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GATE Physics-2025

Complete Solution

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Solution-Mathematical Physics

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Q.11 – Q.35 Carry ONE mark Each

Q21. Consider the function $f(z) = \frac{1}{z^2(z-2)^3}$ of a complex variable z . The residues of the function at $z=0$ and $z=2$, respectively, are

- (A) $-\frac{3}{8}$ and $\frac{3}{8}$ (B) $\frac{3}{8}$ and $-\frac{3}{16}$
(C) $-\frac{3}{16}$ and $\frac{3}{16}$ (D) $-\frac{3}{8}$ and $\frac{3}{16}$

Ans.: (C)

Solution.: $\because f(z) = \frac{1}{z^2(z-2)^3} \Rightarrow z=0$ is a pole of order 2, $z=2$ is a pole of order 3.

Residue at pole $z=a$ of order n : $a_{-1} = \frac{1}{(n-1)!} \lim_{z \rightarrow a} \frac{d^{n-1}}{dz^{n-1}} [(z-a)^n f(z)]$

Residue at $z=0$: $a_{-1} = \frac{1}{(2-1)!} \lim_{z \rightarrow 0} \frac{d}{dz} \left[(z-0)^2 \frac{1}{z^2(z-2)^3} \right] = \lim_{z \rightarrow 0} \left[\frac{-3}{(z-2)^4} \right] = -\frac{3}{16}$

Residue at $z=2$: $a_{-1} = \frac{1}{(3-1)!} \lim_{z \rightarrow 2} \frac{d^2}{dz^2} \left[(z-2)^3 \frac{1}{z^2(z-2)^3} \right] = \frac{1}{2} \lim_{z \rightarrow 2} \frac{d^2}{dz^2} \left[\frac{1}{z^2} \right] = \frac{1}{2} \lim_{z \rightarrow 2} \frac{d}{dz} \left[\frac{-2}{z^3} \right]$

$$\Rightarrow a_{-1} = \frac{1}{2} \lim_{z \rightarrow 2} \left[\frac{6}{z^4} \right] = \frac{1}{2} \times \frac{6}{2^4} = \frac{3}{16}$$

Q.36 – Q.65 Carry TWO marks Each

Q55. Consider the integral

$$I = \frac{1}{2\pi i} \oint \frac{z^4 - 1}{\left(z - \frac{a}{b}\right) \left(z - \frac{b}{a}\right)} dz,$$

where z is a complex variable and a, b are positive real numbers. The integral is taken over a unit circle with center at the origin. Which of the following option(s) is/are correct?

- (A) $I = \frac{5}{8}$ when $a=1, b=2$ (B) $I = \frac{10}{3}$ when $a=1, b=3$
(C) $I = \frac{5}{8}$ when $a=2, b=1$ (D) $I = \frac{5}{8}$ when $a=3, b=2$

Ans.: (A), (C)

Solution.: We have to find $I = \frac{1}{2\pi i} \oint \frac{z^4 - 1}{\left(z - \frac{a}{b}\right)\left(z - \frac{b}{a}\right)} dz = \frac{1}{2\pi i} \times 2\pi i \sum_i a_{-1,i} = \sum_i a_{-1,i}$

Let $f(z) = \frac{z^4 - 1}{\left(z - \frac{a}{b}\right)\left(z - \frac{b}{a}\right)}$ then $z = \frac{a}{b}$ and $z = \frac{b}{a}$ is a simple pole of $f(z)$.

$$\text{At } z = \frac{a}{b}: a_{-1} = \lim_{z \rightarrow \frac{a}{b}} \left(z - \frac{a}{b}\right) \frac{z^4 - 1}{\left(z - \frac{a}{b}\right)\left(z - \frac{b}{a}\right)} = \frac{a^4/b^4 - 1}{\frac{a}{b} - \frac{b}{a}} = \frac{a^4 - b^4}{b^4} \times \frac{ab}{a^2 - b^2} = \frac{a(a^2 + b^2)}{b^3}$$

$$\text{At } z = \frac{b}{a}: a_{-1} = \lim_{z \rightarrow \frac{b}{a}} \left(z - \frac{b}{a}\right) \frac{z^4 - 1}{\left(z - \frac{a}{b}\right)\left(z - \frac{b}{a}\right)} = \frac{b^4/a^4 - 1}{\frac{b}{a} - \frac{a}{b}} = \frac{b^4 - a^4}{a^4} \times \frac{ab}{b^2 - a^2} = \frac{b(a^2 + b^2)}{a^3}$$

(A) When $a = 1, b = 2$; $\frac{a}{b} = \frac{1}{2}$ (inside), $\frac{b}{a} = 2$ (outside)

$$\Rightarrow I = a_{-1} = \frac{a(a^2 + b^2)}{b^3} = \frac{1(1^2 + 2^2)}{2^3} = \frac{5}{8}$$

(B) When $a = 1, b = 3$; $\frac{a}{b} = \frac{1}{3}$ (inside), $\frac{b}{a} = 3$ (outside)

$$\Rightarrow I = a_{-1} = \frac{a(a^2 + b^2)}{b^3} = \frac{1(1^2 + 3^2)}{3^3} = \frac{10}{27}$$

(C) When $a = 2, b = 1$; $\frac{a}{b} = 2$ (outside), $\frac{b}{a} = \frac{1}{2}$ (inside)

$$\Rightarrow I = a_{-1} = \frac{b(a^2 + b^2)}{a^3} = \frac{1(2^2 + 1^2)}{2^3} = \frac{5}{8}$$

(D) When $a = 3, b = 2$; $\frac{a}{b} = \frac{3}{2}$ (outside), $\frac{b}{a} = \frac{2}{3}$ (inside)

$$\Rightarrow I = a_{-1} = \frac{b(a^2 + b^2)}{a^3} = \frac{2(3^2 + 2^2)}{3^3} = \frac{26}{27}$$

Q61. Consider the set $\{1, x, x^2\}$. An orthonormal basis in $x \in [-1, 1]$ is formed from these three terms, where the normalization of a function $f(x)$ is defined via $\int_{-1}^1 x^2 [f(x)]^2 dx = 1$. If the orthonormal basis set is $\left(\sqrt{\frac{3}{2}}, \sqrt{\frac{5}{2}}x, \frac{1}{2}\sqrt{\frac{21}{N}}(5x^2 - 3)\right)$, then the value of N (in integer) is

Ans.: 6

Solution.: $\because \int_{-1}^1 x^2 [f(x)]^2 dx = 1 \Rightarrow \int_{-1}^1 x^2 \left[\frac{1}{2} \sqrt{\frac{21}{N}} (5x^2 - 3) \right]^2 dx = 1 \Rightarrow \frac{21}{4N} \int_{-1}^1 x^2 (5x^2 - 3)^2 dx = 1$

$$\Rightarrow N = \frac{21}{4} \int_{-1}^1 x^2 (5x^2 - 3)^2 dx = \frac{21}{4} \int_{-1}^1 x^2 (25x^4 + 9 - 30x^2) dx$$

$$\Rightarrow N = \frac{21}{4} \left[25 \frac{x^7}{7} + 9 \frac{x^3}{3} - 30 \frac{x^5}{5} \right]_{-1}^1 = \frac{21}{4} \left[25 \left(\frac{2}{7} \right) + 9 \left(\frac{2}{3} \right) - 30 \left(\frac{2}{5} \right) \right]$$

$$\Rightarrow N = \frac{21}{4} \left[\frac{50}{7} + 6 - 12 \right] = \frac{21}{4} \left(\frac{50 - 42}{7} \right) = \frac{3}{4} \times 8 = 6$$



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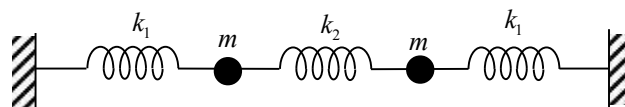
GATE Physics-2025
Solution-Classical Mechanics

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Q.36 – Q.65 Carry TWO marks Each

Q38. The figure shows a system of two equal masses m and three massless horizontal springs with spring constants k_1, k_2, k_1 . Ignore gravity. The masses can move only in the horizontal direction and there is no dissipation. If $m = 1, k_1 = 2$ and $k_2 = 3$ (all in appropriate units), the frequencies of the normal modes of the system in the same system of units are



(A) $\sqrt{2}, \sqrt{8}$

(B) $\sqrt{2}, \sqrt{6}$

(C) $\sqrt{3}, \sqrt{10}$

(D) $\sqrt{3}, \sqrt{8}$

Ans.: (A)

Solution.: Let x_1 and x_2 be displacements of the left and right masses from equilibrium.

Analyze Forces on Mass 1: (a) Left Spring Force on Mass 1

If mass 1 moves to the right by x_1 , the left spring stretches by x_1 .

The spring pulls mass 1 back to the left (restoring force): $F_{\text{left}} = -k_1 x_1$

(b) Middle Spring Force on Mass 1

The middle spring's extension depends on the **relative displacement** between the two masses.

Right end of middle spring moves by x_2 , Left end of middle spring moves by x_1

Extension of middle spring: $\Delta x_{\text{middle}} = x_2 - x_1$

If $x_2 > x_1$, the spring is stretched, and it pulls mass 1 to the **right** (positive direction):

$$F_{\text{middle}} = +k_2 (x_2 - x_1)$$

Equation of Motion for Mass 1: $m\ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1) \Rightarrow m\ddot{x}_1 = -(k_1 + k_2)x_1 + k_2 x_2$

Analyze Forces on Mass 2:

(a) Middle Spring Force on Mass 2

The middle spring has the same extension $x_2 - x_1$ as before.

If $x_2 > x_1$, the spring is stretched and pulls mass 2 to the **left** (negative direction):

$$F_{\text{middle}} = -k_2 (x_2 - x_1)$$

(b) Right Spring Force on Mass 2

If mass 2 moves to the right by x_2 , the right spring stretches by x_2 .

The spring pulls mass 2 back to the left (restoring force): $F_{\text{right}} = -k_1 x_2$

Equation of Motion for Mass 2: $m\ddot{x}_2 = -k_2 (x_2 - x_1) - k_1 x_2 \Rightarrow m\ddot{x}_2 = k_2 x_1 - (k_1 + k_2)x_2$

Substitute $m = 1, k_1 = 2$ and $k_2 = 3$: $\ddot{x}_1 = -5x_1 + 3x_2$ and $\ddot{x}_2 = 3x_1 - 5x_2$

Assume normal modes: $x_1 = A_1 e^{i\omega t}$, $x_2 = A_2 e^{i\omega t}$

$$-\omega^2 A_1 = -5A_1 + 3A_2 \text{ and } -\omega^2 A_2 = 3A_1 - 5A_2$$

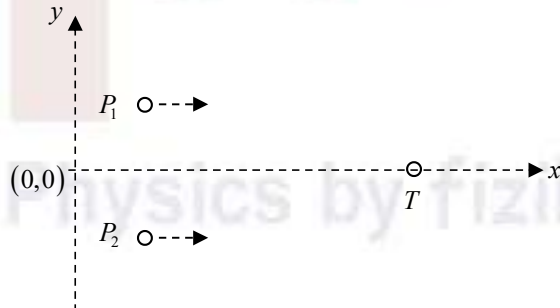
Rearranging: $(5 - \omega^2)A_1 - 3A_2 = 0$

$$-3A_1 + (5 - \omega^2)A_2 = 0$$

$$\Rightarrow \begin{vmatrix} 5 - \omega^2 & -3 \\ -3 & 5 - \omega^2 \end{vmatrix} = 0 \Rightarrow (5 - \omega^2)^2 - 9 = 0 \Rightarrow (5 - \omega^2)^2 = 9 \Rightarrow 5 - \omega^2 = \pm 3$$

$$\Rightarrow \omega^2 = 2 \text{ and } \omega^2 = 8 \Rightarrow \omega = \sqrt{2} \text{ and } \omega = \sqrt{8}$$

Q39. Two projectile protons P_1 and P_2 both with spin up (along the $+z$ direction) are scattered from another fixed target proton T with spin up at rest in the xy plane, as shown in the figure. They scatter one at a time. The nuclear interaction potential between both the projectiles and the target proton is $\lambda \vec{L} \cdot \vec{S}$, where $\vec{L}$ is the orbital angular momentum of the system with respect to the target, $\vec{S}$ is the spin angular momentum of the system and λ is a negative constant in appropriate units. Which one of the following is correct?



