Physics by fiziks

Learn Physics in Right Way

IIT-JAM Physics-2025

Solution-Mathematical Methods

Be Part of Disciplined Learning

Section A: Q.1-Q.10 Carry ONE mark each.

Q1. Consider a volume V enclosed by a closed surface S having unit surface normal $\hat{n}$. For $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, the value of the surface integral $\frac{1}{9}\oiint \vec{r}\cdot\hat{n}\,dS$ is

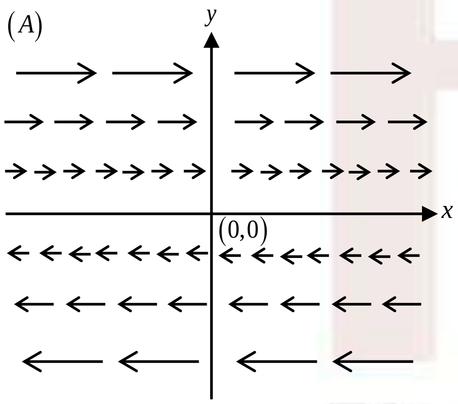
(A) V (B) $3V$ (C) $\frac{V}{3}$ (D) $\frac{V}{9}$

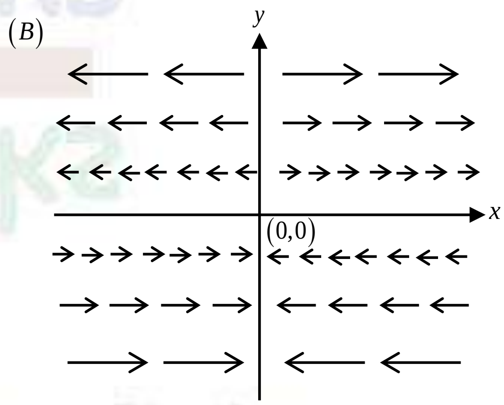
Ans.: (C)

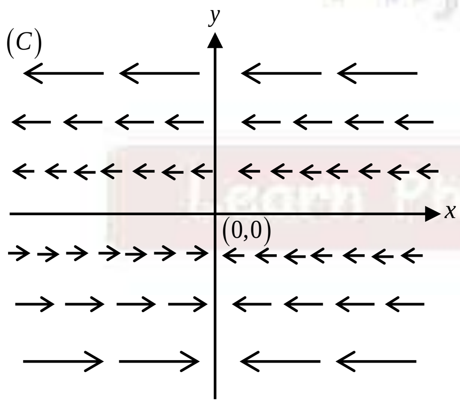
Solution.: $\frac{1}{9}\oiint \vec{r}\cdot\hat{n}\,dS = \int_V \left(\vec{\nabla}.\vec{r}\right)d\tau = \frac{1}{9}\times 3V = \frac{1}{3}V$

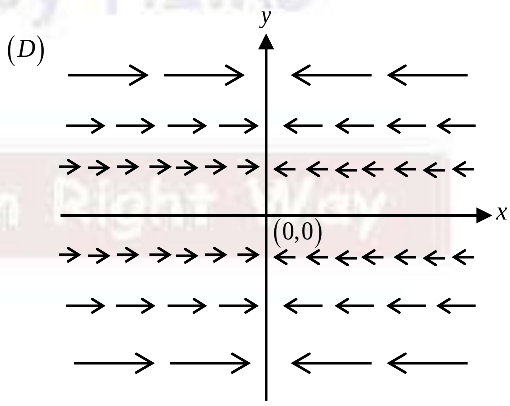
Q3. Which one of the following figures represents the vector field $\vec{A} = y\hat{i}$?

($\hat{i}$ is the unit vector along the x-direction)









Ans.: (A)

Solution.: $\because \vec{A} = y\hat{i} \Rightarrow y > 0,\ \vec{A} \to \hat{i}$ and $y < 0,\ \vec{A} \to -\hat{i}$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Section A: Q.11-Q.30 Carry TWO marks each.

Q12. Given a function $f(x,y)=\frac{x}{a}e^{y}+\frac{y}{b}e^{x}$, where $x=at$ and $y=bt$ (a and b are non-zero constants), the value of $\frac{df}{dt}$ at $t=0$ is

(A) –1 (B) 0 (C) 1 (D) 2

Ans.: (D)

Solution.: $\because f(x,y)=\frac{x}{a}e^{y}+\frac{y}{b}e^{x}$ where $x=at$ and $y=bt$

$$\Rightarrow \frac{df}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}=\left(\frac{1}{a}e^{y}\right)a+\left(\frac{1}{b}e^{x}\right)b=e^{x}+e^{y}=e^{at}+e^{bt}\Rightarrow \left.\frac{df}{dt}\right|_{t=0}=e^{0}+e^{0}=1+1=2$$

Q13. If the system of linear equations

$$x+my+az=0$$
$$2x+ay+mz=0$$
$$ax+2y-z=0$$

with m and a as non-zero constants, admits a non-trivial solution, then which one of the following conditions is correct?

(A) $m^2-a^2=3$ (B) $m^2-a^2=-3$ (C) $a^2-2m^2=-3$ (D) $m^2-2a^2=3$

Ans.: (B)

Solution.: For non-trivial solution $\begin{vmatrix}1 & m & a\\ 2 & a & m\\ a & 2 & -1\end{vmatrix}=0$

$$\Rightarrow 1(-a-2m)-m(-2-am)+a(4-a^2)=0\Rightarrow am^2+3a-a^3=0\Rightarrow m^2-a^2=-3.$$

Q14. If $\left(\frac{1-i}{1+i}\right)^{\frac{n}{2}}=-1$, where $i=\sqrt{-1}$, one possible value of n is

(A) 2 (B) 4 (C) 6 (D) 8

Ans.: (B)

Solution.: $1-i=\sqrt{2}e^{-i\pi/4}$ and $1+i=\sqrt{2}e^{+i\pi/4}\Rightarrow \left(\frac{1-i}{1+i}\right)^{\frac{n}{2}}=\left(\frac{\sqrt{2}e^{-i\pi/4}}{\sqrt{2}e^{+i\pi/4}}\right)^{\frac{n}{2}}=\left(e^{-i\pi/2}\right)^{\frac{n}{2}}=(-i)^{\frac{n}{2}}$

$$\because \left(\frac{1-i}{1+i}\right)^{\frac{n}{2}}=(-i)^{\frac{n}{2}}=-1\Rightarrow n=4$$

Q15. In Cartesian coordinates, consider the functions $u(x,y)=\frac{1}{2}(x^2-y^2)$ and $v(x,y)=xy$. If (r,θ) are the polar coordinates, the Jacobian determinant $\left|\frac{\partial(u,v)}{\partial(r,\theta)}\right|$

(A) r (B) $\frac{1}{r}$ (C) r^2 (D) r^3

Ans.: (D)

Solution.: $\because \frac{\partial(u,v)}{\partial(r,\theta)}=\frac{\partial(u,v)}{\partial(x,y)}.\frac{\partial(x,y)}{\partial(r,\theta)}$

$u(x,y)=\frac{1}{2}(x^2-y^2)$ and $v(x,y)=xy$; $x=r\cos\theta$ $y=r\cos\theta$

$$\frac{\partial(u,v)}{\partial(x,y)}=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}=\begin{vmatrix}x & -y\\ y & x\end{vmatrix}=x^2+y^2=r^2$$

$$\frac{\partial(x,y)}{\partial(r,\theta)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta}\end{vmatrix}=\begin{vmatrix}\cos\theta & -r\sin\theta\\ \sin\theta & r\cos\theta\end{vmatrix}=r \Rightarrow \frac{\partial(u,v)}{\partial(r,\theta)}=r^2.r=r^3$$

