is given below where symbols  $\nu, h$  and  $k_B$  have their standard meaning. The specific heat ( $C_V$ ) of this photon gas obeys

$$E = \int_0^\infty d\nu g_{2d}(\nu) \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

- (a)  $C_V \propto T$  (b)  $C_V \propto T^2$  (c)  $C_V \propto T^3$  (d)  $C_V \propto T^4$

**Ans.: (b)**

**Solution.:** In a 2D system, the density of states for photons is  $g_{2d}(\nu) \propto \nu$ .

The total internal energy  $E$  is given by:

$$E = \int_0^\infty d\nu g_{2d}(\nu) \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \propto \int_0^\infty \frac{\nu^2}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} d\nu$$

$$\text{Let } x = \frac{h\nu}{k_B T} \Rightarrow d\nu = \frac{k_B T}{h} dx \Rightarrow E \propto T^3 \int_0^\infty \frac{x^2}{\exp(x) - 1} dx \Rightarrow E \propto T^3$$

$$\text{Specific heat } C_V = \frac{\partial E}{\partial T} \propto T^2.$$

**NOTE:** For photons in  $d$  spatial dimensions,  $g_{2d}(\nu) \propto \nu^{d-1}$ ,  $E \propto \nu^{d+1}$ ,  $C_V \propto T^d$

**Q20.** A gas of non-interacting  ${}^4\text{He}$  atoms (of mass  $m$ ) is in a three-dimensional trap whose energy levels can be approximated by those of a harmonic oscillator potential  $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$ . The chemical potential of the gas at  $T = 0\text{ K}$  is

- (a) 0 (b)  $\frac{1}{2}\hbar\omega$  (c)  $\frac{3}{2}\hbar\omega$  (d)  $3\hbar\omega$

**Ans.: (c)**

**Solution.:** The atoms are  ${}^4\text{He}$ , which are **bosons** because the total spin of  ${}^4\text{He}$  is integer (zero).

The particles are confined in a three-dimensional harmonic oscillator potential:

$$V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$$

The energy levels of a 3D harmonic oscillator are

$$E_{n_x, n_y, n_z} = \hbar\omega\left(n_x + n_y + n_z + \frac{3}{2}\right)$$

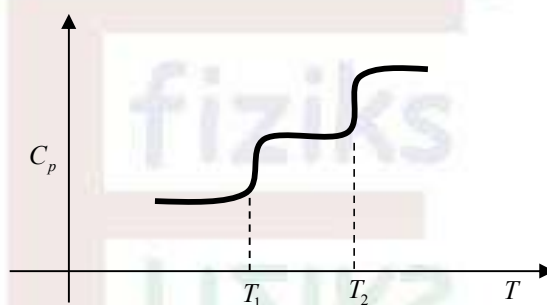
At  $T = 0\text{ K}$ , all bosons occupy the **lowest energy state**.

For  $n_x = 0, n_y = 0, n_z = 0$ ;  $E_0 = \hbar\omega \left(0 + 0 + 0 + \frac{3}{2}\right) = \frac{3}{2} \hbar\omega$

For an ideal Bose gas at  $T = 0\text{ K}$ , the chemical potential approaches the ground-state energy:

$$\mu(T = 0) = E_0 = \frac{3}{2} \hbar\omega$$

**Q25.** Schematic variation of the specific heat  $C_p$  of an ideal gas of diatomic molecules with temperature  $T$  is shown in the figure below. For rotational energy  $E_R$  and vibrational energy  $E_v$  of the molecule, which of the following options is/are correct? Here  $k_B$  is the Boltzmann constant.



(a)  $E_R \cong k_B T_1$

(b)  $E_R \cong k_B T_2$

(c)  $E_v \cong k_B T_1$

(d)  $E_v \cong k_B T_2$

**Ans.: (a) and (d)**

**Solution.:**

For a diatomic molecule, different degrees of freedom become thermally accessible at different temperatures.

At low temperature:

- Translational degrees of freedom contribute.
- Rotational and vibrational modes may be frozen out.

As temperature increases, **rotational states** become populated first because their energy spacing is relatively small.

Thus, the first rise in  $C_p$ , at  $T_1$ , corresponds to rotational excitation so  $E_R \cong k_B T_1$ .

At still higher temperature, vibrational states become populated. The second rise, at  $T_2$ , corresponds to vibrational excitation so  $E_v \cong k_B T_2$ .

**Q33.** The specific heat  $C_p(T)$  of one mole of a material as a function of temperature  $T$  is given as  $C_p(T) = AT + BT^3$ , where  $A = 0.695 \text{ mJ.mol}^{-1}.\text{K}^{-2}$  and  $B = 0.045 \text{ mJ.mol}^{-1}.\text{K}^{-4}$ . When  $T$  is changed from  $1\text{K}$  to  $10\text{K}$  at constant pressure, then the change in entropy  $\Delta S$  in  $\text{mJ.mol}^{-1}.\text{K}^{-1}$  (rounded off to one decimal place) is \_\_\_\_\_

**Ans.: 21.1 to 21.3**

**Solution.:** At constant pressure,  $dS = \frac{C_p(T)dT}{T}$

$$\Rightarrow \Delta S = \int_1^{10} \frac{1}{T} (AT + BT^3) dT = \left[ AT + B \frac{T^3}{3} \right]_1^{10} = 9A + \frac{999}{3} B$$

$$\Rightarrow \Delta S = 9(0.695) + \frac{999}{3}(0.045) = 6.255 + 14.985 = 21.240 \text{ mJ.mol}^{-1}.\text{K}^{-2}$$