- (A) P_1 will be scattered in the $+y$ direction (upward) and P_2 will be scattered in the $-y$ direction (downward)
- (B) P_1 will be scattered in the $+y$ direction (upward) and P_2 will be scattered in the $+y$ direction (upward)
- (C) P_1 will be scattered in the $-y$ direction (downward) and P_2 will be scattered in the $+y$ direction (upward)
- (D) P_1 will be scattered in the $-y$ direction (downward) and P_2 will be scattered in the $-y$ direction (downward)

Ans.: (B)

Q50. In coordinates (t, x) , a contra variant second rank tensor A has non-zero diagonal components $A^{tt} = P$ and $A^{xx} = Q$, with all other components vanishing, and P, Q being real constants. Here, t is time and x is space coordinate. Consider a Lorentz transformation $(t, x) \rightarrow (t', x')$ to another frame that moves with relative speed v in the $+x$ direction, so that $A \rightarrow A'$. If A'^{tt} and A'^{xx} are the diagonal components of A' , then setting the speed of light $c = 1$, and with $\gamma = \frac{1}{\sqrt{1-v^2}}$, which of the following option(s) is/are correct?

- (A) $A'^{tt} = \gamma^2 P + \gamma^2 v^2 Q$ (B) $A'^{tt} = \gamma^2 v^2 P + v^2 Q$
 (C) $A'^{xx} = \gamma^2 v^2 P + \gamma^2 Q$ (D) $A'^{xx} = v^2 P + \gamma^2 Q$

Ans.: (A), (C)

Q51. The Lagrangian of a particle of mass m and charge q moving in a uniform magnetic field of magnitude $2B$ that points in the z direction, is given by $L = \frac{m}{2} v^2 + qB(xv_y - yv_x)$ where v_x, v_y, v_z are the components of its velocity $\vec{v}$. If p_x, p_y, p_z denote the conjugate momenta in the x, y, z directions and H is the Hamiltonian, which of the following options(s) is/are correct?

- (A) $\frac{dx}{dt} = \frac{1}{m}(p_x - qBy)$ (B) $\frac{dp_x}{dt} = \frac{qB}{m}(p_y - qBx)$
 (C) $\frac{dp_y}{dt} = -\frac{qB}{m}(p_x + qBy)$ (D) $H = \frac{1}{2m}[(p_x + qBy)^2 + (p_y - qBx)^2 + p_z^2]$

Ans.: (B), (C), (D)

Solution.: The given Lagrangian is $L = \frac{m}{2} v^2 + qB(xv_y - yv_x)$ where $v^2 = v_x^2 + v_y^2 + v_z^2$

The conjugate momenta are $p_x = \frac{\partial L}{\partial v_x} = mv_x - qBy \Rightarrow v_x = \frac{p_x + qBy}{m}$

Similarly $p_y = \frac{\partial L}{\partial v_y} = mv_y + qBx \Rightarrow v_y = \frac{p_y - qBx}{m}$ also $p_z = \frac{\partial L}{\partial v_z} = mv_z$

(A) Given $\frac{dx}{dt} = v_x = \frac{1}{m}(p_x - qBy)$ is not correct. $\therefore v_x = \frac{p_x + qBy}{m}$

(D) The Hamiltonian, obtained by the Legendre transformation, is

$$H = \frac{1}{2m}[(p_x + qBy)^2 + (p_y - qBx)^2 + p_z^2]$$

So option (D) is correct.

$$(B) \dot{p}_x = -\frac{\partial H}{\partial x} = \frac{qB}{m}(p_y - qBx). \text{ So option (B) is correct.}$$

$$(C) \dot{p}_y = -\frac{\partial H}{\partial y} = -\frac{qB}{m}(p_x + qBy). \text{ So option (C) is correct.}$$

Q52. A bead is constrained to move along a long, massless, frictionless horizontal rod parallel to the x axis. The rod itself is moving vertically upward along the z direction against gravity with a constant speed, starting from $z=0$ at $t=0$, and remains horizontal. The conjugate momenta are denoted by p_x, p_y, p_z and the Hamiltonian by H . Which of the following option(s) is/are correct?

- (A) H is the total energy of the system and is conserved
- (B) H is the total energy of the system and is not conserved
- (C) H is not the total energy of the system, but it is conserved
- (D) H is not the total energy of the system and is not conserved

Ans.: (D)

Solution.: The rod moves upward with constant speed. Therefore, the constraint itself is explicitly time-dependent.

The bead's vertical coordinate is prescribed by $z = ut$

Consequently, the Hamiltonian associated with the reduced system is **not simply the total mechanical energy**.

Because the constraint is explicitly time-dependent, the Hamiltonian need not be conserved.

Thus, H is not the total energy of the system and is not conserved

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Q53. In a one-dimensional Hamiltonian system with position q and momentum p , consider the canonical transformation $(q, p) \rightarrow \left(Q = \frac{1}{p}, P = qp^2 \right)$, where Q and P are the new position and momentum, respectively. Which of the following option(s) regarding the generating function F is/are correct?

(A) $F = F_1(q, Q) = \frac{q}{Q}$

(B) $F = F_2(q, P) = \sqrt{Pq}$

(C) $F = F_3(p, Q) = 2\frac{P}{Q}$

(D) $F = F_4(p, P) = \frac{P}{p}$

Ans.: (A), (D)

Solution.: The canonical transformation is $Q = \frac{1}{p}, P = qp^2$

(A) $F = F_1(q, Q) = \frac{q}{Q} : p = \frac{\partial F_1}{\partial q}, P = -\frac{\partial F_1}{\partial Q}$

$p = \frac{\partial F_1}{\partial q} = \frac{1}{Q} \Rightarrow Q = \frac{1}{p}, P = -\frac{\partial F_1}{\partial Q} = \frac{q}{Q^2} \Rightarrow P = qp^2$. So option (A) is correct.

(B) $F = F_2(q, P) = \sqrt{Pq} : p = \frac{\partial F_2}{\partial q}, Q = \frac{\partial F_2}{\partial P}$

$p = \frac{\partial F_2}{\partial q} = \frac{1}{2} \sqrt{\frac{P}{q}} \Rightarrow P \neq qp^2$. So option (B) is incorrect.

(C) $F = F_3(p, Q) = 2\frac{P}{Q} : q = -\frac{\partial F_3}{\partial p}, P = -\frac{\partial F_3}{\partial Q}$

$q = -\frac{\partial F_3}{\partial p} = -\frac{2}{Q}$. So option (C) is incorrect.

(D) $F = F_4(p, P) = \frac{P}{p} : q = -\frac{\partial F_4}{\partial p}, Q = \frac{\partial F_4}{\partial P}$

$q = -\frac{\partial F_4}{\partial p} = \frac{P}{p^2} \Rightarrow P = qp^2, Q = \frac{\partial F_4}{\partial P} = \frac{1}{p}$. So option (D) is correct.

Q54. The energy of a free, relativistic particle of rest mass m moving along the x axis in one dimension, is denoted by T . When moving in a given potential $V(x)$, its Hamiltonian is $H = T + V(x)$. In the presence of this potential, its speed is v , conjugate momentum p , and the Lagrangian L . Then, which of the following option(s) is/are correct?

(A) $H = c^2 \sqrt{m^2 + \frac{p^2}{c^2}} + V(x)$

(B) $v = \frac{pc}{\sqrt{p^2 + m^2 c^2}}$

(C) $L = mc^2 \sqrt{1 - \frac{v^2}{c^2}} - V(x)$

(D) $L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - V(x)$

Ans.: (A), (B), (D)

Solution.: Relativistic Energy and Hamiltonian:

$$H = \sqrt{p^2 c^2 + m^2 c^4} + V(x) = c^2 \sqrt{m^2 + \frac{p^2}{c^2}} + V(x)$$

$$\text{Velocity: } v = \frac{\partial H}{\partial p} = \frac{pc^2}{\sqrt{p^2 c^2 + m^2 c^4}} = \frac{pc}{\sqrt{p^2 + m^2 c^2}}$$

$$\text{Lagrangian: } L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - V(x)$$

Q62. The Hamiltonian for a one dimensional system with mass m , position q and momentum p is $H(p, q) = \frac{p^2}{2m} + q^2 A(q)$, where $A(q)$ is a real function of q . If $m \frac{d^2 q}{dt^2} = -5qA(q)$, then

$\frac{dA(q)}{dq} = n \frac{A(q)}{q}$. The value of n (in integer) is _____

Ans.: 3

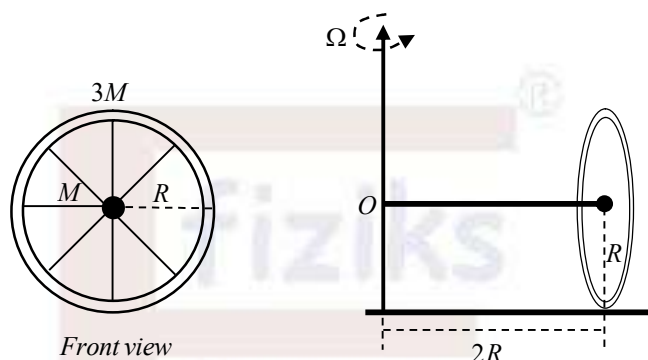
Solution.: Hamilton's Equation:

$$\dot{p} = -\frac{\partial H}{\partial q} \Rightarrow m\ddot{q} = -\frac{\partial}{\partial q} \left[\frac{p^2}{2m} + q^2 A(q) \right] = - \left[2qA(q) + q^2 \frac{dA(q)}{dq} \right]$$

$$\Rightarrow m\ddot{q} = - \left[2qA(q) + q^2 n \frac{A(q)}{q} \right] \Rightarrow m\ddot{q} = -(2+n)qA(q) \quad \therefore \frac{dA(q)}{dq} = n \frac{A(q)}{q}$$

$$\text{Compare with } m \frac{d^2 q}{dt^2} = -5qA(q) \Rightarrow 2+n=5 \Rightarrow n=3$$

Q64. A wheel of mass $4M$ and radius R is made of a thin uniform distribution of mass $3M$ at the rim and a point mass M at the center. The spokes of the wheel are massless. The center of mass of the wheel is connected to a horizontal massless rod of length $2R$, with one end fixed at O , as shown in the figure. The wheel rolls without slipping on horizontal ground with angular speed Ω . If $\vec{L}$ is the total angular momentum of the wheel about O , then the magnitude $\left| \frac{d\vec{L}}{dt} \right| = N(MR^2\Omega^2)$. The value of N (in integer) is ____



Ans.: 6

Solution.: Mass of wheel: $M_{Total} = 4M$, The center of the wheel is at $OC = 2R$

The center moves in a horizontal circle with angular speed Ω , so $v_C = 2R\Omega$

Orbital angular momentum about O : $L_{orb} = r_C (M_{Total} v_C)$ $\because \vec{r}_C \perp \vec{v}_C$

$\Rightarrow L_{orb} = (2R)(4M)(2R\Omega) = 16MR^2\Omega$. This component is along the vertical direction.