Section B: Q.31 - Q.40 Carry TWO marks arks each. (No Question)

Section C: Q.41 - Q.50 Carry ONE mark each. (No Question)

Section C: Q.51-Q60 Carry TWO marks each. (No Question)

Q54. Consider a vector $\vec{F}=\frac{1}{\pi}\left[-\sin y\hat{i}+x(1-\cos y)\hat{j}\right]$. The value of the integral $\oint\vec{F}\cdot d\vec{r}$ over a circle $x^2+y^2=1$ evaluated in the anti-clockwise direction is __________. (in integer)

Ans.: 1

Solution.: $\oint_C \vec{f}\cdot d\vec{l}=\oint_C\left[P(x,y)dx+Q(x,y)dy\right]$

$$\Rightarrow \underbrace{\oint_C\left[P(x,y)dx+Q(x,y)dy\right]}_{\text{Anti-clockwise}}=\iint_R\left[\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right]dxdy$$

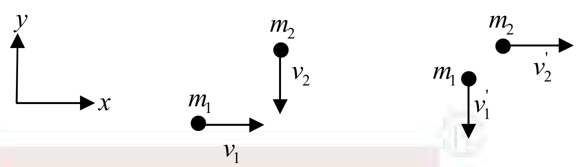
$$\because P=-\frac{\sin y}{\pi},\ Q=\frac{x}{\pi}(1-\cos y)\Rightarrow \iint_R\left[\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right]dxdy=\frac{1}{\pi}\iint_R\left[(1-\cos y)-(-\cos y)\right]dxdy$$

$$\Rightarrow \iint_R\left[\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right]dxdy=\frac{1}{\pi}\iint_R dxdy=\frac{1}{\pi}\times\pi(1)^2=1$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Section A: Q.1-Q.10 Carry ONE mark each.

Q2. Two point-particles having masses m_1 and m_2 approach each other in perpendicular directions with speeds v_1 and v_2, respectively, as shown in the figure below. After an elastic collision, they move away from each other in perpendicular directions with speeds $v_1^{'}$ and $v_2^{'}$, respectively.



The ratio $\frac{v_2^{'}}{v_1^{'}}$ is

(A) $\frac{m_1^2 v_1}{m_2^2 v_2}$ (B) $\frac{m_1 v_1}{m_2 v_2}$ (C) $\frac{m_1^2 v_2}{m_2^2 v_1}$ (D) $\frac{m_1 v_2}{m_2 v_1}$

Ans.: (A)

Solution.:

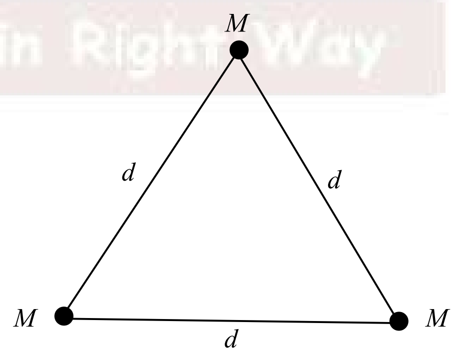
Ans. (a)

Solution: $m_2 v_2^1 = m_1 v_1 \Rightarrow v_2^1 = \frac{m_1 v_1}{m_2}$ and $m_1 v_1^1 = m_2 v_2 \Rightarrow v_1^1 = \frac{m_2 v_2}{m_1} \Rightarrow \frac{v_2^1}{v_1^1} = \frac{m_1^2 v_1}{m_2^2 v_2}$

Section A: Q.11-Q.30 Carry TWO marks each.

Q16. Three particles of equal mass M, interacting via gravity, lie on the vertices of an equilateral triangle of side d, as shown in the figure below. The whole system is rotating with an angular velocity ω about an axis perpendicular to the plane of the system and passing through the center of mass. The value of ω, for which the distance between the masses remains d, is

(G is the universal gravitational constant)



(A) $\sqrt{\frac{2GM}{d^3}}$

(B) $\sqrt{\frac{3GM}{d^3}}$

(C) $\sqrt{\frac{GM}{3d^3}}$

(D) $\sqrt{\frac{GM}{d^3}}$

Ans.: (B)

Solution.:

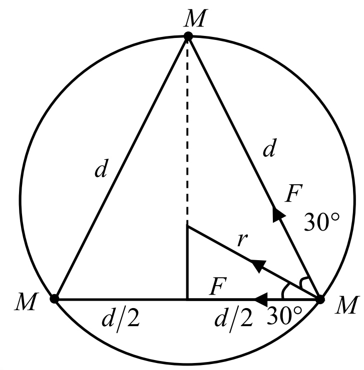
$$F_R = 2F\cos 30^\circ = \frac{2\sqrt{3}}{2}\frac{GM^2}{d^2}$$

For circular motion

$$M\omega^2 r = \sqrt{3}\frac{GM^2}{d^2} \Rightarrow \omega^2 \frac{d}{\sqrt{3}} = \sqrt{3}\frac{GM}{d^2}$$

$$\Rightarrow \omega = \sqrt{\frac{3GM}{d^3}}$$

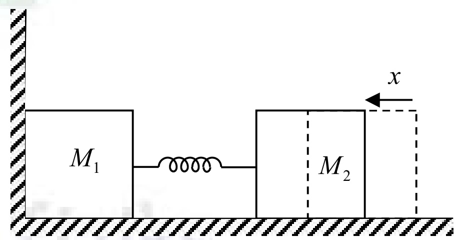
$$\because r\cos 30^0 = \frac{d}{2} \Rightarrow r = \frac{d}{\sqrt{3}}$$



Q17. Two masses, M_1 and M_2, are connected through a massless spring of spring constant k, as shown in the figure below. The mass M_1 is at rest against a rigid wall. Both M_1 and M_2 are on a frictionless surface. The mass M_2 is pushed towards M_1 by a distance x from its equilibrium position and then released. After M_1 leaves the wall, the speed of the center of mass of the composite system is

(A) $\sqrt{\frac{k}{M_2}}x$ (B) $\sqrt{\frac{k}{M_1+M_2}}x$

(C) $\frac{\sqrt{kM_2}}{M_1+M_2}x$ (D) $\frac{\sqrt{kM_1}}{M_1+M_2}x$



Ans.: (C)

Solution.: As no external force is acting on the system, so total momentum of the system will remain constant.

$$\vec{p}_{System} = \text{constant}$$

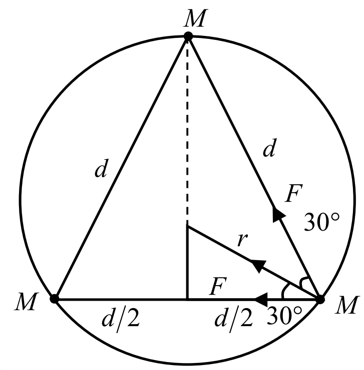
(i) If mass M_2 is shifted by a distance x and then released, then it will move towards equilibrium point O. M_1 is fixed, only M_2 is moving, so, $\omega = \sqrt{\frac{k}{M_2}}$

(ii) At equilibrium its velocity will be maximum $v_{\max} = A\omega = x\sqrt{\frac{k}{M_2}}$

(iii) As mass M_2 crosses equilibrium and moves toward right, M_1 leaves the wall.