**Q.36 – Q.55 Carry TWO marks Each**

**Q42.** Two identical particles with a fixed total energy  $E = 2\hbar\omega$  are in thermal equilibrium in a one-dimensional harmonic oscillator potential  $\frac{1}{2}m\omega^2 x^2$ . Let the entropy of the particles be denoted by  $S_F$  if they are fermions with spin  $\frac{1}{2}\left(s_z = \pm \frac{\hbar}{2}\right)$  and  $S_B$  if they are bosons with spin 0. Then, which of the following options is correct? ( $k_B$  is the Boltzmann constant)

- (a)  $S_F = k_B \ln 2, S_B = k_B \ln 2$  (b)  $S_F = 2k_B \ln 2, S_B = 0$   
 (c)  $S_F = 4k_B \ln 2, S_B = 0$  (d)  $S_F = 2k_B \ln 2, S_B = k_B \ln 2$

**Ans.: (b)**

**Solution.:** Two identical particles in a 1D harmonic oscillator

The total energy is  $E = 2\hbar\omega$

For a one-dimensional harmonic oscillator  $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

So,  $E_0 = \frac{1}{2}\hbar\omega, E_1 = \frac{3}{2}\hbar\omega, E_2 = \frac{5}{2}\hbar\omega, \dots$

For two particles, the lowest possible total energy is  $E_0 + E_0 = \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \hbar\omega$

We need  $E_{total} = 2\hbar\omega$

The only possibility is  $E_0 + E_1 = \frac{1}{2}\hbar\omega + \frac{3}{2}\hbar\omega = 2\hbar\omega$ .

Thus, one particle must be in  $n = 0$  and the other in  $n = 1$ .

We have to compare fermions with spin  $\frac{1}{2}$  and bosons with spin 0.

**Fermions:** Each spatial level has two possible spin states:  $s_z = \pm \frac{\hbar}{2}$

One fermion occupies  $n = 0$ , and the other occupies  $n = 1$ .

The particle in  $n = 0$  has two spin possibilities. The particle in  $n = 1$  also has two spin possibilities.

Therefore, the number of allowed many-particle states is  $\Omega_F = 2 \times 2 = 4$

The entropy is  $S_F = k_B \ln \Omega_F = k_B \ln 4 = 2k_B \ln 2$

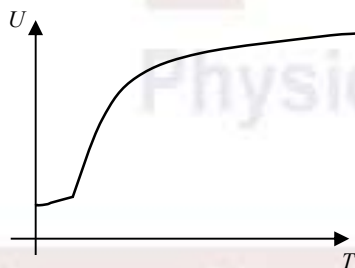
**Bosons:** For spin-0 bosons, there is no spin degeneracy.

At total energy  $2\hbar\omega$ , the configuration is  $\{n_0, n_1\} = \{1, 1\}$ . There is only **one** allowed occupation configuration.

Thus  $\Omega_B = 1$ , entropy is  $S_B = k_B \ln \Omega_B = k_B \ln 1 = 0$ .

**Q46.** A gas of  $N$  classical particles that can occupy energy levels,  $\epsilon_1$  and  $\epsilon_2 = \epsilon_1 + \Delta$  is in equilibrium with a reservoir at temperature  $T$ . From the schematics shown below, choose the correct dependence of the internal energy  $U$  on  $T$ .

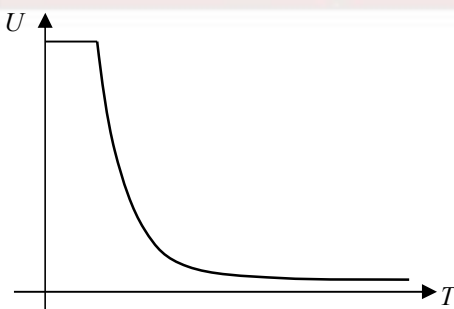
(a)



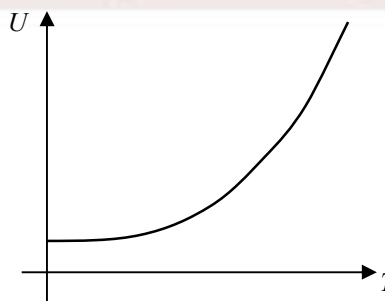
(b)



(c)



(d)



**Ans.: (a)**

**Solution.:** For a single particle, the partition function is

$$Z_1 = e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2} = e^{-\beta\epsilon_1} + e^{-\beta(\epsilon_1+\Delta)} = e^{-\beta\epsilon_1} (1 + e^{-\beta\Delta})$$

The probability of finding a particle in the upper state is

$$P_2 = \frac{e^{-\beta\epsilon_2}}{e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2}} = \frac{e^{-\beta\epsilon_1} e^{-\beta\Delta}}{e^{-\beta\epsilon_1} (1 + e^{-\beta\Delta})} = \frac{e^{-\beta\Delta}}{1 + e^{-\beta\Delta}} = \frac{1}{e^{\beta\Delta} + 1}$$

The average energy of one particle is

$$\begin{aligned} \langle E \rangle &= \epsilon_1 P_1 + \epsilon_2 P_2 = \epsilon_1 (1 - P_2) + \epsilon_2 P_2 = \epsilon_1 + (\epsilon_2 - \epsilon_1) P_2 = \epsilon_1 + \Delta P_2 \quad \because P_1 + P_2 = 1 \\ \Rightarrow \langle E \rangle &= \epsilon_1 + \frac{\Delta}{e^{\beta\Delta} + 1} \text{ thus for } N \text{ particles } U = N \langle E \rangle = N \epsilon_1 + \frac{N\Delta}{e^{\beta\Delta} + 1} \end{aligned}$$