Spin angular velocity of the wheel

For rolling without slipping, $\omega_s R = v_C \Rightarrow \omega_s R = 2R\Omega \Rightarrow \omega_s = 2\Omega$

The $3M$ mass is distributed at the rim, so its M.I. about the wheel's axle is $I_{axle} = 3MR^2$

Therefore, the spin angular momentum about the CM is

$$L_{spin} = I_{axle} \omega_s = 3MR^2 (2\Omega) = 6MR^2\Omega$$

This vector is along the horizontal radial direction. This radial direction continuously rotates

about O . Hence $\frac{d\vec{L}_{spin}}{dt} = \Omega L_{spin} = 6MR^2\Omega^2$

The orbital angular momentum $\vec{L}_{orb}$ is vertical and has constant magnitude and direction, so

$$\frac{d\vec{L}_{orb}}{dt} = 0 \Rightarrow \left| \frac{d\vec{L}}{dt} \right| = 6MR^2\Omega^2 = N(MR^2\Omega^2) \Rightarrow N = 6.$$



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Solution-Electromagnetic Theory

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Q31. A linear dielectric sphere of radius R has uniform frozen-in polarization along the z -axis. The center of the sphere initially coincides with the origin, about which the electric dipole moment is $\vec{p}_1$. When the sphere is shifted to the point $(2R, 0, 0)$, the corresponding dipole moment with respect to the origin is $\vec{p}_2$. The value of $\frac{|\vec{p}_1|}{|\vec{p}_2|}$ (in integer)

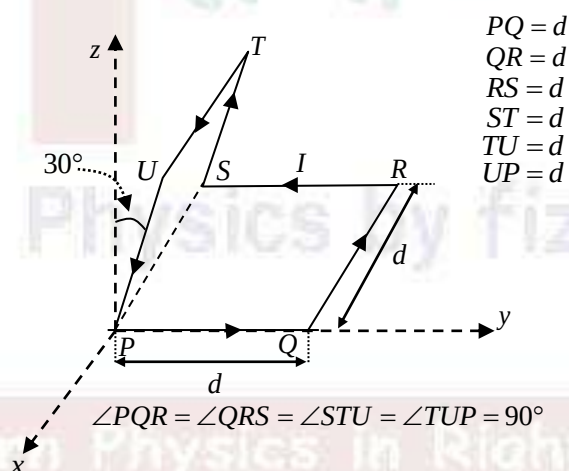
Ans.: 1

Solution.: Polarization $\vec{P} = \text{constant}$, then electric dipole moment $\vec{p}_1 = \vec{P} \times \frac{4}{3}\pi R^3$.

Total charge $q_T = 0$, so dipole moment is independent of origin and $\vec{p}_1 = \vec{p}_2 \Rightarrow \frac{|\vec{p}_1|}{|\vec{p}_2|} = 1$.

Q.36 – Q.65 Carry TWO marks Each

Q37. The wire loop shown in the figure carries a steady current I . Each straight section of the loop has length d . A part of the loop lies in xy plane and the other part is tilted at 30° with respect to the xz plane. The magnitude of the magnetic dipole moment of the loop (in appropriate units) is



(A) $\sqrt{2}Id^2$

(B) $2Id^2$

(C) $\sqrt{3}Id^2$

(D) Id^2

Ans.: (D)

Solution.: For loop $PQRSP$: $\vec{m}_1 = Id^2 \hat{z}$

For loop $STUPS$: $\vec{m}_2 = Id^2 \hat{n} = Id^2 [\cos(90^\circ + 30^\circ) \hat{z} + \sin(90^\circ + 30^\circ) \hat{y}]$

$$\Rightarrow \vec{m}_2 = Id^2 \left[(-\sin 30^\circ) \hat{z} + (\cos 30^\circ) \hat{y} \right] = Id^2 \left[-\frac{1}{2} \hat{z} + \frac{\sqrt{3}}{2} \hat{y} \right]$$

$$\text{Thus } \vec{m} = \vec{m}_1 + \vec{m}_2 = Id^2 \hat{z} + Id^2 \left[-\frac{1}{2} \hat{z} + \frac{\sqrt{3}}{2} \hat{y} \right] = \frac{Id^2}{2} \hat{z} + \frac{\sqrt{3}}{2} Id^2 \hat{y}$$

$$\Rightarrow |\vec{m}| = \frac{Id^2}{2} \sqrt{3+1} = \frac{Id^2}{2} \times 2 = Id^2$$

Q40. A thin circular ring of radius R lies in the xy plane with its center coinciding with the origin. The ring carries a uniform line charge density λ . The quadrupole contribution to the electrostatic potential at the point $(0,0,d)$, where $d \gg R$, is

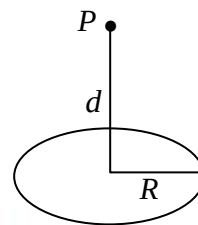
- (A) $-\frac{\lambda R^3}{4\epsilon_0 d^3}$ (B) 0 (C) $\frac{\lambda R^3}{4\epsilon_0 d^3}$ (D) $-\frac{\lambda R^3}{2\epsilon_0 d^3}$

Ans.: (A)

Solution.: Electric field at a distance z above the center of loop

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda(2\pi R)z}{(R^2 + z^2)^{3/2}} \hat{z} = \frac{\lambda R}{2\epsilon_0} \frac{z}{(R^2 + z^2)^{3/2}} \hat{z}$$

$$V(P) = -\int_{\infty}^d \vec{E} \cdot d\vec{l} = -\frac{\lambda R}{2\epsilon_0} \int_{\infty}^d \frac{z}{(R^2 + z^2)^{3/2}} dz$$



$$\text{Let } R^2 + z^2 = t^2 \Rightarrow z dz = t dt$$

$$\Rightarrow V(P) = -\frac{\lambda R}{2\epsilon_0} \int_{\infty}^{\sqrt{R^2+d^2}} \frac{1}{t^3} t dt = \frac{\lambda R}{2\epsilon_0 \sqrt{R^2+d^2}} = \frac{\lambda R}{2\epsilon_0 d \sqrt{1+R^2/d^2}} = \frac{\lambda R}{2\epsilon_0 d} \left[1 + \frac{R^2}{d^2} \right]^{-1/2}$$

$$\Rightarrow V(P) = \frac{\lambda R}{2\epsilon_0 d} \left[1 - \frac{1}{2} \frac{R^2}{d^2} \right]. \text{ So quadrupole contribution is } V_{quad.}(P) = -\frac{\lambda R^3}{4\epsilon_0 d^3}.$$

Q49. A point charge q is placed at a distance d above an infinite, grounded conducting plate placed on the xy plane at $z=0$. The electrostatic potential in $z>0$ region is given by

$$\phi = \phi_1 + \phi_2, \text{ where } \phi_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{x^2 + y^2 + (z-d)^2}} \text{ and } \phi_2 = -\frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{x^2 + y^2 + (z+d)^2}}. \text{ Which}$$

of the following option(s) is/are correct?

(A) The magnitude of the force experienced by the point charge q is $\frac{1}{16\pi\epsilon_0} \frac{q^2}{d^2}$

(B) The electrostatic energy of the system is $\frac{1}{8\pi\epsilon_0} \frac{q^2}{d}$

(C) The induced surface charge density on the plate is proportional to $\frac{1}{\sqrt{x^2 + y^2 + d^2}}$

(D) The electrostatic potential ϕ_1 satisfies Poisson's equation for $z>0$

Ans.: (A), (D)

Solution.:

(A) The magnitude of the force experienced by the point charge q is

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{(2d)^2} = \frac{1}{16\pi\epsilon_0} \frac{q^2}{d^2}$$

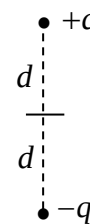
(B) The electrostatic energy of the system is $\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{q^2}{2d} = \frac{1}{16\pi\epsilon_0} \frac{q^2}{d}$

(C) The induced surface charge density on the plate is proportional to

$$\sigma(x, y) = -\epsilon_0 \left[\frac{\partial}{\partial z} (\phi_1 + \phi_2) \right]_{z=0} = \frac{-qd}{2\pi(x^2 + y^2 + d^2)^{3/2}}$$

(D) The electrostatic potential ϕ_1 satisfies Poisson's equation for $z > 0$:

$$\nabla^2 \phi_1 = \frac{-q}{\epsilon_0} \delta(x) \delta(y) \delta(z-d)$$



Q56. A neutral conducting sphere of radius R is placed in a uniform electric field of magnitude E_0 , that points along the z axis. The electrostatic potential at any point $\vec{r}$ outside the sphere is given by $V(r, \theta) = V_0 - E_0 r \left(1 - \frac{R^3}{r^3} \right) \cos \theta$, where V_0 is the constant potential of the sphere.

Which of the following option(s) is/are correct?

(A) The induced surface charge density on the sphere is proportional to $\sin \theta$

(B) As $r \rightarrow \infty$, $\vec{E} = E_0 \cos \theta \hat{r}$

(C) The electric field at any point is curl free for $r > R$

(D) The electric field at any point is divergence free for $r > R$

Ans.: (C), (D)

Solution.: $r > R$: $V(r, \theta) = V_0 - E_0 r \left(1 - \frac{R^3}{r^3} \right) \cos \theta$

(A) The induced surface charge density on the sphere is $\sigma = -\epsilon_0 \frac{\partial V}{\partial r} \Big|_{r=R}$

$$\Rightarrow \sigma = -\epsilon_0 \left[0 - E_0 \left(1 - \frac{R^3}{r^3} \right) \cos \theta - E_0 r \left(0 + \frac{3R^3}{r^4} \right) \cos \theta \right]_{r=R}$$

$$\Rightarrow \sigma = -\epsilon_0 [-3E_0 \cos \theta] = 3\epsilon_0 E_0 \cos \theta$$

$$(B) \because \vec{E} = E_0 \hat{z} \Rightarrow -\frac{dV}{dz} = E_0 \Rightarrow V = -E_0 z + C = -E_0 r \cos \theta + C$$

$\therefore V = 0$ in xy plane so $C = 0$. Now $V = -E_0 r \cos \theta$ for $r \gg R$.

(C) $\because \vec{E} = E_0 \hat{z}$ for $r > R$, then electric field at any point is curl free for $r > R$

(D) $\because \vec{E} = E_0 \hat{z}$ for $r > R$, then electric field at any point is divergence free for $r > R$

Q57. A point charge q is placed at the origin, inside a linear dielectric medium of infinite extent, having relative permittivity ϵ_r . Which of the following option(s) is/are correct?

(A) The magnitude of the polarization varies as $\frac{1}{r^2}$

(B) The magnitude of the polarization varies as $\frac{1}{r^3}$

(C) The magnitude of the screened charge due to the dielectric medium is less than the magnitude of the point charge q for $\epsilon_r > 1$

(D) The magnitude of the screened charge due to the dielectric medium is more than the magnitude of the point charge q for $\epsilon_r = 1$

Ans.: (A), (C)

Solution.: The electric field inside is $\vec{E} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q}{r^2} \hat{r}$

Polarization $\vec{P} = \epsilon_0 \chi_e \vec{E} = \epsilon_0 (\epsilon_r - 1) \vec{E} = \frac{(\epsilon_r - 1)}{4\pi \epsilon_r} \frac{q}{r^2} \hat{r}$

Bound Charge at the origin $q_b = -\frac{(\epsilon_r - 1)q}{\epsilon_r}$

So the total effective (screened) charge is: $q_{eff} = q + q_b = \frac{q}{\epsilon_r}$

Q58. A linear magnetic material in the form of a cylinder of radius R and length L is placed with its axis parallel to the z axis. The cylinder has uniform magnetization $M\hat{k}$. Which of the following option(s) is/are correct?