Solution.:

$$F_R = 2F\cos 30° = \frac{2\sqrt{3}}{2}\frac{GM^2}{d^2}$$



For circular motion

$$M\omega^2 r = \sqrt{3}\frac{GM^2}{d^2} \Rightarrow \omega^2 \frac{d}{\sqrt{3}} = \sqrt{3}\frac{GM}{d^2}$$

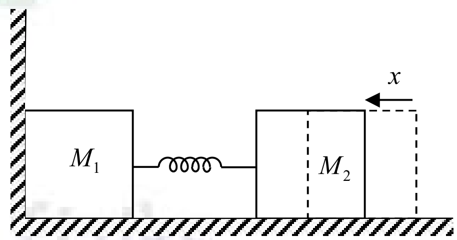
$$\Rightarrow \omega = \sqrt{\frac{3GM}{d^3}}$$

$$\because r\cos 30^0 = \frac{d}{2} \Rightarrow r = \frac{d}{\sqrt{3}}$$

Q17. Two masses, M_1 and M_2, are connected through a massless spring of spring constant k, as shown in the figure below. The mass M_1 is at rest against a rigid wall. Both M_1 and M_2 are on a frictionless surface. The mass M_2 is pushed towards M_1 by a distance x from its equilibrium position and then released. After M_1 leaves the wall, the speed of the center of mass of the composite system is

(A) $\sqrt{\frac{k}{M_2}}x$ (B) $\sqrt{\frac{k}{M_1+M_2}}x$

(C) $\frac{\sqrt{kM_2}}{M_1+M_2}x$ (D) $\frac{\sqrt{kM_1}}{M_1+M_2}x$



Ans.: (C)

Solution.: As no external force is acting on the system, so total momentum of the system will remain constant.

$$\vec{p}_{System} = \text{constant}$$

(i) If mass M_2 is shifted by a distance x and then released, then it will move towards equilibrium point O. M_1 is fixed, only M_2 is moving, so, $\omega = \sqrt{\frac{k}{M_2}}$

(ii) At equilibrium its velocity will be maximum $v_{\max} = A\omega = x\sqrt{\frac{k}{M_2}}$

(iii) As mass M_2 crosses equilibrium and moves toward right, M_1 leaves the wall.

At this moment, total momentum of the system

$$P_{sys} = M_2 v_{\max} = M_2 x\sqrt{\frac{k}{M_2}} = x\sqrt{kM_2}$$

$$P_{sys} = \left(M_1 + M_2\right) v_{CM} = x\sqrt{kM_2} \Rightarrow v_{CM} = \frac{\sqrt{kM_2}}{M_1 + M_2} x$$

Q18. One end of a long chain is lifted vertically from flat ground to a height H with constant speed v by a force of magnitude F. Assume that the length of the chain is greater than H and that it has a uniform mass per unit length ρ. The magnitude of the force F at height H is

(g is the acceleration due to gravity)

(A) $\rho\left(gH + v^2\right)$ (B) $\rho\left(gH + 2v^2\right)$ (C) $\rho\left(2gH + v^2\right)$ (D) $\frac{\rho}{2}\left(gH + v^2\right)$

Ans.: (A)

Solution.: Mass of the moving part of the chain, $m = \rho H$

Force acting on the lifted part

(i) Weight of the lifted chain, $W = mg = \rho Hg$

(ii) Force required to given momentum to new elements of the chain

$$\frac{dm}{dt} = \rho v, \text{ Rate at which mass is picked up}$$

Rate of change of momentum $\frac{dp}{dt} = v\frac{dm}{dt} = v\left(\rho v\right) = \rho v^2$

This acts downward, so the applied force must overcome

So total applied force required, $F = \rho Hg + \rho v^2 \Rightarrow F = \rho\left(gH + v^2\right)$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q32. Two particles of masses m_1 and m_2, interacting via gravity, rotate in circular orbits about their common center of mass with the same angular velocity ω.

For masses m_1 and m_2, respectively,

- r_1 and r_2 are the constant distances from the center of mass,
- L_1 and L_2 are the magnitudes of the angular momenta about the center of mass, and
- K_1 and K_2 are the kinetic energies.

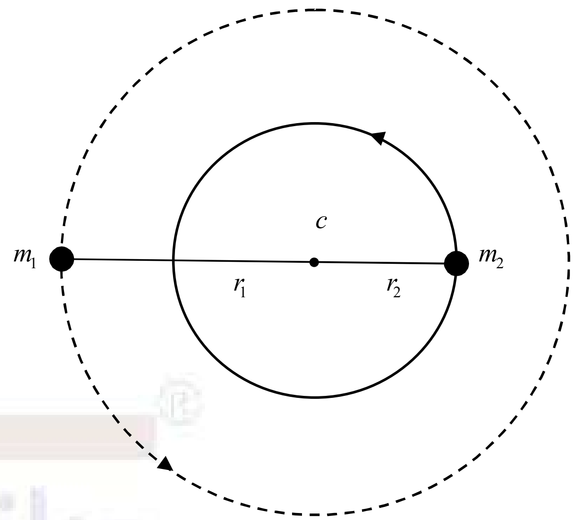
Which of the following is(are) correct? (G is the universal gravitational constant)

(A) $\frac{L_1}{L_2} = \frac{m_2}{m_1}$ (B) $\frac{K_1}{K_2} = \frac{m_2}{m_1}$ (C) $\omega = \sqrt{\frac{G\left(m_1 + m_2\right)}{\left(r_1 + r_2\right)^3}}$ (D) $m_2 r_1 = m_1 r_2$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Ans.: (A), (B), (C)

Solution.: c is the center of mass, so



$$m_1 r_1 = m_2 r_2$$

(a) $\dfrac{L_1}{L_2} = \dfrac{m_1\omega_1 r_1^2}{m_2\omega_2 r_2^2} = \dfrac{m_1 r_1 \omega r_1}{m_2 r_2 \omega r_2} \quad (\because \omega_1 = \omega_2 = \omega)$

$$\frac{L_1}{L_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1} \quad (\because m_1 r_1 = m_2 r_2)$$

(b) $\dfrac{K_1}{K_2} = \dfrac{\frac{1}{2}m_1 r_1^2\omega^2}{\frac{1}{2}m_2 r_2^2\omega^2} = \dfrac{m_1}{m_2}\dfrac{m_2^2}{m_1^2} = \dfrac{m_2}{m_1}$

(c) $\cancel{m_1} r_1\omega^2 = \dfrac{G\cancel{m_1}m_2}{(r_1+r_2)^2} \Rightarrow \omega = \sqrt{\dfrac{Gm_2}{r_1(r_1+r_2)^2}}$

$$\because r_1 = \frac{m_2}{m_1+m_2} r = \frac{m_2}{m_1+m_2}(r_1+r_2) \Rightarrow \omega = \sqrt{\frac{G(m_1+m_2)}{(r_1+r_2)^3}}$$

Section C: Q.41 - Q.50 Carry ONE mark each.

Q41. Two solid cylinders of the same density are found to have the same moment of inertia bout their respective principal axes. The length of the second cylinder is 16 times that of the first cylinder. If the radius of the first cylinder is $4\,cm$, the radius of the second cylinder is __________ cm. (in integer)

Ans.: 2

Solution.: $L_2 = 16L_1, R_1 = 4\,cm$

$$\frac{1}{2}M_1R_1^2 = \frac{1}{2}M_2R_2^2 \Rightarrow \frac{1}{2}\left(\pi R_1^2 L_1\rho\right)R_1^2 = \frac{1}{2}\left(\pi R_2^2 L_2\rho\right)R_2^2 \Rightarrow R_1^4 L_1 = R_2^4 L_2$$

$$\Rightarrow (4)^4 L_1 = R_2^4(16L_1) \Rightarrow R_2^4 = 16 \Rightarrow R_2 = 2\,cm$$

Section C: Q.51-Q60 Carry TWO marks each.