Low-temperature limit:  $T \rightarrow 0$ ,  $\beta\Delta = \frac{\Delta}{k_B T} \rightarrow \infty \Rightarrow e^{\beta\Delta} \rightarrow \infty \Rightarrow U \rightarrow N \epsilon_1 = \text{const.}$

High-temperature limit:  $T \rightarrow \infty$ ,  $\beta\Delta = \frac{\Delta}{k_B T} \rightarrow 0 \Rightarrow e^{\beta\Delta} \rightarrow 1 \Rightarrow U \rightarrow \left( N \epsilon_1 + \frac{N\Delta}{2} \right) = \text{const.}$

Thus  $U(T)$ , begins at  $N \epsilon_1$ , rises as temperature increases and eventually saturates at

$$\left( N \epsilon_1 + \frac{N\Delta}{2} \right).$$

**Q53.** Consider the distribution of outcomes generated by  $N(N \gg 1)$  independent throws of

(i) a coin or (ii) a six-sided dice. A coin (dice) is unbiased if both (all) its sides have equal probability to show up in a throw; it is biased otherwise. For the cases (i) and (ii) above, which of the following statements is/are true?

- (a) The entropy of an unbiased coin is smaller than that of an unbiased dice
- (b) The entropy of an unbiased coin is greater than that of an unbiased dice
- (c) The entropy of a biased dice is smaller than that of an unbiased dice
- (d) The entropy of a biased coin is greater than that of an unbiased coin

**Ans.: (a) and (c)**

**Solution.: Entropy of a coin and a six-sided die**

For a discrete probability distribution,  $S = -Nk_B \sum_i p_i \ln p_i$

**(a) Unbiased coin:** There are two outcomes:  $p_1 = p_2 = \frac{1}{2}$

$$\text{Thus } S_{\text{coin}} = -Nk_B \left[ \frac{1}{2} \ln \frac{1}{2} + \frac{1}{2} \ln \frac{1}{2} \right] = Nk_B \ln 2$$

**Unbiased six-sided die:** There are six equally likely outcomes:  $p_i = \frac{1}{6}$

$$\text{Thus } S_{\text{die}} = -Nk_B \sum_i p_i \ln p_i = Nk_B \ln 6$$

$$\Rightarrow S_{\text{die}} > S_{\text{coin}} \text{ or } S_{\text{coin}} < S_{\text{die}}$$

**(b) Die is biased**

For a fixed number of possible outcomes, entropy is maximum for a uniform distribution.

Therefore, if the die becomes biased,  $S_{\text{biased die}} < S_{\text{unbiased die}}$

**Coin is biased:**  $S_{\text{biased coin}} < S_{\text{unbiased coin}}$

**Q63.** Copper has an electron number density of  $8.3 \times 10^{28} \text{ m}^{-3}$ . Its Fermi energy in  $eV$  (rounded off to one decimal place) is \_\_\_\_\_

( $\hbar = 1.06 \times 10^{-34} \text{ J.s}$ , mass of electron  $m_e = 9.10 \times 10^{-31} \text{ kg}$ , charge of electron =  $1.60 \times 10^{-19} \text{ C}$ )

**Ans.: 6.5 to 7.5**

$$\text{Solution.: } E_F = \frac{\hbar^2}{2m_e} (3\pi^2 n)^{2/3},$$

$$3\pi^2 n = 3 \times 9.8696 \times 8.3 \times 10^{28} = 2.457 \times 10^{30} \Rightarrow (3\pi^2 n)^{2/3} = 1.82 \times 10^{20} \text{ m}^{-2}$$

$$\text{Thus } E_F = \frac{(1.06 \times 10^{-34})^2}{2 \times 9.10 \times 10^{-31}} \times 1.82 \times 10^{20} = \frac{1.1236 \times 10^{-68}}{1.82 \times 10^{-30}} \times 1.82 \times 10^{20} = 1.1236 \times 10^{-18} \text{ J}$$

$$E_F = \frac{1.1236 \times 10^{-18}}{1.6 \times 10^{-19}} \text{ eV} = 7.02 \text{ eV}$$

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**Extra (Buffer) Questions: Carry ONE mark only**

**Q1D.** A spin  $\frac{1}{2}$  immobile particle is placed in a magnetic field of  $1T$  pointing in the  $+z$  direction at a temperature of  $1K$ . Then after repeated measurements, probability of finding the particle in a spin up state (rounded off to two decimal places) is \_\_\_\_\_  
( $\mu_B = 9.27 \times 10^{-24} J.T^{-1}$ ,  $k_B = 1.38 \times 10^{-23} J.K^{-1}$ )

**Ans.: 0.79**

**Solution.:** For a magnetic moment in a magnetic field, the two spin states have energies

$$E_{\uparrow} = -\mu_B B \quad \text{and} \quad E_{\downarrow} = +\mu_B B$$

Therefore,  $E_{\uparrow} = -\mu_B B$  is the lower-energy state.