(A) The magnetic field at any point outside the cylinder can be expressed as the gradient of a scalar function

(B) The bound volume current density is zero

(C) The surface current density on the curved surface is non-zero

(D) The surface current densities on the flat surfaces (top and bottom) are non-zero

Ans.: (A), (B), (C)

Solution.: Cylinder has uniform magnetization $\vec{M} = M\hat{k} \Rightarrow \vec{J}_b = \vec{\nabla} \times \vec{M} = 0$.

At $r = R$: $\vec{K}_b = \vec{M} \times \hat{n} = M\hat{k} \times \hat{r} = M\hat{\phi}$

At $z = L$: $\vec{K}_b = \vec{M} \times \hat{n} = M\hat{k} \times (+\hat{k}) = 0$; At $z = 0$: $\vec{K}_b = \vec{M} \times \hat{n} = M\hat{k} \times (-\hat{k}) = 0$.

For $r > R$: $\vec{B} = 0 \Rightarrow \vec{\nabla} \times \vec{B} = 0 \Rightarrow \vec{B} = \vec{\nabla}V$

Q59. Cyclotrons are used to accelerate ions like deuterons (d) and α particles. Keeping the magnetic field same for both, d and α are extracted with energies 10 MeV and 20 MeV with extraction radii r_d and r_α , respectively. Taking the masses $M_d = 2000 \text{ MeV}/c^2$ and $M_\alpha = 4000 \text{ MeV}/c^2$, the value of $\frac{r_\alpha}{r_d}$ (in integer) is _____

Ans.: 1

Solution.: $\because K.E. = K = \frac{q^2 B^2 R^2}{2m} \Rightarrow \frac{K_\alpha}{K_d} = \frac{q_\alpha^2 R_\alpha^2}{q_d^2 R_d^2} \times \frac{m_d}{m_\alpha}$. Magnetic field remains same.

$$\Rightarrow \frac{20 \text{ MeV}}{10 \text{ MeV}} = \frac{(2e)^2 r_\alpha^2}{e^2 r_d^2} \times \frac{2000}{4000} \Rightarrow \frac{r_\alpha^2}{r_d^2} = 2 \times \frac{4}{8} = 1 \Rightarrow \frac{r_\alpha}{r_d} = 1$$



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Q13. A particle of mass m is in a potential $V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and $V(x) = \infty$ for $x \leq 0$,

where ω is the angular frequency. The ratio of the energies corresponding to the lowest energy level to the next higher level is

- (A) $\frac{3}{7}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{3}{5}$

Ans.: (A)

Solution.: Energy levels of a truncated (half) harmonic oscillator:

Due to the infinite boundary condition at $x = 0$, the wave function must vanish at the origin. This restricts the allowed energy levels to only the **odd** quantum states of the standard 1D harmonic oscillator:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \text{ for } n = 1, 3, 5, \dots$$

Ground state (lowest energy level, $n = 1$): $E_1 = \left(1 + \frac{1}{2}\right)\hbar\omega = \frac{3}{2}\hbar\omega$

First excited state (next higher level, $n = 3$): $E_3 = \left(3 + \frac{1}{2}\right)\hbar\omega = \frac{7}{2}\hbar\omega$

Thus $\frac{E_1}{E_3} = \frac{3}{7}$

Q14. A particle is scattered from a potential $V(\vec{r}) = g\delta^3(\vec{r})$, where g is a positive constant.

Using the first-Born approximation, the angular (θ, ϕ) dependence of differential scattering cross section $\frac{d\sigma}{d\Omega}$ is

- (A) Independent of θ but dependent on ϕ (B) Dependent on θ but independent of ϕ
(C) Dependent on both θ and ϕ (D) Independent of both θ and ϕ

Ans.: (D)

Solution.: Scattering amplitude in Born approximation: $f(\theta, \phi) = -\frac{m}{2\pi\hbar^2} \int V(r) e^{-iqr} d^3r$

$$\Rightarrow f(\theta, \phi) = -\frac{m}{2\pi\hbar^2} \int g\delta^3(\vec{r}) e^{-iqr} d^3r = -\frac{mg}{2\pi\hbar^2}$$

Differential cross section: $\frac{d\sigma}{d\Omega} = |f(\theta, \phi)|^2 = \left(-\frac{mg}{2\pi\hbar^2}\right)^2$

This value is a constant, containing no dependence on the scattering angles θ or ϕ .

Q16. Which one of the following is correct for the phase velocity v_p and group velocity v_g ?

(c is the speed of light in vacuum)

- (A) For matter waves in the relativistic case, $v_p v_g > c^2$
- (B) For electromagnetic waves in a medium, v_p represents the speed with which energy propagates
- (C) For electromagnetic waves in a medium, both v_p and v_g can be more than c
- (D) For matter waves in free space, $v_p \neq v_g$

Ans.: (D)

Solution.:

Analyze Relativistic Matter Waves:

Total energy: $E = \gamma mc^2 = \hbar\omega$, Momentum: $p = \gamma mv = \hbar k$

Phase velocity: $v_p = \frac{\omega}{k} = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v}$, Group velocity: $v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = v$

$$\Rightarrow v_p v_g = c^2 \Rightarrow v_p > c \quad \because v_g < c$$

For matter waves in free space: $E = \frac{\hbar^2 k^2}{2m} \Rightarrow v_p = \frac{\hbar k}{2m}$ and $v_g = \frac{\hbar k}{m} \Rightarrow v_p = \frac{1}{2} v_g$

Q.36 – Q.65 Carry TWO marks Each

Q43. Let $|m\rangle$ and $|n\rangle$ denote the energy eigenstates of a one-dimensional simple harmonic oscillator. The position and momentum operators are $\hat{X}$ and $\hat{P}$, respectively. The matrix element $\langle m | \hat{P} \hat{X} | n \rangle$ is non-zero when

- (A) $m = n \pm 2$ only (B) $m = n$ or $m = n \pm 2$
- (C) $m = n \pm 3$ only (D) $m = n \pm 1$ only

Ans.: (B)

Solution.: $\because \hat{X} = \sqrt{\frac{\hbar}{2m\omega}} (a + a^\dagger)$ and $\hat{P} = -i\sqrt{\frac{m\omega\hbar}{2}} (a - a^\dagger)$

$$\hat{P} \hat{X} \propto (a - a^\dagger)(a + a^\dagger) = a^2 + aa^\dagger - a^\dagger a - (a^\dagger)^2$$

Note: $a^2 |n\rangle \propto |n-2\rangle$, $aa^\dagger |n\rangle \propto |n\rangle$, $a^\dagger a |n\rangle \propto |n\rangle$, $(a^\dagger)^2 |n\rangle \propto |n+2\rangle$

$$\langle m | \hat{P} \hat{X} | n \rangle \propto \langle m | n-2 \rangle + \langle m | n \rangle + \langle m | 2 \rangle + \langle m | n+2 \rangle$$

The matrix element $\langle m | \hat{P} \hat{X} | n \rangle$ will be non-zero only when $m = n$ or $m = n \pm 2$

Q44. A two-level quantum system has energy eigenvalues E_1 and E_2 . A perturbing potential $H' = \lambda \Delta \sigma_x$ is introduced, where Δ is a constant having dimensions of energy, λ is a small dimensionless parameter, and $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The magnitudes of the first and the second order corrections to E_1 due to H' , respectively, are

- (A) 0 and $\frac{\lambda^2 \Delta^2}{|E_1 - E_2|}$ (B) $\frac{|\lambda \Delta|}{2}$ and $\frac{\lambda^2 \Delta^2}{|E_1 - E_2|}$
 (C) $|\lambda \Delta|$ and $\frac{\lambda^2 \Delta^2}{|E_1 - E_2|}$ (D) 0 and $\frac{1}{2} \frac{\lambda^2 \Delta^2}{|E_1 - E_2|}$

Ans.: (A)

Solution.:

First-order energy correction $E_1^{(1)} = \langle 1 | H' | 1 \rangle$

Since $|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $H' = \lambda \Delta \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$: $E_1^{(1)} = \langle 1 | H' | 1 \rangle = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \lambda \Delta \\ \lambda \Delta & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$

Second-order energy correction $E_1^{(2)} = \frac{|\langle 2 | H' | 1 \rangle|^2}{E_2 - E_1}$

Calculate the off-diagonal matrix element:

$$\langle 2 | H' | 1 \rangle = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & \lambda \Delta \\ \lambda \Delta & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \lambda \Delta \Rightarrow |E_1^{(2)}| = \frac{\lambda^2 \Delta^2}{|E_2 - E_1|}$$

Q45. An electron with mass m and charge q is in the spin up state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ at time $t = 0$.

A constant magnetic field is applied along the y -axis, $\vec{B} = B_0 \hat{j}$, where B_0 is a constant. The Hamiltonian of the system is $H = -\hbar \omega \sigma_y$, where $\omega = \frac{q B_0}{2m} > 0$ and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. The minimum

time after which the electron will be in the spin down state along the x -axis, i.e., $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, is

- (A) $\frac{\pi}{8\omega}$ (B) $\frac{\pi}{4\omega}$
 (C) $\frac{\pi}{2\omega}$ (D) $\frac{\pi}{\omega}$

Ans.: (B)

Solution.: Time evolution operator: $U(t) = e^{-iHt/\hbar} = e^{i\omega\sigma_y} = \cos(\omega t)I + i\sin(\omega t)\sigma_y$

$$U(t) = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix}$$

State at time t : $|\psi(t)\rangle = U(t) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\omega t) \\ -\sin(\omega t) \end{bmatrix}$

We require $|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \cos(\omega t) = \frac{1}{\sqrt{2}}, \sin(\omega t) = \frac{1}{\sqrt{2}}$

Solve for minimum t : $\omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{4\omega}$

Q46. A system of three non-identical spin $\frac{1}{2}$ particles has the Hamiltonian $H = \frac{A}{\hbar^2} (\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$,

where $\vec{S}_1, \vec{S}_2$ and $\vec{S}_3$ are the spin operators of particles labelled 1, 2 and 3 respectively and A is a constant with appropriate dimensions. The set of possible energy eigenvalues of the system is

- (A) $0, \frac{A}{2}, -A$ (B) $0, \frac{A}{2}, -\frac{A}{2}$ (C) $0, \frac{3A}{2}, -\frac{A}{2}$ (D) $0, -\frac{3A}{2}, \frac{A}{2}$

Ans.: (A)

Solution.: Hamiltonian: $H = \frac{A}{\hbar^2} (\vec{S}_1 + \vec{S}_2) \cdot \vec{S}_3$. Let $\vec{S}_{12} = \vec{S}_1 + \vec{S}_2$ then $H = \frac{A}{\hbar^2} \vec{S}_{12} \cdot \vec{S}_3$

Total spin: $\vec{S} = \vec{S}_{12} + \vec{S}_3 \Rightarrow S^2 = S_{12}^2 + S_3^2 + 2\vec{S}_{12} \cdot \vec{S}_3 \Rightarrow \vec{S}_{12} \cdot \vec{S}_3 = \frac{1}{2}(S^2 - S_{12}^2 - S_3^2)$

Possible values: $S_{12} = 1, 0, S_3 = \frac{1}{2}, S = \frac{3}{2}$ or $\frac{1}{2} \left(S_{12} = \frac{1}{2} \right)$ or $\frac{1}{2} (S_{12} = 0)$