Q55. A particle is moving with a constant angular velocity 2 rad/s in an orbit on a plane. The radial distance of the particle from the origin at time t is given by $r = r_0 e^{2\beta t}$ where r_0 and β are positive constants. The radial component of the acceleration vanishes for β = _______ rad/s. (in integer)

Ans.: 1

Solution.: $r = r_0 e^{2\beta t} \Rightarrow \dot{r} = 2\beta r_0 e^{2\beta t} \Rightarrow \ddot{r} = 4\beta^2 r_0 e^{2\beta t} \Rightarrow \ddot{r} = 4\beta^2 r$

Radial component of acceleration: $a_r = \ddot{r} - r\dot{\theta}^2 = 0 \Rightarrow a_r = 4\beta^2 r - r(2)^2 = 0$

$\Rightarrow 4r(\beta^2 - 1) = 0$.

$\boxed{\beta = +1}$ as β is a positive constant.

Q56. A planet rotates in an elliptical orbit with a star situated at one of the foci. The distance from the center of the ellipse to any foci is half of the semi-major axis. The ratio of the speed of the planet when it is nearest (perihelion) to the star to that at the farthest (aphelion) is _________. (in integer)

Ans.: 3

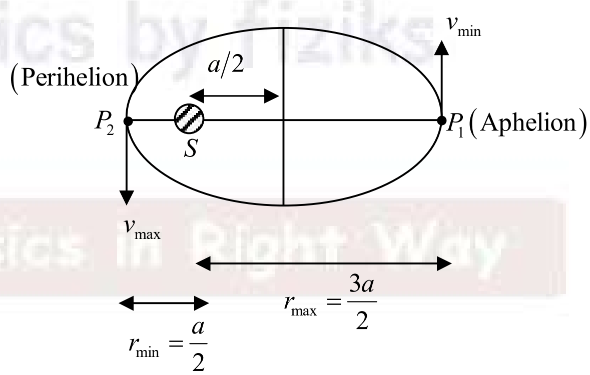
Solution.:

Solution.:

$v_{max} r_{min} = v_{min} r_{max}$

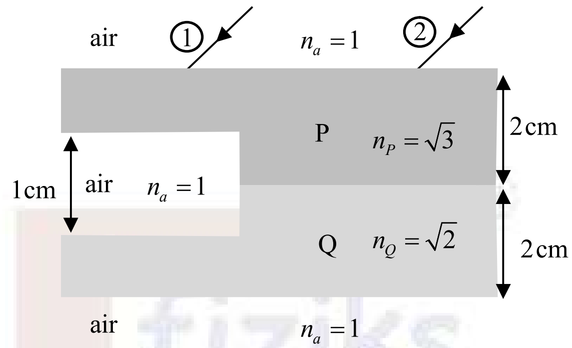
$v_{max} \dfrac{a}{2} = v_{min} \dfrac{3a}{2}$

$\dfrac{v_{max}}{v_{min}} = 3$



Section A: Q.1-Q.10 Carry ONE mark each.

Q4. Two parallel light rays ① and ② are incident from air on a system consisting of media P,Q and air, as shown in the figure below. The incident angle is 45^0. Ray ① passes through medium P, air and medium Q and ray ② passes through media P and Q before leaving the system. After passing through the system, the angular deviation (in radians) between the two rays is



[The dimensions of the media and their refractive indices (n_a, n_P and n_Q) are shown in the figure]

(A) 0 (B) $\tan^{-1}\sqrt{\frac{3}{2}}$ (C) $\tan^{-1}\sqrt{\frac{2}{3}}$ (D) $\tan^{-1}\sqrt{\frac{1}{3}}$

Ans.: (A)

Solution.:

As first and last medium are same for ray-1 and ray-2. So, emergent rays will be parallel to incident rays in both cases. Incident waves are parallel to each other. So emergent rays will also be parallel. As a result angular deviation will be zero between the two rays.

Section A: Q.11-Q.30 Carry TWO marks each.

Q19. For a two-slit Fraunhofer diffraction, each slit is 0.1mm wide and separation between the two slits is 0.8 mm. The total number of interference minima between the first diffraction minima on both sides of the central maxima is

(A) 16 (B) 18 (C) 8 (D) 9

Ans.: (A)

Solution.:

Missing order: $\frac{e+d}{e} = \frac{n}{m} \Rightarrow \frac{0.8}{0.1} = \frac{n}{1} \Rightarrow n = 8$

So, $n = \pm 8$ order principal maximas are absent.

No. of principal maximas in central envelope = $2n+1 = 17$

No. of interference minimas in central envelope = $2n = 16$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q20. Consider the superposition of two orthogonal simple harmonic motions $y_1 = a\cos 2\omega t$ and $y_2 = b\cos(\omega t + \phi)$. If $\phi = \pi$, the resultant motion will represent

(a, b and ω are constants with appropriate dimensions)

(A) a parabola (B) a hyperbola (C) an ellipse (D) a circle

Ans.: (A)

Solution.:

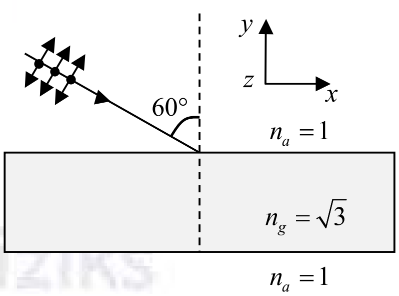
$$y_1 = a\cos 2\omega t,\ y_2 = b\cos(\omega t + \pi) = -b\cos\omega t \Rightarrow \cos\omega t = -\frac{y_2}{b}$$

$$y_1 = a\left[2\cos^2\omega t - 1\right]$$

$$y_1 + a = 2a\frac{y_2^2}{b^2} \Rightarrow y_2^2 = \frac{b^2}{2a}(y_1 + a)$$

So, the trajectory of resultant path will be a parabola.

Q21. An unpolarized light ray passing through air (refractive index $n_a = 1$) is incident on a glass slab (refractive index $n_g = \sqrt{3}$) at an angle of 60°, as shown in the figure below. The amplitude of the in-plane $(x - y)$ electric field component of the incident light is $4V/m$ and amplitude of the out of plane (z) electric field component is $3V/m$. After passing through the glass slab, the electric field amplitude (in V/m) of the light is



(A) 5 (B) 4 (C) 7 (D) 3

Ans.: (B)

Solution.: $n_1 = 1, n_2 = \sqrt{3}, n_3 = 1, \theta_i = 60°$, $E_{p,i} = 4V/m, E_{si} = 3V/m$

First interface: $n_i \sin\theta_i = n_2 \sin\theta_t \Rightarrow 1 \times \sin 60° = \sqrt{3}\sin\theta_t \Rightarrow \sin\theta_t = \frac{1}{2} \Rightarrow \theta_t = 30°$

$$t_{p12} = \frac{2n_1\cos\theta_i}{n_2\cos\theta_i + n_1\cos\theta_t} = \frac{2(1)\cos 60°}{\sqrt{3}\cos 60° + 1\cos 30°} \Rightarrow t_{p12} = \frac{1}{\sqrt{3}}$$

$$t_{s12} = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} = \frac{2(1)\cos 60°}{1\cos 60° + \sqrt{3}\cos 30°} = \frac{1}{2}$$