In thermal equilibrium, the probability is given by the Boltzmann distribution:  $P_i = \frac{e^{-E_i/k_B T}}{Z}$

The partition function is  $Z = e^{-E_{\uparrow}/k_B T} + e^{-E_{\downarrow}/k_B T} = e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}$

$$\text{Hence } P_{\uparrow} = \frac{e^{\mu_B B/k_B T}}{e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}} = \frac{1}{1 + e^{-2\mu_B B/k_B T}}$$

$$\therefore \frac{\mu_B B}{k_B T} = \frac{(9.27 \times 10^{-24})(1)}{(1.38 \times 10^{-23})(1)} \approx 0.672 \Rightarrow P_{\uparrow} = \frac{1}{1 + e^{-2(0.672)}} = \frac{1}{1 + e^{-1.344}} = \frac{1}{1 + 0.261} = \frac{1}{1.261} \approx 0.79$$

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**GATE Physics-2026**  
**Solution-Electronics**

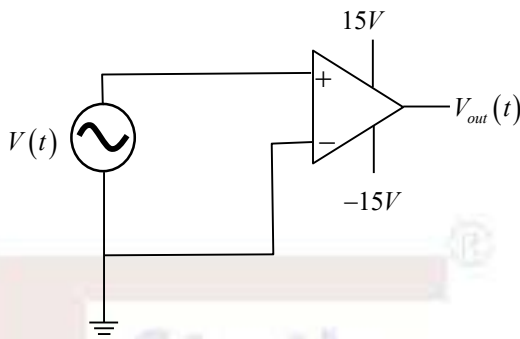
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Q.11 – Q.35 Carry ONE mark Each

Q.36 – Q.55 Carry TWO marks Each

Q43. In the circuit shown,  $V(t) = 2 \sin(2000\pi t)$  Volts, where  $t$  is in seconds, which of the following options is correct? (take the opamp to be ideal)



- (a)  $V_{out}(t)$  is square wave with peak-to-peak voltage =  $30V$  and time period is  $1\text{ ms}$
- (b)  $V_{out}(t)$  is sine wave with peak-to-peak voltage =  $4V$  and time period is  $1\text{ ms}$
- (c)  $V_{out}(t)$  is sine wave with peak-to-peak voltage =  $30V$  and time period of  $1\text{ ms}$
- (d)  $V_{out}(t)$  is square wave with peak-to-peak voltage =  $4V$  and time period is  $1\text{ ms}$

Ans.: (a)

**Solution.:** For  $V(t) > 0$ ;  $V_+ > V_- = 0$  so  $V_{out}(t) = +15V$

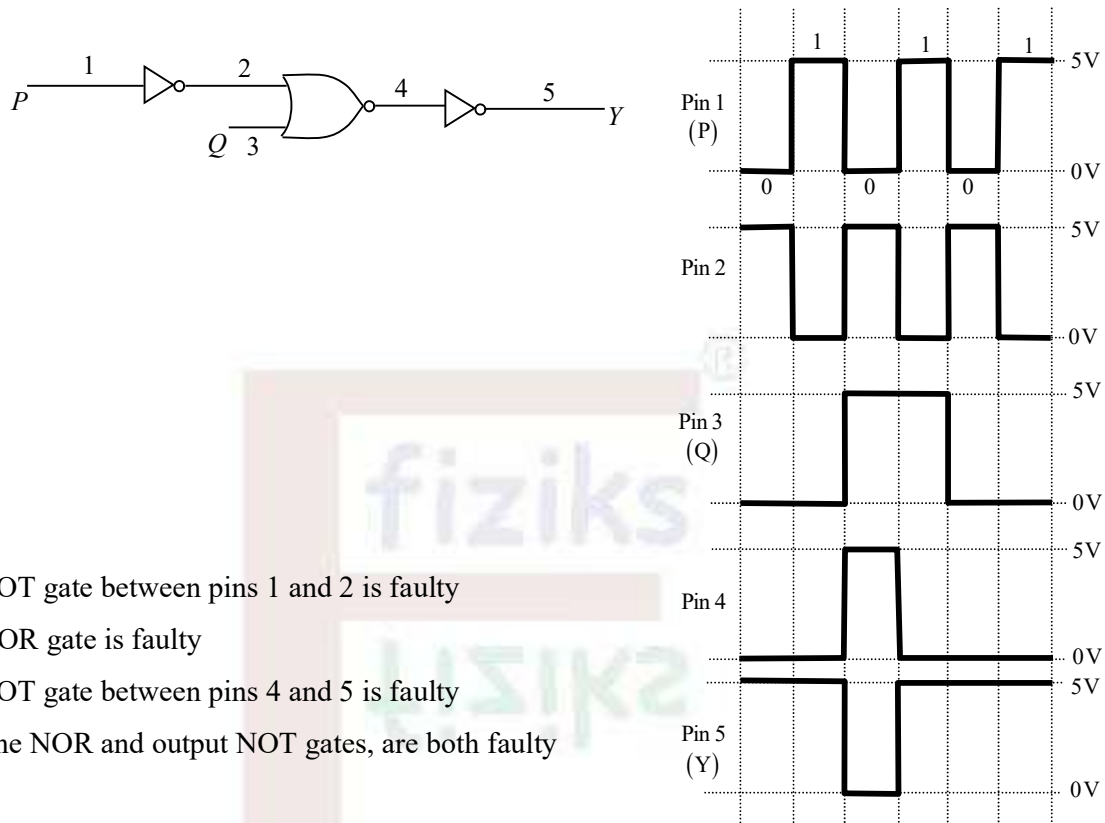
For  $V(t) < 0$ ;  $V_+ < V_- = 0$  so  $V_{out}(t) = -15V$

Thus, the output is a **square wave**.

The angular frequency is  $\omega = 2000\pi$  and  $T = \frac{2\pi}{\omega} = \frac{2\pi}{2000\pi} = 1\text{ ms}$ .