Eigenvalues:

$$E = \frac{A}{\hbar^2} \vec{S}_{12} \cdot \vec{S}_3 = \frac{A}{2\hbar^2} (S^2 - S_{12}^2 - S_3^2) = \frac{A}{2\hbar^2} [S(S+1)\hbar^2 - S_{12}(S_{12}+1)\hbar^2 - S_3(S_3+1)\hbar^2]$$

$$\Rightarrow E = \frac{A}{2} [S(S+1) - S_{12}(S_{12}+1) - S_3(S_3+1)]$$

For $S_{12} = 1, S = \frac{3}{2}, S_3 = \frac{1}{2}$: $E = \frac{A}{2} \left[\frac{15}{4} - 2 - \frac{3}{4} \right] = \frac{A}{2}$

For $S_{12} = 1, S = \frac{1}{2}, S_3 = \frac{1}{2}$: $E = \frac{A}{2} \left[\frac{3}{4} - 2 - \frac{3}{4} \right] = -A$

For $S_{12} = 0, S = \frac{1}{2}, S_3 = \frac{1}{2}$: $E = \frac{A}{2} \left[\frac{3}{4} - 0 - \frac{3}{4} \right] = 0$

So, eigenvalues: $0, \frac{A}{2}, -A$

Q63. A system of five identical, non-interacting particles with mass m and spin $\frac{3}{2}$ is confined to a one-dimensional potential well of length L . If the lowest energy of the system is $N \frac{\pi^2 \hbar^2}{2mL^2}$, the value of N (in integer) is _____

Ans.: 8

Solution.: For spin $s = \frac{3}{2}$, the number of spin projection states is: $2s + 1 = 3 + 1 = 4$

By the Pauli exclusion principle, a maximum of **4 particles** can occupy any single spatial energy level n .

Distribution of 5 particles:

- $n = 1$ level: Occupied by 4 particles.
- $n = 2$ level: Occupied by the remaining 1 particle.

Total ground state energy:

Energy of a particle in level n is $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} = n^2 E_0$.

$$\text{So, } E_{\text{Total}} = 4E_1 + E_2 = 4(1^2 E_0) + E_2(2^2 E_0) = 8E_0 \Rightarrow E_{\text{Total}} = 8 \frac{\pi^2 \hbar^2}{2mL^2} = N \frac{\pi^2 \hbar^2}{2mL^2} \Rightarrow N = 8$$



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Q.11 – Q.35 Carry ONE mark Each**Q15.** The Joule-Thomson expansion of a gas is

- (A) Isentropic (B) Isenthalpic (C) Isobaric (D) Isochoric

Ans.: (B)**Solution.:**

Q22. Consider one mole of a monovalent metal at absolute zero temperature, obeying the free electron model. Its Fermi energy is E_F . The energy corresponding to the filling of $\frac{N_A}{2}$ electrons, where N_A is the Avogadro number, is $2^n E_F$. The value of n is

- (A)
- $-\frac{2}{3}$
- (B)
- $+\frac{2}{3}$
- (C)
- $-\frac{1}{3}$
- (D)
- -1

Ans.: (A)**Solution.:** In the free electron model, the fermi energy for a gas of N electrons in volume V is:

$$E_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{2/3}.$$

For $\frac{N_A}{2}$ electrons, the electron density becomes $\frac{N_A}{2V}$. Then the fermi energy is

$$E'_F = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N_A}{2V} \right)^{2/3} = \left(\frac{1}{2} \right)^{2/3} E_F = 2^{-2/3} E_F$$

$$\therefore 2^n E_F = 2^{-2/3} E_F \Rightarrow n = -\frac{2}{3}$$

Q23. A paramagnetic material containing paramagnetic ions with total angular momentum $J = \frac{1}{2}$ is kept at absolute temperature T . The ratio of the magnetic field required for 80% of the ions to be in the lowest energy state to that required for having 60% of the ions to be in the lowest energy state at the same temperature is

- (A)
- $\frac{2 \ln 2}{\ln\left(\frac{3}{2}\right)}$
- (B)
- $\frac{\ln 2}{\ln\left(\frac{3}{2}\right)}$
- (C)
- $\frac{3 \ln 2}{\ln\left(\frac{3}{2}\right)}$
- (D)
- $\frac{\ln 3}{\ln\left(\frac{3}{2}\right)}$

Ans.: (A)

Solution.: With total angular momentum $J = \frac{1}{2}$ there are two orientations of dipole $+1$ in the direction of magnetic field with energy $E_1 = -\mu B$ and -1 in the direction opposite to magnetic field with energy $E_2 = +\mu B$.

$$\text{Thus } \frac{N_2}{N_1} = \frac{e^{-\beta E_2}}{e^{-\beta E_1}} = e^{-\beta(E_2 - E_1)} = e^{-\beta(2\mu B)}$$

$$\text{Case-I: } N_1 = \frac{80}{100} N, \quad N_2 = \frac{20}{100} N \Rightarrow \frac{N_2}{N_1} = \frac{1}{4} = e^{-\beta(2\mu B_1)} \Rightarrow \beta(2\mu B_1) = \ln 4$$

$$\text{Case-II: } N_1 = \frac{60}{100} N, \quad N_2 = \frac{40}{100} N \Rightarrow \frac{N_2}{N_1} = \frac{2}{3} = e^{-\beta(2\mu B_2)} \Rightarrow \beta(2\mu B_2) = \ln \frac{3}{2}$$

$$\text{Thus } \frac{\beta(2\mu B_1)}{\beta(2\mu B_2)} = \frac{\ln 4}{\ln \frac{3}{2}} \Rightarrow \frac{B_1}{B_2} = \frac{2 \ln 2}{\ln 3/2}$$

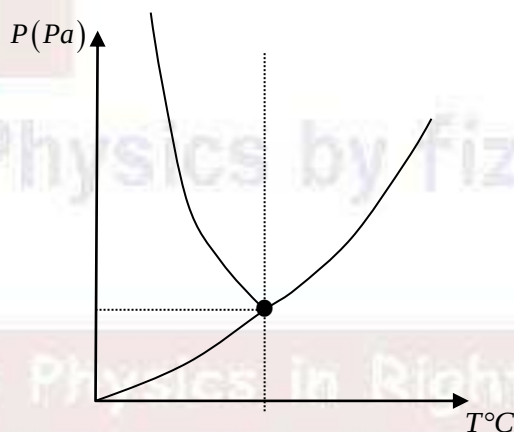
Q25. Which of the following option(s) is/are correct for photons?

- (A) Its rest mass is zero, but its energy is non-zero
- (B) It carries non-zero linear momentum
- (C) It carries zero spin angular momentum
- (D) It has two linearly independent states of polarization

Ans.: (A), (B), (D)

Solution.:

Q26. A schematic Pressure-Temperature diagram of water is shown in the figure. Which of the following option(s) is/are correct?



- (A) Clausius-Clapeyron equation is valid across the melting curve and the vaporization curve
- (B) Melting curve has the highest slope
- (C) The critical point exists only for the vaporization curve
- (D) Clausius-Clapeyron equation is not valid across the melting curve and the vaporization curve

Ans.: (A), (B), (C)

Solution.:

Q30. One mole of an ideal monatomic gas at absolute temperature T undergoes free expansion to double its original volume, so that the entropy change is ΔS_1 . An identical amount of the same gas at absolute temperature $2T$ undergoes isothermal expansion to double its original volume, so that the entropy change is ΔS_2 . The value of $\frac{\Delta S_1}{\Delta S_2}$ (in integer) is _____

Ans.: 1

Solution.: In Free Expansion: $\Delta S = k_B \ln \frac{V_2}{V_1}$.

At temp. T : $\Delta S_1 = k_B \ln \frac{2V}{V} = k_B \ln 2$ and At temp. $2T$: $\Delta S_2 = k_B \ln \frac{2V}{V} = k_B \ln 2$

Thus $\frac{\Delta S_1}{\Delta S_2} = 1$

Q.36 – Q.65 Carry TWO marks Each

Q42. Consider a two-level system with energy states $+\varepsilon$ and $-\varepsilon$. The number of particles at $+\varepsilon$ level is N_+ and the number of particles at $-\varepsilon$ level is N_- . The total energy of the system is E and the total number of particles is $N = N_+ + N_-$. In the thermodynamic limit, the inverse of the absolute temperature of the system is

(Given: $\ln N! \approx N \ln N - N$)

(A) $\frac{k_B}{2\varepsilon} \ln \left[\frac{N - \frac{E}{\varepsilon}}{N + \frac{E}{\varepsilon}} \right]$ (B) $\frac{k_B}{\varepsilon} \ln N$ (C) $\frac{k_B}{2\varepsilon} \ln N$ (D) $\frac{k_B}{\varepsilon} \ln \left[\frac{N - \frac{E}{\varepsilon}}{N + \frac{E}{\varepsilon}} \right]$

Ans.: (A)

Solution.: $N = N_+ + N_- \dots (1)$

$$E = N_+ (+\varepsilon) + N_- (-\varepsilon) = \varepsilon (N_+ - N_-) \Rightarrow \frac{E}{\varepsilon} = N_+ - N_- \dots (2)$$

From (1) and (2): $N_+ = \frac{1}{2} \left(N + \frac{E}{\varepsilon} \right)$ and $N_- = \frac{1}{2} \left(N - \frac{E}{\varepsilon} \right)$

Thermodynamic Probability $\Omega = \frac{N!}{N_+! N_-!}$

Entropy $S = k_B \ln \Omega = k_B \ln \left(\frac{N!}{N_+! N_-!} \right) = k_B [\ln N! - \ln N_+! - \ln N_-!]$

$\Rightarrow S = k_B [N \ln N - N - N_+ \ln N_+ + N_+ - N_- \ln N_- + N_-]$

$$\Rightarrow S = k_B [N \ln N - N_+ \ln N_+ - N_- \ln N_-]$$

$$\Rightarrow \frac{\partial S}{\partial E} = k_B \left[0 - \frac{\partial N_+}{\partial E} \ln N_+ - \frac{N_+}{N_+} \frac{\partial N_+}{\partial E} - \frac{\partial N_-}{\partial E} \ln N_- - \frac{N_-}{N_-} \frac{\partial N_-}{\partial E} \right]$$

$$\Rightarrow \frac{\partial S}{\partial E} = -k_B \left[\frac{1}{2\varepsilon} \ln N_+ + \frac{1}{2\varepsilon} + \left(-\frac{1}{2\varepsilon} \ln N_- \right) - \frac{1}{2\varepsilon} \right] = -\frac{k_B}{2\varepsilon} \ln \frac{N_+}{N_-} = \frac{k_B}{2\varepsilon} \ln \frac{N_-}{N_+}$$

$$\Rightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial E} \right)_V = \frac{k_B}{2\varepsilon} \ln \left(\frac{N - E/\varepsilon}{N + E/\varepsilon} \right)$$



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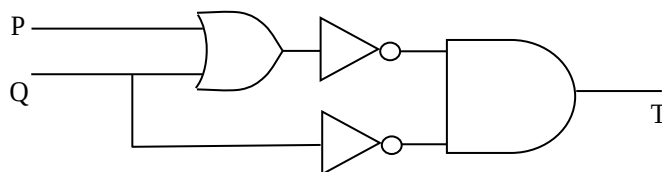
Solution-Electronics

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Q.11 – Q.35 Carry ONE mark Each