Second interface: $\theta_i = 30°, \theta_t = 60°$

$$t_{p23} = \frac{2n_2\cos\theta_i}{n_3\cos\theta_2 + n_2\cos\theta_2} = \frac{2\sqrt{3}\left(\sqrt{3}/2\right)}{1\left(\sqrt{3}/2\right) + \sqrt{3}(1/2)} = \sqrt{3}$$

$$t_{s23}=\frac{2n_2\cos\theta_i}{n_2\cos\theta_i+n_3\cos\theta_t}=\frac{2\sqrt{3}\left(\sqrt{3}/2\right)}{\sqrt{3}\left(\sqrt{3}/2\right)+\left(1/2\right)}=\frac{3}{2}$$

Net transmission coefficient: $t_p=t_{p12}t_{p23}=\frac{1}{\sqrt{3}}\sqrt{3}=1$; $t_s=t_{s12}t_{s23}=\frac{1}{2}\times\frac{3}{2}=\frac{3}{4}$

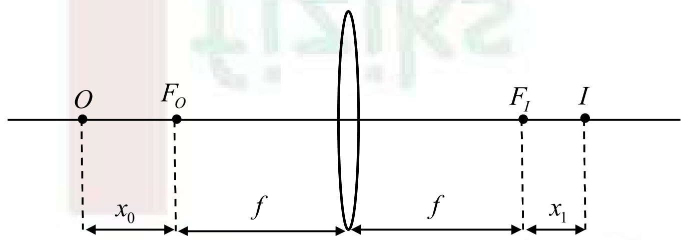
Transmitted field amplitude: $E_{p,t}=t_pE_{pi}=1\times 4=4V/m$

$$E_{s,t}=t_sE_{si}=\frac{3}{4}\times 3=\frac{9}{4}V/m;\quad E_t=\sqrt{E_{Pt}^2+E_{st}^2}=\sqrt{4^2+\left(\frac{9}{4}\right)^2}=4.6V/m$$

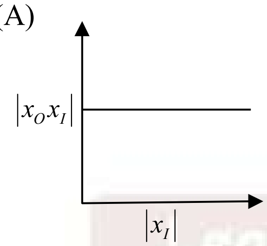
As E_t should be less than $E_i=\sqrt{4^2+3^2}=5$, so, best possible answer is $E_t=4V/m$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

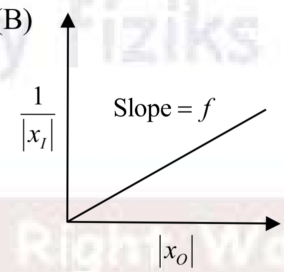
Q34. For a thin convex lens of focal length f, the image of an object at O is formed at I, as shown in the figure below. The distances of object and image from the two focal points (F_O and F_I) are x_O and x_I, respectively. Which of the following graphs correctly represent(s) the variation of the quantities shown in the figure?



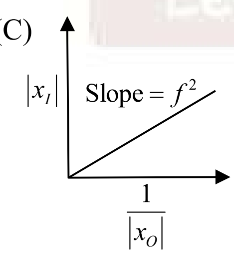
(A)



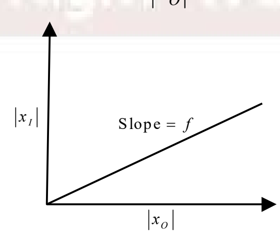
(B)



(C)



(D)



Ans.: (A), (C)

Solution.: $u=-\left(f+x_0\right)$; $v=\left(f+x_I\right)$

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{f+x_I}-\frac{1}{-\left(f+x_0\right)}=\frac{1}{f}\Rightarrow\frac{f+x_0+f+x_I}{\left(f+x_I\right)\left(f+x_0\right)}=\frac{1}{f}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

$2f^2 + x_0 f + x_I f = f^2 + f x_0 + f x_I + x_0 x_I$

$\boxed{x_0 x_I = f^2 = \text{constant}}$. So, option (a) is correct.

$\boxed{|x_I| = f^2 \frac{1}{|x_0|}}$. So, option (c) is correct.

Q35. Identify which of the following wave functions describe(s) travelling waves(s). (A_0, B_0, a, and b are positive constants of appropriate dimensions)

(A) $\psi(x,t) = A_0 (x+t)^2$ (B) $\psi(x,t) = A_0 \sin(ax^2 + bt^2)$

(C) $\psi(x,t) = \frac{A_0}{B_0 (x-t)^2 + 1}$ (D) $\psi(x,t) = A_0 e^{(ax+bt)^2}$

Ans.: (A), (C), (D)

Section C: Q.41 - Q.50 Carry ONE mark each.

Q42. The shortest distance between an object and its real image formed by a thin convex lens of focal length $20\,cm$ is ____________ cm. (in integer)

Ans.: 80

Solution.:

Minimum distance between object and its real image formed by a thin convex lens occurs when $|u| = |v| = 2f$

So minimum distance = $|u| + |v| = 2f + 2f = 4f = 4 \times 2 = 80\,cm$

Section C: Q.51-Q60 Carry TWO marks each.

Q57. A light beam given by $\vec{E}(z,t)=E_{01}\sin(kz-\omega t)\hat{i}+E_{02}\sin\left(kz-\omega t+\frac{\pi}{6}\right)\hat{j}$ passes through an ideal linear polarizer whose transmission axis is titled by 60° from x-axis (in $x-y$ plane). If $E_{01}=4V/m$ and $E_{02}=2V/m$, the electric field amplitude of the emerging light beam from the polarizer is __________ V/m. (up to two decimal places)

Ans.: 3.59 to 3.63

Solution.: Unit vector along polarizer axis: $\hat{n}=\cos 60^0\hat{i}+\sin 60^0\hat{j}=\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$

Electric field component along polarizer $=\vec{E}.\hat{n}$

$$E_{out}=\frac{E_{01}}{2}\sin\omega t-\frac{\sqrt{3}}{2}E_{02}\sin\left(\omega t-\frac{\pi}{6}\right)\Rightarrow E_{out}=2\sin\omega t-\sqrt{3}\sin\left(\omega t-\frac{\pi}{6}\right)$$

So resultant amplitude of two sinusoids: $E_R=\sqrt{(2)^2+(\sqrt{3})^2+2(2)(\sqrt{3})\cos\frac{\pi}{6}}$

$$\Rightarrow E_R=\sqrt{4+3+6}=\sqrt{13}=3.61V/m$$

Q58. A wedge-shaped thin film is formed using soap-water solution. The refractive index of the film is 1.25. At near normal incidence, when the film is illuminated by a monochromatic light of wavelength $600\,nm$, 10 interference fringes per cm are observed. The wedge angle (in radians) is _________ $\times 10^{-5}$. (in integer)

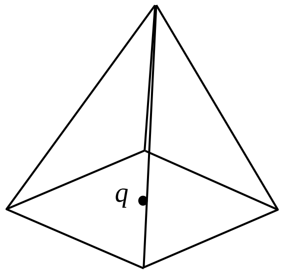
Ans.: 24

Solution.: Fringe width $\beta=\frac{\lambda}{2\mu\theta}$

$$\frac{1\times 10^{-2}}{10}=\frac{600\times 10^{-9}}{2\times 1.25\theta}\Rightarrow \theta=\frac{600\times 10^{-6}}{2.50}=24\times 10^{-5}\text{ rad}$$

Section A: Q.1-Q.10 Carry ONE mark each.