The output switches between  $+15V$  to  $-15V$  so  $V_{pp} = +15V - (-15V) = 30V$

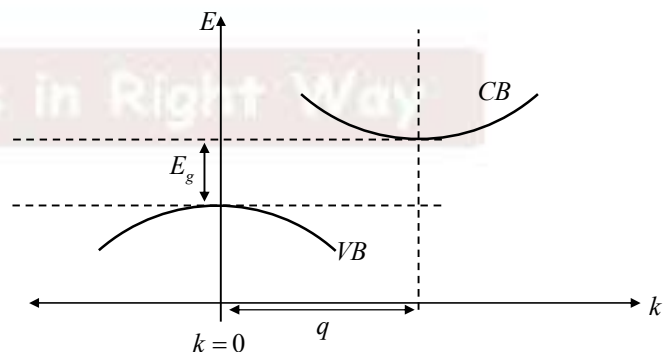
**Q44.** Considering the circuit and the associated signals measured at different pins (numbered as 1, 2, 3, 4, 5) shown in figure, the correct option is



- (a) NOT gate between pins 1 and 2 is faulty
- (b) NOR gate is faulty
- (c) NOT gate between pins 4 and 5 is faulty
- (d) The NOR and output NOT gates, are both faulty

**Ans.: (b)**

**Q54.** The dispersion ( $E(k)$ ) of the conduction band (CB) and valence band (VB) for a semiconductor are shown schematically in the figure. Considering the possibility of an electron making a transition from the bottom of the CB to the top of the VB, which of the following options is/are correct?



- (a) The transition is forbidden
- (b) A photon can be emitted with an energy exactly equal to  $E_g$
- (c) A photon can be emitted with an energy less than  $E_g$
- (d) A phonon can be created with a crystal momentum  $\hbar q$

**Ans.: (c) and (d)**

**Solution.:** The top of the valence band occurs at  $k = 0$  and the bottom of the conduction band occurs at  $k = q$ . Therefore, this is an indirect band-gap semiconductor.

### Momentum conservation

Initially, the electron is at  $k_i = q$  and finally it must reach  $k_f = 0$ .

Thus, the electron must undergo a crystal momentum change  $\Delta p = \hbar(k_f - k_i) = \hbar(0 - q) = -\hbar q$ .

A photon involved in an optical transition carries very little crystal momentum compared with  $\hbar q$ . Therefore, a photon alone cannot conserve crystal momentum. A phonon is required.

### Phonon emission

If a phonon is created, it can supply the required crystal momentum.

Its momentum magnitude is  $p_{\text{phonon}} = \hbar q$ .

Hence A phonon can be created with a crystal momentum  $\hbar q$ .

Hence option (d) is possible:

### Photon energy

The band-gap energy is  $E_g = E_C(q) - E_V(0)$

If a phonon of energy  $\hbar\omega_{\text{phonon}}$  is emitted, energy conservation gives  $E_g = E_\gamma + \hbar\omega_{\text{phonon}}$ .

$$\Rightarrow E_\gamma = E_g - \hbar\omega_{\text{phonon}} \Rightarrow E_\gamma < E_g \quad \because \hbar\omega_{\text{phonon}} > 0$$

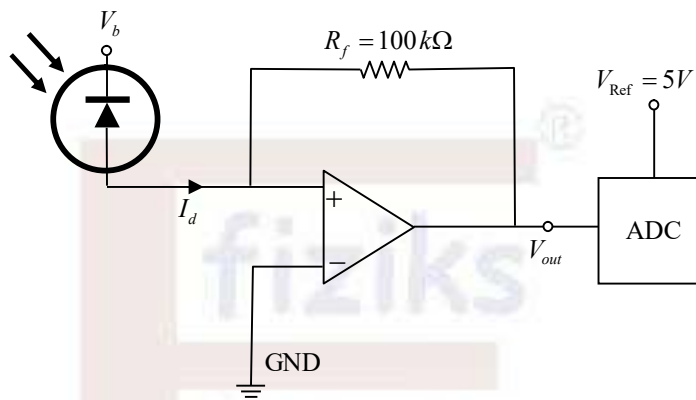
Thus option (c) is also correct.

### Why option (b) is incorrect?

A photon having exactly energy  $E_g$  would leave no energy for the phonon:  $E_\gamma = E_g$ .

But a phonon is required to satisfy crystal-momentum conservation. Therefore, the transition cannot occur by emitting only a photon of energy exactly  $E_g$ .

**Q60.** The output signal (current  $I_d$ ) of a reversed biased (with a voltage  $V_b$ ) photodiode, on which light is incident, is fed to an amplifier (see figure). The output voltage is digitized by a 10 bit Analogue to Digital convertor (ADC) which has a reference voltage of  $5V$ . The smallest current  $I_d$  which can be measured by the circuit in nano-Amperes (rounded off to one decimal place) is \_\_\_\_\_



**Ans.: 47.5 to 50.0**

**Solution.:** The op-amp configuration with photodiode current  $I_d$  entering the non-inverting terminal creates an output voltage  $V_{out} = I_d \times R_f$ .