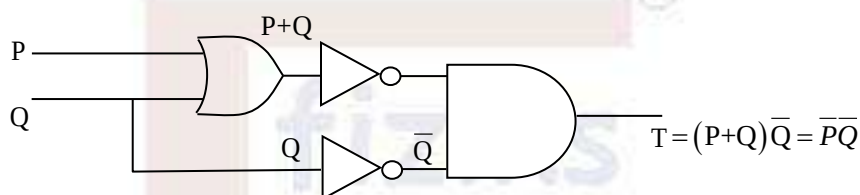
Q19. A logic gate circuit is shown in the figure below. The correct combination for the input (P, Q) for which the output $T = 1$ is



- (A) (0,0) (B) (0,1) (C) (1,1) (D) (1,0)

Ans.: (A)

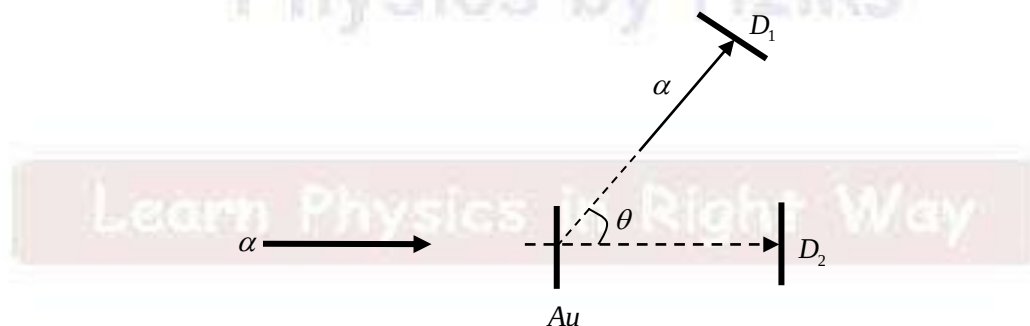
Solution.:



If $(P, Q) = (0, 0) \Rightarrow T = \bar{0}\bar{0} = 1$

Q.36 – Q.65 Carry TWO marks Each

Q41. An α particle is scattered from an Au target at rest as shown in the figure. D_1 and D_2 are the detectors to detect the scattered α particle at an angle θ and along the beam direction, respectively, as shown. The signals from D_1 and D_2 are converted to logic signals and fed to logic gates. When a particle is detected, the signal is 1 and is 0 otherwise. Which one of the following circuits detects the particle scattered at the angle θ only?



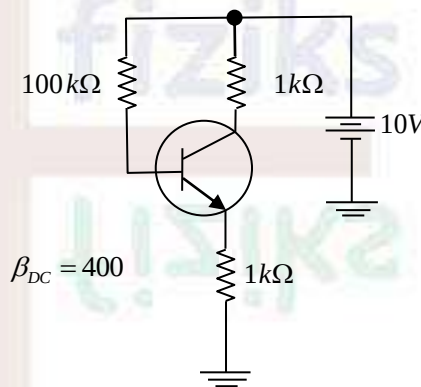
- (A) (B) (C) (D)

Ans.: (A)

Solution.:

	$D_1 = 1, D_2 = 0$	$D_1 = 1, D_2 = 1$
(A) $Y = D_1 \bar{D}_2$	$Y = 1$	$Y = 0$
(B) $Y = \bar{D}_1 D_2$	$Y = 0$	$Y = 0$
(C) $Y = \bar{D}_1 + \bar{D}_2$	$Y = 0$	$Y = 0$
(D) $Y = D_1 + \bar{D}_2$	$Y = 1$	$Y = 1$

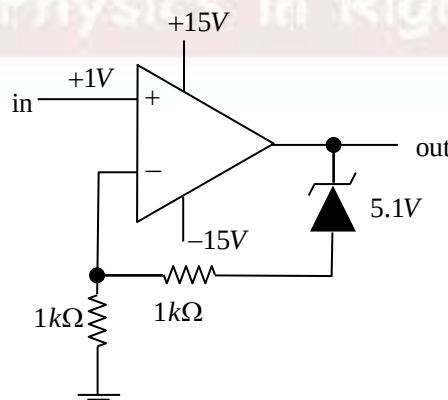
Q60. In the transistor circuit shown in the figure, $V_{BE} = 0.7V$ and $\beta_{DC} = 400$. The value of the base current in μA (rounded off to one decimal place) is _____



Ans.: 18 to 19

Solution.: $I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{10V - 0.7V}{100k + (400 + 1) \times 1k} = \frac{9.3V}{501k} = 0.01856mA = 18.6\mu A$

Q65. The figure shows an op-amp circuit with a 5.1V Zener diode in the feedback loop. The op-amp runs from $\pm 15V$ supplies. If a +1V is applied at the input, the output voltage (rounded off to one decimal place) is _____



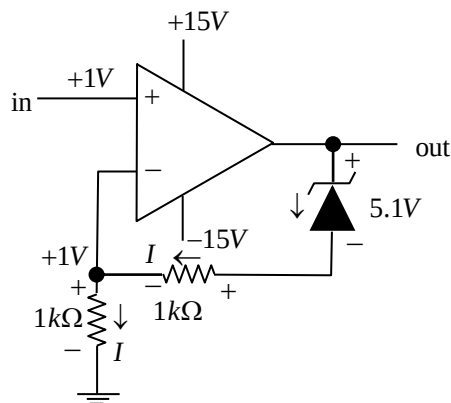
Ans.: 7 to 7.2

Solution.:

$$\text{Here } I = \frac{1V}{1k} = 1mA$$

$$-v_0 + 5.1V + 1k \times 1mA + 1V = 0$$

$$\Rightarrow v_0 = 7.1V$$



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Solution-Atomic and Molecular Physics

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Q.11 – Q.35 Carry ONE mark Each

Q24. Which of the following option(s) is/are correct for the ground state of a hydrogen atom?

- (A) Linear Stark effect is zero (B) It has definite parity
(C) Spin-orbit coupling is zero (D) Hyperfine splitting is zero

Ans.: (A), (B), (C)

Solution.:

(A) The ground state of hydrogen ($n=1, l=0$) is non-degenerate (ignoring spin). Linear Stark effect requires degenerate states of opposite parity, so the linear Stark shift is **zero** (it exhibits a quadratic Stark effect). **So (A) is Correct.**

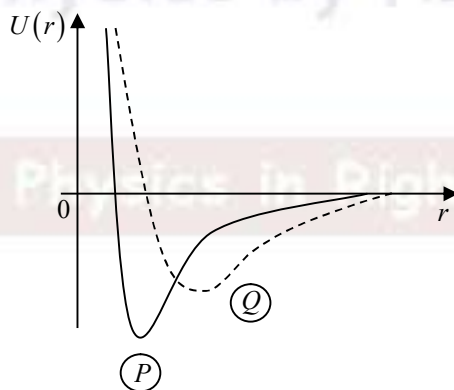
(B) The ground state wave function is spherically symmetric ($\psi_{100} \propto e^{-r/a_0}$), corresponding to orbital angular momentum $l=0$. Parity is given by $(-1)^l$ (even parity). Thus, it has definite parity. **So (B) is Correct.**

(C) Spin-orbit coupling energy is proportional to LS . For the ground state, $l=0 \Rightarrow L=0$ so $LS=0$. **So (C) is Correct.**

(D) Hyperfine splitting arises from the interaction between the electron and the proton's magnetic moment. It is nonzero for the hydrogen ground state (the famous 21 cm line).

So (D) is Incorrect.

Q28. Potential energy of two diatomic molecules P and Q of the same reduced mass is shown in the figure. According to this diagram, which of the following option(s) is/are correct?



- (A) The equilibrium inter-nuclear distance of Q is more than that of P
(B) The total energy $E=0$ separates bound and unbound states of the molecules
(C) The lowest vibrational frequency of P is larger than that of Q
(D) Dissociation energy of Q is more than that of P

Ans.: (A), (B), (C)

Solution.: From the potential energy curves $U(r)$:

(A) The minimum of curve Q occurs at a larger value of r compared to curve P , meaning $r_{eq,Q} > r_{eq,P}$. **So (A) is Correct.**

(B) As $r \rightarrow \infty$, $U(r) \rightarrow 0$. Therefore, states with total energy $E < 0$ are bound states (trapped in the potential well), while states with $E \geq 0$ are unbound (free states).

So (B) is Correct

(C) The vibrational frequency near the minimum is $\omega = \sqrt{\frac{k}{m}}$, where $k = \left. \frac{d^2U}{dr^2} \right|_{r_{eq}}$ is the curvature (stiffness) of the potential well. Curve P is sharper/narrower (higher curvature) at its minimum than Q , so $k_P > k_Q$. Since reduced masses are equal ($\mu_P = \mu_Q$), the lowest vibrational frequency of P is larger than that of Q . **So (C) is Correct**

(D) The depth of the potential well for P is greater than that of Q ($|U_{\min,P}| > |U_{\min,Q}|$). Therefore, the dissociation energy of P is greater than that of Q . **So (D) is Incorrect**

Q32. The effective magnetic moment (in units of Bohr magneton) for the ground state of an isolated $4f$ ion with 6 unpaired electrons in the $4f$ shell according to Hund's rules is (in integer) _____

Ans.: 0

Solution.: For a $4f$ shell ($l=3$, max electrons = 14), having **6 unpaired electrons** means the electronic configuration has $n=6$ electrons ($4f^6$).

Using Hund's rules to fill the sub-levels: $m_l = +3, +2, +1, 0, -1, -2, -3$

Spin $S = 6 \times \frac{1}{2} = 3$

Orbital angular momentum

For $4f$ orbitals, $l=3$ for each electron. With 6 electrons, the maximum L (Hund's rule) is

$$L = (+3) + (+2) + (+1) + (0) + (-1) + (-2) = 3$$

Total angular momentum:

Since the shell is less than half-filled ($6 < 7$), $J = |L - S| = |3 - 3| = 0$.

The effective magnetic moment is $\mu_{\text{eff}} = g_J \sqrt{J(J+1)} \mu_B$. Since $J = 0$, the ground state magnetic moment is 0.

Q.36 – Q.65 Carry TWO marks Each

Q36. The screened nuclear charge of neutral Helium atom is given as $1.7e$, where e is the magnitude of the electronic charge. Assuming the Bohr model of the atom for which the energy levels are $E_n = -\frac{Z^2}{2} \frac{1}{n^2}$ atomic units (Z is the atomic number), the first ionization potential of Helium in atomic units is

- (A) 0.89 (B) 1.78 (C) 0.94 (D) 3.16

Ans.: (A)

Solution.: To find the first ionization potential of the neutral Helium atom, we compare the total energy of the atom in its ground state with the energy of the system after one electron is removed.