Q5. A charge q is placed at the center of the base of a square pyramid. The net outward electric flux across each of the slanted faces is (Consider permittivity as ε_0)



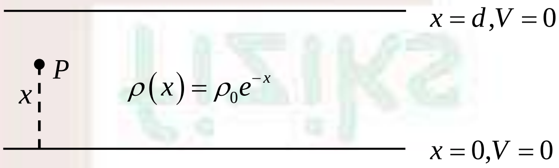
(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{2\varepsilon_0}$

(C) $\frac{q}{4\varepsilon_0}$ (D) $\frac{q}{8\varepsilon_0}$

Ans.: (D)

Solution.: The net outward electric flux across each of the slanted faces is $=\frac{1}{4}\left(\frac{q}{2\varepsilon_0}\right)=\frac{q}{8\varepsilon_0}$

Q6. Consider a parallel plate capacitor (distance between the plates d, and permittivity ε_0) as shown in the figure below. The space charge density between the plates varies as $\rho(x)=\rho_0 e^{-x}$. Voltage $V=0$ both at $x=0$ and $x=d$.



The voltage $V(x)$ at point P between the plates is [ρ_0 is a constant of appropriate dimensions]

(A) $\frac{\rho_0}{\varepsilon_0}\left[e^{-x}+\frac{1-e^{-d}}{d}x-1\right]$ (B) $\frac{2\rho_0}{\varepsilon_0}\left[e^{-x}+\frac{1-e^{-d}}{d}x-1\right]$

(C) $\frac{\rho_0}{2\varepsilon_0}\left[e^{-x}+\frac{1-e^{-d}}{d}x-1\right]$ (D) $\frac{3\rho_0}{\varepsilon_0}\left[e^{-x}+\frac{1-e^{-d}}{d}x-1\right]$

Ans.: (A)

Solution.: $\rho(x)=\rho_0 e^{-x}$. Voltage $V=0$ both at $x=0$ and $x=d$.

From Poisson's Equation: $\frac{d^2V}{dx^2}=-\frac{\rho}{\varepsilon_0}\Rightarrow\frac{d^2V}{dx^2}=-\frac{\rho_0 e^{-x}}{\varepsilon_0}\Rightarrow\frac{dV}{dx}=\frac{\rho_0}{\varepsilon_0}e^{-x}+A$

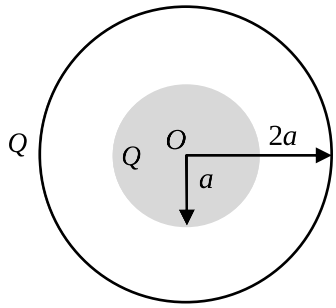
$$\Rightarrow V(x)=-\frac{\rho_0}{\varepsilon_0}e^{-x}+Ax+B.$$

$$\because V=0 \text{ at } x=0:\ 0=-\frac{\rho_0}{\varepsilon_0}e^{-0}+0+B\Rightarrow B=\frac{\rho_0}{\varepsilon_0}$$

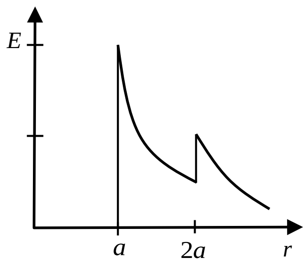
$$\because V=0 \text{ at } x=d:\ 0=-\frac{\rho_0}{\varepsilon_0}e^{-d}+Ad+B\Rightarrow A=\frac{1}{d}\left(\frac{\rho_0}{\varepsilon_0}e^{-d}-B\right)=\frac{\rho_0}{d\varepsilon_0}e^{-d}-\frac{\rho_0}{d\varepsilon_0}$$

$$\Rightarrow V(x) = -\frac{\rho_0}{\varepsilon_0}e^{-x} + \left(\frac{\rho_0}{d\varepsilon_0}e^{-d} - \frac{\rho_0}{d\varepsilon_0}\right)x + \frac{\rho_0}{\varepsilon_0} \Rightarrow V(x) = -\frac{\rho_0}{\varepsilon_0}\left[e^{-x} + \frac{1-e^{-d}}{d}x - 1\right]$$

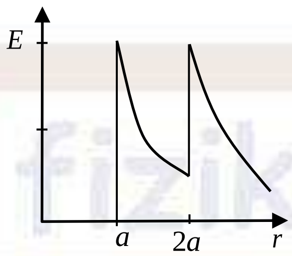
Q7. Consider a metal sphere enclosed concentrically within a spherical shell. The inner sphere of radius a carries charge Q. The outer shell of radius $2a$ also has charge Q. The variation of the magnitude E of the electric field as a function of distance r from the center O is



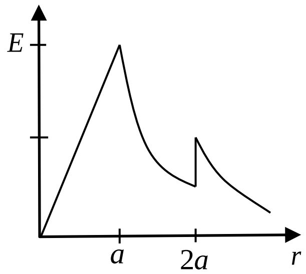
(A)



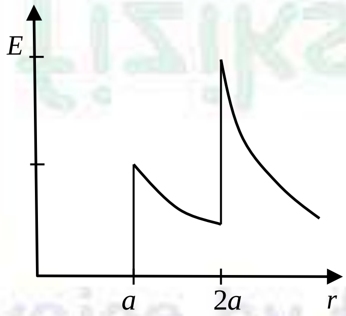
(B)



(C)



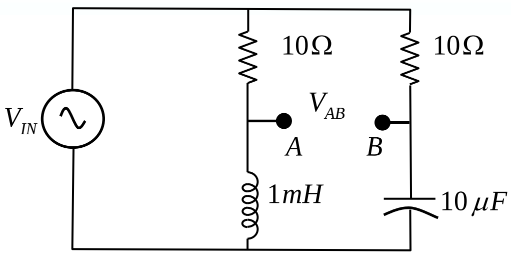
(D)



Ans.: (A)

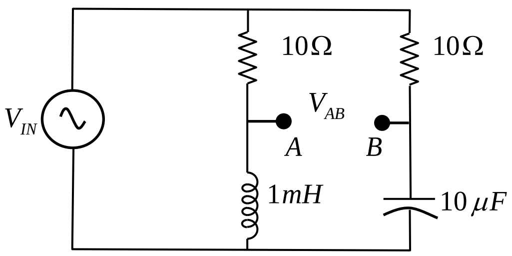
Section A: Q.11-Q.30 Carry TWO marks each.

Q11. In the circuit given below, the frequency of the input voltage V_{IN} is $\omega = 10^4\, rad/s$. The output voltage V_{AB} leads V_{IN} by



(A) 0° (B) 45° (C) 90° (D) –90°

Ans.: (C)



Solution.: Here $\omega = 10^4\ rad/s$

$$V_A = \frac{X_L}{R + X_L} V_{IN} = \frac{i\omega L}{R + i\omega L} V_{IN}$$

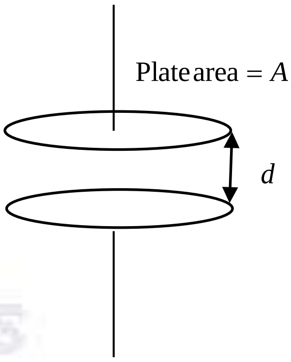
$$\Rightarrow V_A = \frac{i \times 10^4 \times 10^{-3}}{10 + i \times 10^4 \times 10^{-3}} V_{IN} = \left(\frac{i}{1+i}\right) V_{IN}$$

$$V_B = \frac{X_C}{R + X_C} V_{IN} = \frac{1/i\omega C}{R + 1/i\omega C} V_{IN} = \frac{1}{1 + i\omega CR} V_{IN}$$

$$\Rightarrow V_B = \frac{1}{1 + i \times 10^4 \times 10^{-5} \times 10} V_{IN} = \frac{1}{1+i} V_{IN}$$

$$\Rightarrow V_{AB} = V_A - V_B = \left(\frac{i}{1+i} - \frac{1}{1+i}\right) V_{IN} = \left(\frac{-1+i}{1+i}\right) V_{IN} \Rightarrow \frac{V_{AB}}{V_{IN}} = \frac{\sqrt{2} e^{i3\pi/4}}{\sqrt{2} e^{i\pi/4}} = e^{i\pi/2}$$