10-bit ADC has  $2^{10} = 1024$  levels.

$$\text{Voltage resolution (LSB)} = \frac{V_{ref}}{2^{10}} = \frac{5V}{1024} \approx 4.8828 \times 10^{-3} V = 4.8828 mV$$

Smallest Measurable Current

$$I_{d,\min} = \frac{V_{out,\min}}{R_f} = \frac{\text{LSB}}{R_f} = \frac{4.8828 \times 10^{-3} V}{100 \times 10^3 \Omega} = 4.8828 \times 10^{-8} A = 48.8 nA$$



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**GATE Physics-2026**  
**Solution-Atomic and Molecular Physics**

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**Q.11 – Q.35 Carry ONE mark Each**

**Q34.** Raman spectrum of a molecule was recorded using a source of wavelength  $5000\text{\AA}$ . The first Stokes line is observed at  $5100\text{\AA}$ . The first anti-Stokes line will appear at a wavelength  $L$  (in  $\text{\AA}$ ). The value of  $L$  (rounded off to nearest integer) is \_\_\_\_\_

**Ans.: 4901 to 4905**

**Solution.:** To find the wavelength of the first anti-Stokes line, we use the fact that the Raman shift  $\Delta\nu$  is equal in magnitude for both Stokes and anti-Stokes lines:

$$\Delta\nu = \frac{1}{\lambda_{\text{source}}} - \frac{1}{\lambda_{\text{stokes}}} = \frac{1}{\lambda_{\text{anti-stokes}}} - \frac{1}{\lambda_{\text{source}}} \Rightarrow \frac{2}{\lambda_{\text{source}}} = \frac{1}{\lambda_{\text{stokes}}} + \frac{1}{L} \Rightarrow \frac{2}{5000} = \frac{1}{5100} + \frac{1}{L}$$

$$\Rightarrow \frac{1}{L} = \frac{2}{5000} - \frac{1}{5100} = \frac{10200 - 5000}{25500000} = \frac{52}{2550000} \Rightarrow L = \frac{2550000}{52} \approx 4904\text{\AA}$$

**Q.36 – Q.55 Carry TWO marks Each**

**Q58.** An atom has two energy levels with energy difference  $2.2eV$  between them. A gas of these atoms is with  $8 \times 10^{20}$  atoms in the upper state and  $5 \times 10^{20}$  atoms in the lower state. Ignoring spontaneous emission, the maximum possible energy released by this gas of atoms by stimulated emission is  $E$  Joules. The value of  $E$  (rounded off to one decimal place) is \_\_\_\_\_  
( $e = 1.6 \times 10^{-19} C$ )

**Ans.: 52.0 to 53.0**

**Solution.:** Stimulated emission occurs until the populations of the upper and lower energy states become equal (saturation condition).

Initial upper state population  $N_2 = 8 \times 10^{20}$ ; Initial lower state population,  $N_1 = 5 \times 10^{20}$

Total population,  $N = N_1 + N_2 = 13 \times 10^{20}$

At saturation,  $N'_1 = N'_2 = \frac{N}{2} = 6.5 \times 10^{20}$ .

The number of transitions from the upper state to the lower state via stimulated emission is

$$\Delta N = N_2 - N'_1 = 8 \times 10^{20} - 6.5 \times 10^{20} = 1.5 \times 10^{20}$$

Each transition releases energy equal to the energy gap

$$\Delta E = 2.2eV = 2.2 \times 1.6 \times 10^{-19} J = 3.52 \times 10^{-19} J$$

The total maximum energy released  $E$  is:  $E = \Delta E \times \Delta N = (3.52 \times 10^{-19})(1.5 \times 10^{20}) J = 52.8 J$

**Q62.** Rotational spectrum of diatomic molecule consists of lines of equal spacing with an interval of  $20.0\text{ cm}^{-1}$ . Its moment of inertia is found to be  $I_0 \times 10^{-47} \text{ kg.m}^2$ , the value of  $I_0$  (rounded off to one decimal place) is \_\_\_\_\_

( $h = 6.6 \times 10^{-34} \text{ J.s}$ , speed of light in vacuum  $c = 3 \times 10^8 \text{ m.s}^{-1}$ )

**Ans.: 2.6 to 3.0**

**Solution.:** The spacing between consecutive rotational lines in a diatomic molecular spectrum is:

$$\Delta \nu = 2B = 20.0 \text{ cm}^{-1} \Rightarrow B = 10.0 \text{ cm}^{-1} = 1000 \text{ m}^{-1}$$

The rotational constant  $B$  in terms of moment of inertia  $I$  is given by:

$$B = \frac{h}{8\pi^2 Ic} \Rightarrow I = \frac{h}{8\pi^2 Bc} = \frac{6.6 \times 10^{-34}}{8 \times 10 (1000) (3 \times 10^8)} \approx 2.8 \times 10^{-47} \text{ kg.m}^2 \Rightarrow I_0 = 2.8$$

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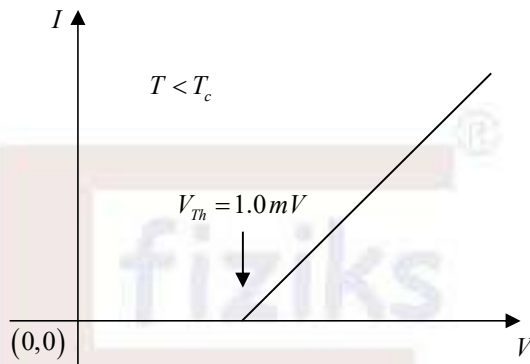
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**Solution-Solid State Physics**

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**Q.11 – Q.35 Carry ONE mark Each**

**Q31.** Consider a metal-superconductor junction connected to a dc voltage  $V$ . At  $T < T_c$ , where  $T_c$  is the superconductor's transition temperature, the current  $I$  versus  $V$  behavior of this junction is shown schematically in the figure below. If the superconducting energy gap is  $D$  meV. The value of  $D$  (rounded off to one decimal place) is \_\_\_\_\_