1. Ground State Energy of Neutral Helium

- The Helium atom has two electrons ($N = 2$) in its ground state ($n = 1$).
- Using the effective screened nuclear charge $Z_{\text{eff}} = 1.7e$, the energy of each electron in

atomic units is: $E_1 = -\frac{Z_{\text{eff}}^2}{2} \frac{1}{1^2} = -\frac{(1.7)^2}{2} = -\frac{2.89}{2} = -1.445 \text{ a.u.}$

- Since both electrons occupy the ground state, the total initial energy of the neutral Helium atom is: $E_{\text{initial}} = 2 \times (-1.445) \text{ a.u.} = -2.89 \text{ a.u.}$

2. Energy After Ionization

- After removing one electron, the remaining ion (He^+) has a single electron orbiting a nucleus with actual atomic number $Z = 2$ (unscreened).
- The energy of the remaining electron in the $n = 1$ state is:

$$E_{\text{final}} = -\frac{Z^2}{2} \frac{1}{1^2} = -\frac{(2)^2}{2} = -2.00 \text{ a.u.}$$

3. First Ionization Potential

The ionization potential (or energy) is the difference between the final energy state and the initial ground state:

$$E_{\text{final}} - E_{\text{initial}} = -2.00 - (-2.89) = 0.89 \text{ a.u.}$$



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GATE Physics-2025
Solution-Solid State Physics

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Q.11 – Q.35 Carry ONE mark Each**Q11.** For a two-dimensional hexagonal lattice with lattice constant a , the atomic density is

- (A) $\frac{1}{\sqrt{3}a^2}$ (B) $\frac{1}{\sqrt{6}a^2}$ (C) $\frac{4}{3\sqrt{3}a^2}$ (D) $\frac{1}{3\sqrt{3}a^2}$

Ans.: (C)

Solution.: In a 2D hexagonal lattice, the total area A of the standard hexagonal unit cell (formed by 6 equilateral triangles of side length a) is given by: $A = 6 \times \left(\frac{\sqrt{3}}{4} a^2 \right) = \frac{3\sqrt{3}}{2} a^2$

The number of effective atoms contained in this conventional hexagonal unit cell is $N = 2$ atoms (from the 6 corners contributing $\frac{1}{3}$ each, plus the central atom contribution depending on cell choice, total $N = 2$).

Calculating the atomic density $\rho = \frac{N}{A} = \frac{2}{\frac{3\sqrt{3}}{2} a^2} = \frac{4}{3\sqrt{3}a^2}$

Q12. Consider a crystal that has a basis of one atom. Its primitive vectors are $\vec{a}_1 = a\hat{i}$, $\vec{a}_2 = a\hat{j}$, $\vec{a}_3 = \frac{a}{2}(\hat{i} + \hat{j} + \hat{k})$, where $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors in the x, y and z directions of the Cartesian coordinate system and a is a positive constant.

- (A) It is BCC and the volume of the primitive cell is $\frac{a^3}{2}$
 (B) It is FCC and the volume of the primitive cell is $\frac{a^3}{4}$
 (C) It is BCC and the volume of the primitive cell is $\frac{a^3}{8}$
 (D) It is FCC and the volume of the primitive cell is a^3

Ans.: (A)

Solution.: Given Primitive Vectors: $\vec{a}_1 = a\hat{i}$, $\vec{a}_2 = a\hat{j}$, $\vec{a}_3 = \frac{a}{2}(\hat{i} + \hat{j} + \hat{k})$.

Volume of the Primitive Cell $V_p = \left| \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) \right|$

$$\vec{a}_2 \times \vec{a}_3 = a\hat{j} \times \frac{a}{2}(\hat{i} + \hat{j} + \hat{k}) = \frac{a^2}{2}(-\hat{k} + \hat{i})$$

$$V_p = \left| \vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) \right| = \left| a\hat{i} \cdot \left[\frac{a^2}{2}(-\hat{k} + \hat{i}) \right] \right| = \frac{a^3}{2}$$

These vectors define a Body-Centered Cubic (BCC) lattice with conventional lattice constant a .

Q17. As per the Drude model of metals, the electrical resistance of a metallic wire of length L and cross-section area A is

(Consider τ as the relaxation time, m as electron mass, n as carrier concentration and e as electronic charge)

- (A) $\frac{mL}{ne^2 A\tau}$ (B) $\frac{2mL}{ne^2 A\tau}$ (C) $\frac{mL}{2ne^2 A\tau}$ (D) $\frac{mL}{4ne^2 A\tau}$

Ans.: (A)

Solution.:

$$\text{Electrical Conductivity } \sigma = \frac{ne^2\tau}{m}$$

$$\text{Electrical Resistivity } \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

$$\text{Resistance } R = \rho \frac{L}{A} = \frac{mL}{ne^2 A\tau}$$

Q33. Powder X-ray diffraction pattern of a cubic solid with lattice constant a has the (111) diffraction peak at $\theta = 30^\circ$. If the lattice expands such that the lattice constant becomes $1.25a$, the angle (in degrees) corresponding to the (111) peak changes to $\sin^{-1}\left(\frac{1}{n}\right)$. The value of n (rounded off to one decimal place) is _____

Ans.: 2.5

Solution.: Bragg's Law: $2d_{hkl} \sin \theta = \lambda$. For cubic lattice $d_{111} = \frac{a}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{a}{\sqrt{3}}$

$$\text{Initial State } (a_1 = a, \theta_1 = 30^\circ): 2\left(\frac{a}{\sqrt{3}}\right) \sin 30^\circ = \lambda \Rightarrow \lambda = 2\left(\frac{a}{\sqrt{3}}\right) \frac{1}{2} = \frac{a}{\sqrt{3}}$$

$$\text{Expanded State } \left(a_2 = 1.25a = \frac{5}{4}a, \theta_2\right): 2\left(\frac{a_2}{\sqrt{3}}\right) \sin \theta_2 = \lambda \Rightarrow 2\left(\frac{5a}{4\sqrt{3}}\right) \sin \theta_2 = \frac{a}{\sqrt{3}}$$

$$\Rightarrow \sin \theta_2 = \frac{2}{5} \Rightarrow \theta_2 = \sin^{-1}\left(\frac{2}{5}\right) \Rightarrow \theta_2 = \sin^{-1}\left(\frac{1}{2.5}\right) = \sin^{-1}\left(\frac{1}{n}\right) \Rightarrow n = 2.5$$

Q34. Consider a monatomic chain of length 30 cm . The phonon density of states is $1.2 \times 10^{-4} \text{ s}$. Assuming the Debye model, the velocity of sound in m/s (rounded off to one decimal place) is

Ans.: 794 - 797

Solution.:

1D Debye Model Density of States:

For a 1D chain of length L , $g(\omega) = \frac{L}{\pi v_s}$, where v_s is the sound velocity.

$$\therefore L = 30\text{ cm} = 0.3\text{ m}, g(\omega) = 1.2 \times 10^{-4} \text{ s} \Rightarrow v_s = \frac{L}{\pi g(\omega)} = \frac{0.3}{3.14 \times (1.2 \times 10^{-4})} = 795.8 \text{ m/s}$$

Q.36 – Q.65 Carry TWO marks Each

Q47. Which of the following option(s) is/are correct for a Type I superconductor?

- (A) The phase transition to the normal state in the absence of a magnetic field is of second order
- (B) With increase in temperature, the critical magnetic field decreases linearly to zero
- (C) Below the critical temperature, the entropy in the superconducting state is less than that in the normal state
- (D) The phase transition to the normal state in the presence of a magnetic field is of first order

Ans.: (A), (C), (D)

Solution.:

(A) Correct: In zero magnetic field ($H = 0$), the transition at T_c is of **second order** (no latent heat, discontinuous specific heat).

(B) Incorrect: The critical magnetic field depends quadratically on temperature:

$$H_c(T) = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right], \text{ not linearly.}$$

(C) Correct: Below T_c , the superconducting state is more ordered than the normal state, so

$$S_{\text{sup}} < S_{\text{normal}}.$$

(D) Correct: In the presence of a non-zero magnetic field ($H \neq 0$), the phase transition is accompanied by latent heat, making it a **first-order** transition.

Correct Options: (A), (C), (D)



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GATE Physics-2025
Solution-Nuclear and Particle Physics

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Q.11 – Q.35 Carry ONE mark Each

Q18. Which one of the following baryons has strangeness quantum number $S = -1$?

- (A) Σ^{*0} (B) n (C) Ξ^{*0} (D) Δ^0

Ans.: (A)

Solution.:

Σ^{*0} (Sigma): Strangeness $S = -1$; n (Neutron): Strangeness $S = 0$

Ξ^{*0} (Cascade resonance): Strangeness $S = -2$; Δ^0 (Delta): Strangeness $S = 0$

Q20. The nuclear energy levels of mirror nuclei are similar. Using this empirical fact alone, the nuclear force can be said to be independent of which one of the following properties of the nucleons?

- (A) Mass (B) Spin (C) Charge (D) Parity

Ans.: (C)

Solution.:

Mirror nuclei are pairs of isobaric nuclei where the number of protons in one equals the number of neutrons in the other (e.g., ${}^{15}_7\text{N}$ and ${}^{15}_8\text{O}$). Since their nuclear energy levels are nearly identical despite swapping protons and neutrons, this demonstrates **charge independence** of the nuclear force.

Q27. Which of the following consideration(s) is/are showing that nuclear beta decay, $n \rightarrow p + e^- + \bar{\nu}_e$, has to be a three-body decay?

- (A) Continuous distribution of the electron energy (B) Spin of the final state
(C) Mass of the electron (D) Mass of the proton

Ans.: (A), (B)

Solution.:

(A) Continuous distribution of electron energy: A two-body decay yields discrete monoenergetic electrons; a three-body decay is required to explain the continuous kinetic energy spectrum.

(B) Spin of the final state: n has spin $1/2$. If decay produced only $p(1/2)$ and $e^-(1/2)$, the final state total spin could only be 0 or 1, violating angular momentum conservation. Adding a neutrino $\bar{\nu}_e(1/2)$ conserves half-integer spin.

Q29. Nuclear radiation emitted from a ^{60}Co radioactive source is detected by a photomultiplier tube (PMT) coupled to a scintillator crystal. Which of the following option(s) is/are correct?

- (A) γ radiation from ^{60}Co will directly hit the photocathode of the PMT without interacting with the scintillator crystal and produce a signal
- (B) β radiation from ^{60}Co source interacts with the scintillator crystal, producing γ radiation, which will hit the photocathode of the PMT and produce a signal
- (C) A mu-metal shield is put all around the PMT to nullify the effect of external electric fields
- (D) A mu-metal shield is put all around the PMT to nullify the effect of external magnetic fields

Ans.: (B), (D)

Solution.:

(B): Gamma radiation interacts with the scintillator crystal to emit visible light (photons), which then hit the PMT photocathode to emit photoelectrons.

(D): Mu-metal is a high-permeability ferromagnetic alloy specifically used to shield the PMT from external **magnetic fields**, which would otherwise deflect electrons moving between dynodes.

Q35. The Δ^+ baryon with spin $\frac{3}{2}$, at rest, decays to a proton and a pion ($\Delta^+ \rightarrow p + \pi^0$). The Δ^+ has positive intrinsic parity and π^0 has negative intrinsic parity. The orbital angular momentum of the proton-pion system (in integer) is _____

Ans.: 1

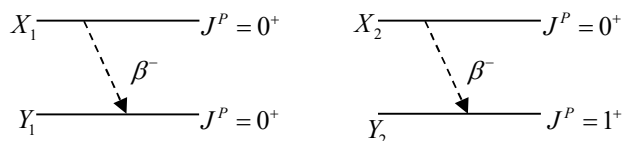
Solution.: Using parity conservation in strong interaction: $\pi_{\Delta^+} = \pi_p \pi_{\pi^0} (-1)^L$

$$\Rightarrow +1 = (+1)(-1)(-1)^L \Rightarrow (-1)^L = -1.$$

Thus, the lowest orbital angular momentum integer is $L = 1$.

Q.36 – Q.65 Carry TWO marks Each

Q48. Consider two hypothetical nuclei X_1 and X_2 undergoing β^- decay, resulting in nuclei Y_1 and Y_2 , respectively. The decay scheme and the corresponding J^P values of the nuclei are given in the figure. Which of the following option(s) is/are correct? (J is the total angular momentum and P is parity)



- (A) $X_1 \rightarrow Y_1$ is Fermi transition and $X_2 \rightarrow Y_2$ is Fermi transition
(B) $X_1 \rightarrow Y_1$ is Fermi transition and $X_2 \rightarrow Y_2$ is Gamow-Teller transition
(C) $X_1 \rightarrow Y_1$ is Gamow-Teller transition and $X_2 \rightarrow Y_2$ is Fermi transition
(D) $X_1 \rightarrow Y_1$ is Gamow-Teller transition and $X_2 \rightarrow Y_2$ is Gamow-Teller transition

Ans.: (B)

Solution.: In beta decay, the selection rules for the change in total angular momentum (ΔJ) and parity ($\Delta\pi$) are:

Fermi transition:

$\Delta J = 0$, $\Delta\pi = \text{no}$ (parity does not change).