Q22. Consider a slowly charging parallel plate capacitor (distance between the plates is d) having circular plates each with an area A, as shown in the figure below. An electric field of magnitude $E = E_0 \sin(\omega t)$ exits between the plates while charging. The associated magnitude of the magnetic field B at the periphery (outer edge) of the capacitor is (Neglect fringe effects)



(A) $\frac{1}{2c^2}\sqrt{\frac{A}{\pi}} E_0 \omega \cos(\omega t)$ (B) $\frac{1}{2c^2}\sqrt{\frac{A}{\pi}} E_0 \omega \sin(\omega t)$

(C) $\frac{1}{c^2}\sqrt{\frac{A}{\pi}} E_0 \omega \cos(\omega t)$ (D) $\frac{1}{c^2}\sqrt{\frac{A}{\pi}} E_0 \omega \sin(\omega t)$

Ans.: (A)

Solution.: $\oint \vec{B}.d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t}.d\vec{a}$

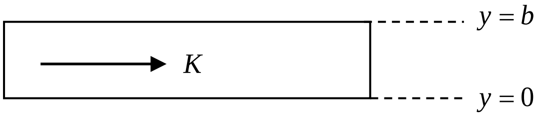
Consider an Amperian loop of radius $r = R$:

$$|\vec{B}| \times 2\pi R = 0 + \mu_0 \varepsilon_0 \times \omega E_0 \cos(\omega t) \times \pi R^2 \quad \because I_{enc} = 0 \text{ and since } E(t) = E_0 \sin(\omega t)$$

$$\Rightarrow |\vec{B}| = \mu_0 \varepsilon_0 \omega E_0 \cos(\omega t) \times \frac{R}{2} \Rightarrow |\vec{B}| = \frac{\omega E_0}{2c^2}\sqrt{\frac{A}{\pi}} \cos(\omega t) \text{ where } A = \pi R^2 \text{ and } c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Q23. A surface current density $K = ae^{-y}$ exists on a thin strip of width b, as shown in the figure below. The associated surface current is



(a is a constant of appropriate dimensions)

(A) $a\left(1-e^{-b}\right)$ (B) $a\left(1+e^{-b}\right)$ (C) $a\left(e^{-b}-1\right)$ (D) $a\left(e^{b}+e^{-b}\right)$

Ans.: (A)

Solution.: $K = \dfrac{dI}{dl_\perp} \Rightarrow dI = Kdl_\perp \Rightarrow I = \int Kdl_\perp = \int_0^b ae^{-y}dy = a\left[\dfrac{e^{-y}}{-1}\right]_0^b$

$\Rightarrow I = a\left[-e^{-b}+1\right] = a\left(1-e^{-b}\right)$

Q24. For an electromagnetic wave, consider an electric field $\vec{E} = E_0 e^{-i\left[a(x+y)-\omega t\right]}\hat{k}$. The corresponding magnetic field $\vec{B}$ is (E_0, a, ω are constants of appropriate dimensions and c is the speed of light)

(A) $\dfrac{1}{c\sqrt{2}}E_0 e^{-i\left[a(x+y)-\omega t\right]}\left(\hat{i}-\hat{j}\right)$ (B) $\dfrac{1}{c\sqrt{2}}E_0 e^{-i\left[a(x+y)-\omega t\right]}\left(\hat{i}+\hat{j}\right)$

(C) $\dfrac{1}{c\sqrt{2}}E_0 e^{-i\left[a(x+y)-\omega t\right]}\left(-\hat{i}-\hat{j}\right)$ (D) $\dfrac{1}{c\sqrt{2}}E_0 e^{-i\left[a(x+y)-\omega t\right]}\left(-\hat{i}+\hat{j}\right)$

Ans.: (A)

Solution.: $\because \vec{E} = E_0 e^{-i\left[a(x+y)-\omega t\right]}\hat{k}$, $\vec{K} = a\left(\hat{i}+\hat{j}\right)$ and $\hat{n} = \hat{k}$. $\vec{B} = \dfrac{1}{\omega}\left(\vec{K}\times\vec{E}\right)$

$$\vec{K}\times\vec{E} = E_0 e^{-i\left[a(x+y)-\omega t\right]}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a & a & 0\\ 0 & 0 & 1\end{vmatrix} = E_0 e^{-i\left[a(x+y)-\omega t\right]}a\left(\hat{i}-\hat{j}\right)$$

$$\Rightarrow \vec{B} = \frac{1}{\omega}\left(\vec{K}\times\vec{E}\right) = \frac{aE_0}{\omega}e^{-i\left[a(x+y)-\omega t\right]}\left(\hat{i}-\hat{j}\right)$$

$$\because c = \frac{\omega}{\left|\vec{K}\right|} = \frac{\omega}{a\sqrt{2}} \Rightarrow \frac{a}{\omega} = \frac{1}{c\sqrt{2}} \Rightarrow \vec{B} = \frac{E_0}{c\sqrt{2}}e^{-i\left[a(x+y)-\omega t\right]}\left(\hat{i}-\hat{j}\right)$$

Q30. A magnetic field is given by $\vec{B}=\vec{\nabla}\times\vec{A}$ where $\vec{A}$ is the magnetic vector potential. If $\vec{A}=\left(ax^2+by^2\right)\hat{i}$, the corresponding current density $\vec{J}$ is (a and b are non-zero constants)

(A) $-\frac{1}{\mu_0}(2a+2b)\hat{i}$ (B) $\frac{1}{\mu_0}(2a+2b)\hat{i}$ (C) $-\frac{1}{\mu_0}(2a)\hat{i}$ (D) $-\frac{1}{\mu_0}(2b)\hat{i}$

Ans.: (D)

Solution.: Given $\vec{B}=\vec{\nabla}\times\vec{A}$, $\vec{A}=\left(ax^2+by^2\right)\hat{i}$

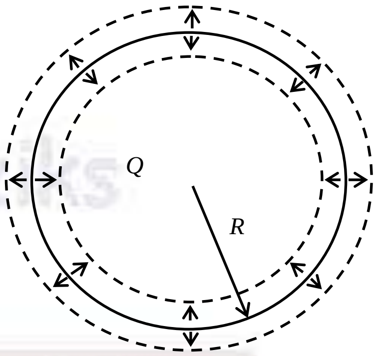
Thus $\vec{\nabla}\times\vec{B}=\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)\Rightarrow\mu_0\vec{J}=\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}\Rightarrow\vec{J}=\frac{1}{\mu_0}\left[\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}\right]$

$$\nabla^2\vec{A}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(ax^2+by^2\right)\hat{i}=(2a+2b)\hat{i},\ \vec{\nabla}.\vec{A}=2ax\Rightarrow\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)=2a\hat{i}$$

$$\Rightarrow\vec{J}=\frac{1}{\mu_0}\left[\vec{\nabla}\left(\vec{\nabla}.\vec{A}\right)-\nabla^2\vec{A}\right]=\frac{1}{\mu_0}\left[2a\hat{i}-(2a+2b)\hat{i}\right]=-\frac{1}{\mu_0}(2b)\hat{i}$$

Section B: Q.31 - Q.40 Carry TWO marks each.