**Ans.: 2**

**Solution.:** Energy associated with the threshold:

$$eV_{th} = \Delta \text{ where } \Delta \text{ is the superconducting gap parameter.}$$

$$\text{Thus, } \Delta = e(1\text{mV}) = 1\text{meV}$$

In BCS superconductivity, the separation between the *two edges of the forbidden-energy* region is

$$D = 2\Delta = 2\text{meV}$$

**Q32.** For the energy dispersion of an electron in a one-dimensional solid  $E(k) = E_0 - 2\gamma \cos(ka)$ , the ratio of the effective mass of the electron in the solid to the free electron mass ( $m_e$ ) at  $k = 0$  is  $R_0$ . Taking  $\gamma = 0.5\text{eV}$  and  $a = 0.5\text{nm}$ , the value of  $R_0$  (rounded off to two decimal place) is \_\_\_\_\_

$$(\hbar = 1.054 \times 10^{-34} \text{ J.s}, m_e = 9.1 \times 10^{-31} \text{ kg}, \text{ electron charge} = 1.6 \times 10^{-19} \text{ C})$$

**Ans.: 0.30 to 0.32**

$$\text{Solution.: } \because E(k) = E_0 - 2\gamma \cos(ka) \Rightarrow \frac{dE}{dk} = 2a\gamma \sin(ka) \Rightarrow \frac{d^2E}{dk^2} = 2a^2\gamma \cos(ka)$$

$$\Rightarrow \left. \frac{d^2E}{dk^2} \right|_{k=0} = 2a^2\gamma.$$

$$\gamma = 0.5\text{eV} = 0.5 \times 1.6 \times 10^{-20} \text{ J} = 8 \times 10^{-21} \text{ J}, a = 0.5 \times 10^{-9} \text{ m}$$

Thus, Effective Mass Formula:  $m^* = \frac{\hbar^2}{\left. \frac{d^2 E}{dk^2} \right|_{k=0}} = \frac{\hbar^2}{2a^2 \gamma} = \frac{(1.054 \times 10^{-34})^2}{2(0.5 \times 10^{-9})^2 (8 \times 10^{-20})}$

$$\Rightarrow m^* = \frac{1.1109 \times 10^{-68}}{4 \times 10^{-38}} = 2.777 \times 10^{-31} \text{ kg} \Rightarrow R_0 = \frac{m^*}{m_e} = \frac{2.777 \times 10^{-31}}{9.1 \times 10^{-31}} \approx 0.305 \approx 0.31$$

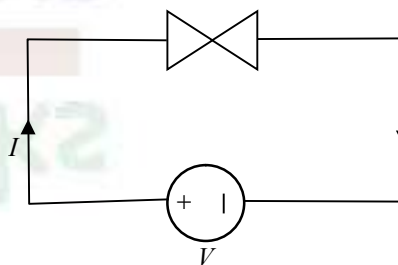
**Q.36 – Q.55 Carry TWO marks Each****Extra (Buffer) Questions: Carry TWO mark only**

**Q2C.** In the circuit shown in the figure the symbol  $\left( \bowtie \right)$  represents a Josephson Junction (JJ).

This circuit exhibits the current ( $I$ ), dc-voltage ( $V_{dc}$ ) relation,  $I = I_0 \sin(\phi)$  and  $\frac{d\phi}{dt} = AV_{dc}$ ,

where  $\phi$  is a phase introduced in the current by the JJ. Given  $A$  is a positive constant parameter,  $t$  is time, the energy of the circuit  $E(\phi)$ :

- (a) Depends on  $\phi$  to leading order
- (b) Depends on  $\phi^2$  to leading order
- (c) Depends on  $\phi^3$  to leading order
- (d) Depends on  $(\phi - \phi^3)$  to leading order



**Ans.: (b)**

**Solution.:** Electrical Power in the Junction:  $P = V \cdot I = V_{dc} I_0 \sin(\phi) = \left( \frac{1}{A} \frac{d\phi}{dt} \right) I_0 \sin(\phi)$

Energy Stored in the Junction:  $E(\phi) = \int P dt = \int \left( \frac{1}{A} \frac{d\phi}{dt} \right) I_0 \sin(\phi) dt = \frac{I_0}{A} \int \sin(\phi) d\phi$

$$\Rightarrow E(\phi) = -\frac{I_0}{A} \cos(\phi) + C \Rightarrow E(\phi) = -\frac{I_0}{A} \left( 1 - \frac{\phi^2}{2} + \dots \right) + C = \text{constant} + \frac{I_0 \phi^2}{2A}$$

The energy of the circuit  $E(\phi)$  depends on  $\phi^2$  to leading order.

**Extra (Buffer) Questions: Carry TWO mark only**

**Q2E.** On heating, a material undergoes a ferroelectric to paraelectric transition associated with the structural change: Tetragonal ( $a = b, c = \sqrt{2}a$ )  $\xrightarrow{120^\circ\text{C}}$  Cubic ( $a = b = c$ ) at  $120^\circ\text{C}$ . Which of the following statement(s) is(are) correct for this transition?