Gamow-Teller transition:

$\Delta J = 0, \pm 1$ (but $0 \rightarrow 0$ is forbidden), $\Delta\pi = \text{no}$ (parity does not change).

Applying this to the given decays:

$X_1 \rightarrow Y_1$: The initial state is $J^P = 0^+$ and the final state is $J^P = 0^+$. Here, $\Delta J = 0$ and there is no parity change. This is a pure Fermi transition.

$X_2 \rightarrow Y_2$: The initial state is $J^P = 0^+$ and the final state is $J^P = 1^+$. Here, $\Delta J = 1$ and there is no parity change. This is a pure Gamow-Teller transition.

Therefore, option **(B)** is correct.



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Solution-General Aptitude

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Section -GA (General Aptitude)**Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each.****Q1.** Is there any good show _____ television tonight?

Select the most appropriate option to complete the above sentence.

- (A) in (B) at (C) within (D) on

Ans.: (D)**Q2.** As the police officer was found guilty of embezzlement, he was _____ dismissed from the service in accordance with the Service Rules.

Select the most appropriate option to complete the above sentence.

- (A) sumptuously (B) brazenly
-
- (C) unintentionally (D) summarily

Ans.: (D)**Q3.** The sum of the following infinite series is: $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$

- (A)
- π
- (B)
- $1+e$
- (C)
- $e-1$
- (D)
- e

Ans.: (C)

Solution.: $e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \Rightarrow e^1 = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$

$$\Rightarrow \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots = e - 1$$

Q4. A thin wire is used to construct all the edges of a cube of $1m$ side by bending, cutting and soldering the wire. If the wire is $12m$ long, what is the minimum number of cuts required to construct the wire frame to form the cube?

- (A) 3 (B) 4 (C) 6 (D) 12

Ans.: (A)**Solution.:** A cube of side $1m$ has 12 edges, each of length $1m$. So total wire length needed is $12m$. Since each cut increases the number of pieces by 1. So

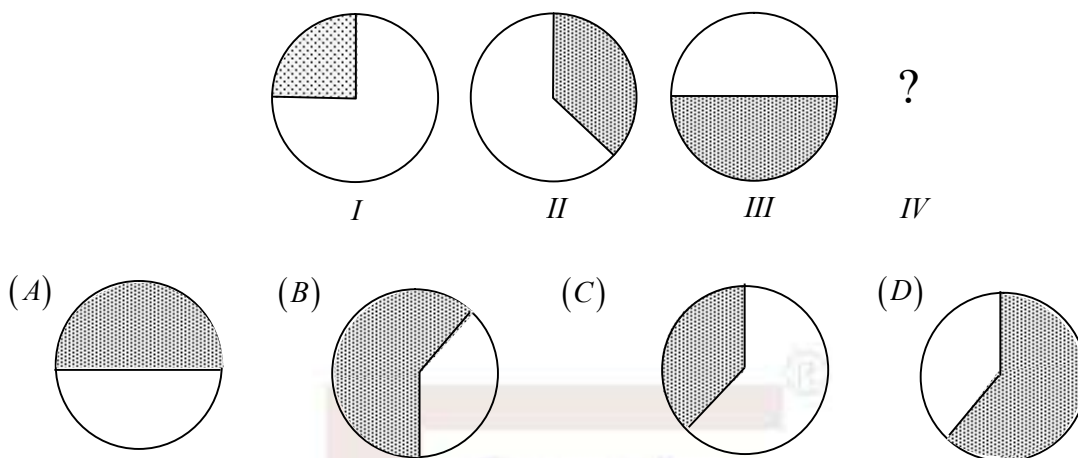
- With no cuts: 1 piece
- With n cuts: $n+1$ pieces

A cube wire-frame can be constructed using **4 continuous bent pieces**, each forming a square face (top, bottom, and two vertical loops), which are then **soldered at joints**.

Thus, minimum number of separate wire pieces needed = 4.

So number of cuts = 3

Q5. The figures I, II and III are parts of a sequence. Which one of the following options comes next in the sequence at IV?



Ans.: (B)

Q. 6 – Q. 10 Multiple Choice Question (MCQ), carry TWO marks each.

Q6. "Why do they pull down and do away with crooked streets, I wonder which are my delight, and hurt no man living? Every day the wealthier nations are pulling down one or another in their capitals and their great towns: they do not know why they do it; neither do I. It ought to be enough, surely, to drive the great broad ways which commerce needs and which are the life-channels of a modern city, without destroying all history and all the humanity in between: the islands of the past."

(From Halaire Belloc's "The Crooked Streets")

Based only on the information provided in the above passage, which one of the following statements is true?

- (A) The author of the passage takes delight in wondering.
- (B) The wealthier nations are pulling down the crooked streets in their capitals.
- (C) In the past, crooked streets were only built on islands.
- (D) Great broad ways are needed to protect commerce and history.

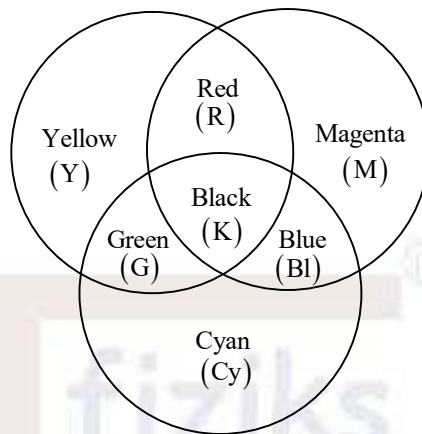
Ans.: (B)

Solution.: Key points from the passage

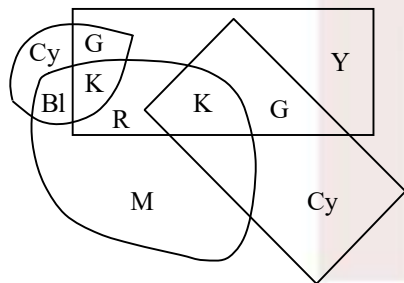
- (i) The author **delights in crooked streets**. (ii) He **wonders why** they are pulled down.
- (iii) He states that **wealthier nations are pulling down crooked streets** in their capitals and great towns.
- (iv) He says **great broad ways are needed for commerce**, but **not at the cost of destroying history and humanity**.

Q7. A color model is shown in the figure with color codes: Yellow (Y), Magenta (M), Cyan (Cy), Red (R), Blue (Bl), Green (G) and Black (K).

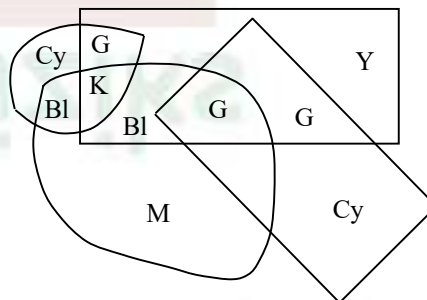
Which one of the following options displays the color codes that are consistent with the color model?



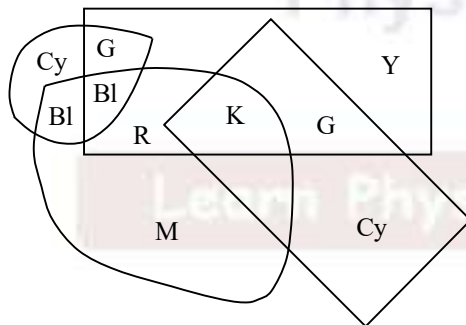
(A)



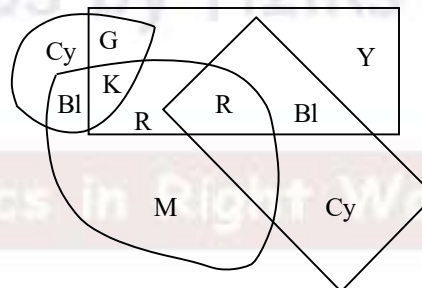
(B)



(C)



(D)



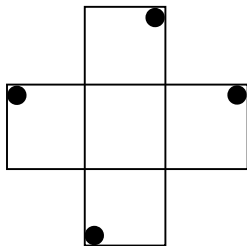
Ans.: (A)

Q8. An object is said to have an n -fold rotational symmetry if the object, rotated by an angle of $\frac{2\pi}{n}$, is identical to the original.

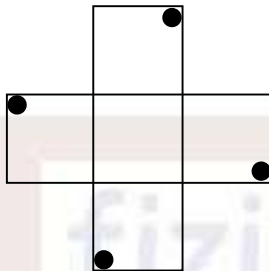
Which one of the following objects exhibits 4-fold rotational symmetry about an axis perpendicular to the plane of the screen?

Note: The figures shown are representative.

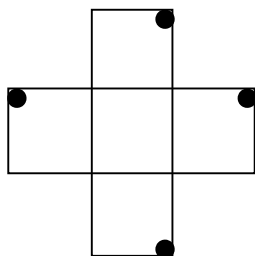
(A)



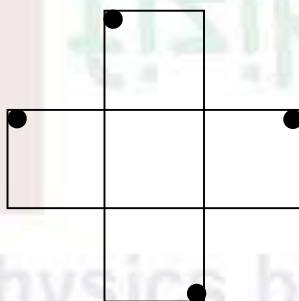
(B)



(C)



(D)



Ans.: (B)

Solution.: 4-fold symmetry means that rotating the object by 90° (i.e., $\frac{2\pi}{4}$) around an axis perpendicular to the plane leaves the object looking exactly the same as the original.

Q9. Rohit goes to a restaurant for lunch at about 1 PM. When he enters the restaurant, he notices that the hour and minute hands on the wall clock are exactly coinciding. After about an hour, when he leaves the restaurant, he notices that the clock hands are again exactly coinciding. How much time (in minutes) did Rohit spend at the restaurant?

(A) $64\frac{6}{11}$

(B) $66\frac{5}{13}$

(C) $65\frac{5}{11}$

(D) $66\frac{6}{13}$

Ans.: (C)

Solution.: We know the hour and minute hands coincide at certain times apart by $\frac{12}{11}$ hours.

The coincidences occur at: 12:00, then $\frac{12}{11}$ hours = $1\frac{1}{11}$ hours after 12. That is 1 hour $\frac{60}{11}$ min.

So he entered at $1:05\frac{5}{11}$. So next coincidence will be after $65\frac{5}{11}$ min.

Q10. A circle with center at $(x, y) = (0.5, 0)$ and radius = 0.5 intersects with another circle with center at $(x, y) = (1, 1)$ and radius = 1 at two points. One of the points of intersection (x, y) is:

- (A) (0,0) (B) (0.2,0.4) (C) (0.5,0.5) (D) (1,2)

Ans.: (B)

Solution.: Equation of the circles:

$$(1) (x-0.5)^2 + y^2 = (0.5)^2 = 0.25 \quad (2) (x-1)^2 + (y-1)^2 = 1$$

Check the option (B) : (0.2,0.4)

$$(1) (0.2-0.5)^2 + (0.4)^2 = 0.09 + 0.16 = 0.25$$

$$(2) (0.2-1)^2 + (0.4-1)^2 = 0.64 + 0.36 = 1$$