( $d_{hkl}$  is the smallest interplanar spacing between planes with Miller indices  $hkl$  and  $\chi$  is the dielectric susceptibility)

(a)  $d_{110}(\text{Cubic}) / d_{110}(\text{Tetragonal}) = 1$

(b)  $d_{110}(\text{Cubic}) / d_{110}(\text{Tetragonal}) = 1/\sqrt{2}$

(c)  $1/\chi$  vs.  $T$  plot is linear above the transition temperature

(d)  $1/\chi$  vs.  $T$  plot is linear below the transition temperature

**Ans.: (a) and (c)**

**Solution.:** For tetragonal crystals,  $\frac{1}{d_{hkl}^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$

$$\Rightarrow d_{110}(\text{tetragonal}) = \frac{a}{\sqrt{2}} \quad \because a = b, c = \sqrt{2}a$$

For cubic crystals:  $\frac{1}{d_{hkl}^2} = \frac{h^2 + k^2 + l^2}{a^2} \Rightarrow d_{110}(\text{cubic}) = \frac{a}{\sqrt{2}}$

$$\Rightarrow \frac{d_{110}(\text{cubic})}{d_{110}(\text{tetragonal})} = \frac{a/\sqrt{2}}{a/\sqrt{2}} = 1. \text{ Thus option (a) is correct.}$$

Above the transition temperature  $T_c = 120^\circ\text{C}$ , the material is in its paraelectric phase and follows

the **Curie-Weiss Law**:  $\chi = \frac{C}{T - T_0} \Rightarrow \frac{1}{\chi} \propto (T - T_0)$

This means the  $\frac{1}{\chi}$  vs.  $T$  plot is linear **above** the transition temperature.

Therefore, **(c) is correct.**



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**GATE Physics-2026**  
**Solution-Nuclear and Particle Physics**

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**Q.11 – Q.35 Carry ONE mark Each****Q23.** Which one of the following is an allowed process?

- (a)  $\pi^- + p \rightarrow \pi^0 + n$  (b)  $\pi^0 \rightarrow \gamma + \gamma + \gamma$   
(c)  $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$  (d)  $p + \bar{p} \rightarrow \gamma$

**Ans.: (a)****Soution.:**

(a)  $\pi^- + p \rightarrow \pi^0 + n$

Charge ( $Q$ ):  $(-1) + (+1) = 0 + 0$  (Conserved)

Baryon Number ( $B$ ):  $0 + 1 = 0 + 1$  (Conserved)

Lepton Number ( $L$ ):  $0 = 0$  (Conserved)

All particles have Strangeness:  $S = 0$  so  $S_f = S_i = 0$

Therefore, this process is allowed. In fact, it is a standard strong-interaction charge-exchange process.

(b)  $\pi^0 \rightarrow \gamma + \gamma + \gamma$

A neutral pion has  $C(\pi^0) = +1$

A photon has  $C = -1$ . Therefore, for three photons,  $C(3\gamma) = (-1)^3 = -1$

Since electromagnetic interaction conserves charge conjugation,  $+1 \neq -1$  so this decay is forbidden.

(c)  $p + \bar{p} \rightarrow \Lambda^0 + \Lambda^0$

Strangeness:  $0 + 0 \neq -1 - 1$ . (Violates Strangeness)

Strong interactions conserve  $S$

(d)  $p + \bar{p} \rightarrow \gamma$

Violates conservation of energy and linear momentum simultaneously in single-photon annihilation without a recoil body. **Forbidden**

**Q24.** Given  $Q$  is the electromagnetic charge and  $S$  is the strangeness quantum number, identify the particle(s) for which  $(Q - S) = 0$  is satisfied.

- (a)  $\Sigma^{*-}$  (b)  $K^+$   
(c)  $\Omega^-$  (d)  $\Delta^{++}$

**Ans.: (a) and (b)**

**Soution.:**

- (a)  $\Sigma^{*-} : Q = -1, S = -1 \Rightarrow Q - S = 0$   
(b)  $K^{+} : Q = +1, S = +1 \Rightarrow Q - S = 0$   
(c)  $\Omega^{-} : Q = -1, S = -3 \Rightarrow Q - S = 2 \neq 0$   
(d)  $\Delta^{++} : Q = +2, S = 0 \Rightarrow Q - S = 2 \neq 0$

So the correct choice is (a) and (b).

**Q.36 – Q.55 Carry TWO marks Each**

**Q38.** Which one of the following statements is true?

- (a) In the decay  $\mu^{+} \rightarrow e^{+} + \nu_e + \bar{\nu}_{\mu}$ , CPT is violated  
(b) The decay  $\Lambda \rightarrow p^{+} + \pi^{-}$  is allowed and strangeness is violated  
(c) The decay  $p^{+} \rightarrow e^{+} + \gamma$  is allowed  
(d) The decay  $\Omega^{-} \rightarrow \Xi^{0} + K^{-}$  is allowed

**Ans.: (b)**

**Soution.:**

**(a)** CPT is a fundamental symmetry conserved in all standard quantum field theories.

So, option (a) is false.

**(b)**  $\Lambda \rightarrow p^{+} + \pi^{-}$

Charge ( $Q$ ):  $0 = 1 - 1$  (Conserved)

Baryon Number ( $B$ ):  $1 = 1 + 0$  (Conserved)

Strangeness:  $-1 \neq 0 + 0$ . So strangeness is violated.

But this does **not** make the decay forbidden. Strangeness need not be conserved in a weak interaction. So, option (b) is true.

**(c)**  $p^{+} \rightarrow e^{+} + \gamma$

This would mean a proton spontaneously emits a photon and remains a proton.

For a free proton, this process cannot satisfy energy-momentum conservation: in the proton's rest frame, there is no available energy for producing a real photon while keeping the final proton on shell. Therefore, this is not an allowed decay.

So, option (c) is false.

**(d)**  $\Omega^{-} \rightarrow \Xi^{0} + K^{-}$  : Strangeness:  $-3 \neq -2 + 1$ .

A  $\Delta S = 2$  transition is not an allowed ordinary weak decay of this type.

Therefore, this option is false.