Q36. A spherical ball having a uniformly distributed charge Q and radius R pulsates with frequency ω such that the radius changes by $\pm 10\%$, as shown in the figure below. Which of the following is(are) correct?



(A) The net outward electric flux across a spherical surface of radius $r>1.5R$ pulsates with a frequency ω

(B) The net outward electric flux across a spherical surface of radius $r=2R$ is $\frac{Q}{\varepsilon_0}$

(C) The potential fluctuates with frequency ω at $r=2R$

(D) The electric field inside the sphere at $r=0.5R$ will not be time dependent

Ans.: (B)

Solution.: The spherical ball has a uniformly distributed charge Q that remains constant and radius R pulsates with frequency ω such that the radius changes by $\pm 10\%$.Thus radius varies between $0.9R$ and $1.1R$.

(A) Since radius $r>1.5R$ so surface encloses total charge Q, so from Gauss Law flux remains constant. Thus, option (A) option is incorrect.

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

(B) Since radius $r = 2R$ so surface encloses total charge Q, so from Gauss Law flux remains constant. Thus, net outward electric flux across a spherical surface of radius $r = 2R$ is $\frac{Q}{\varepsilon_0}$. Thus, option (B) option is correct.

(C) Since charge distribution is spherically symmetric and $r = 2R$ is always outside the sphere, the potential is same as that of point charge Q at center. Thus, potential does not fluctuate with frequency ω at $r = 2R$.So option (C) option is incorrect.

(D) Since $r = 0.5R < 0.9R$, the point is always inside the sphere. For uniformly distributed charge Q and radius R, field inside is $E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$. As R pulsates with frequency ω, E varies with time. So option (D) option is incorrect.

Q37. Which of the following relations is(are) valid for linear dielectrics?

[$\vec{E}$ = Electric field, $\vec{P}$ = Polarization, $\vec{D}$ = Electric displacement, ε_0 = Permittivity of free space, ε = Dielectric permittivity, χ_e = Electric susceptibility, ρ_f = Free charge density, ρ_b = Bound charge density]

(A) $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ (B) $\varepsilon = \varepsilon_0(1 + \chi_e)$ (C) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (D) $\vec{\nabla} \cdot \vec{D} = \rho_f + \rho_b$

Ans.: (A), (B), (C)

Section C: Q.41 - Q.50 Carry ONE mark each.

Q43. Consider two media 1 and 2 having permittivities ε_0 and $\varepsilon_1(= 2\varepsilon_0)$, respectively. The interface between the two media aligns with the $x - y$ plane. An electric field $\vec{E}_1 = 4\hat{i} - 5\hat{j} - \hat{k}$ exists in medium 1. The magnitude of the displacement vector $\vec{D}_2$ in medium 2 is ________ ε_0. (up to two decimal places)

Ans.: 12.65 to 13.05

Solution.: Media 1 and 2 having permittivities ε_0 and $\varepsilon_1(= 2\varepsilon_0)$.

The interface between the two media aligns with the $x - y$ plane

$$\because \vec{E}_1 = 4\hat{i} - 5\hat{j} - \hat{k} \Rightarrow \vec{E}_1^{\parallel} = 4\hat{i} - 5\hat{j} = \vec{E}_2^{\parallel} \text{ and } \vec{E}_1^{\perp} = -\hat{k} \Rightarrow \vec{E}_2^{\perp} = \frac{\varepsilon_0}{2\varepsilon_0}\vec{E}_1^{\perp} = -\frac{1}{2}\hat{k}$$

$$\text{Thus } \vec{E}_2 = 4\hat{i} - 5\hat{j} - \frac{1}{2}\hat{k} \Rightarrow \vec{D}_2 = 2\varepsilon_0 \vec{E}_2 = \varepsilon_0\left(8\hat{i} - 10\hat{j} - \hat{k}\right)$$

$$\Rightarrow \left|\vec{D}_2\right| = \varepsilon_0\sqrt{64 + 100 + 1} = \varepsilon_0\sqrt{165} = 12.85\varepsilon_0$$

Section C: Q.51-Q60 Carry TWO marks each. (No Question)

Section A: Q.1-Q.10 Carry ONE mark each. (No Question)

Section A: Q.11-Q.30 Carry TWO marks each.

Q25. Consider Maxwell's relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$. The equation of state of a thermodynamic system is given as $P = \frac{AT}{V^2} + \frac{BT^3}{V}$, where A and B are constants of appropriate dimensions. Then $\left(\frac{\partial C_V}{\partial V}\right)_T$ of the system varies with temp. as (C_V is the heat capacity at constant volume)

(A) T^2 (B) T (C) T^{-1} (D) T^3

Ans.: (A)

Solution.: $\because C_V = \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial}{\partial V} T\left(\frac{\partial S}{\partial T}\right)_V\right)_T = T\left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)_V\right)_T = T\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V = T\left(\frac{\partial}{\partial T}\left(\frac{\partial P}{\partial T}\right)_V\right)_V$$

$$\Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

$$\because P = \frac{AT}{V^2} + \frac{BT^3}{V} \Rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{A}{V^2} + \frac{3BT^2}{V} \Rightarrow \left(\frac{\partial^2 P}{\partial T^2}\right)_V = 0 + \frac{6BT}{V} = \frac{6BT}{V}$$

$$\Rightarrow \left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V = \frac{6B}{V}T^2$$

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q38. Three gaseous systems, G_1, G_2, and G_3 with pressure and volume $(P_1, V_1), (P_2, V_2)$, and (P_3, V_3), respectively, are such that

[I] when G_1 and G_2 are in thermal equilibrium $P_1V_1 - P_2V_2 + \alpha P_2 = 0$, is satisfied, and

[II] when G_1 and G_3 are in thermal equilibrium, $P_3V_3 - P_1V_1 + \frac{\beta P_1 V_1}{V_3} = 0$, is satisfied.

The relation(s) valid at thermal equilibrium is(are)

(α and β are constants of appropriate dimensions)

(A) $P_3V_3 - (P_2V_2 - \alpha P_2)\left(1 - \frac{\beta}{V_3}\right) = 0$

(B) $P_3V_3 + (P_2V_2 + \alpha P_2)\left(1 + \frac{\beta}{V_3}\right) = 0$

(C) $P_1V_1 = P_2V_2 = P_3V_3$

(D) $P_3V_3 + P_1V_1\left(\frac{\beta}{V_3} - 1\right) = 0$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Ans.: (A), (D)

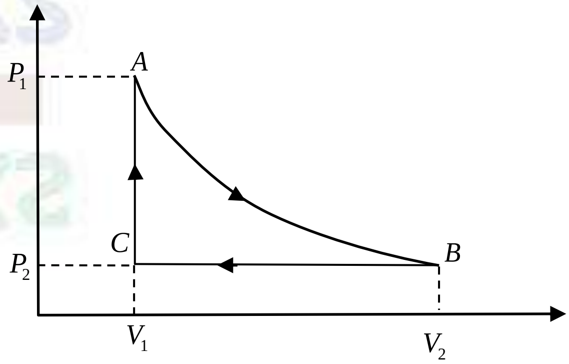
Solution.: [I] when G_1 and G_2 are in thermal equilibrium $P_1V_1 - P_2V_2 + \alpha P_2 = 0$, is satisfied, and

[II] when G_1 and G_3 are in thermal equilibrium, $P_3V_3 - P_1V_1 + \frac{\beta P_1V_1}{V_3} = 0$, is satisfied.

Thus $P_3V_3 - (P_2V_2 - \alpha P_2) + \frac{\beta}{V_3}(P_2V_2 - \alpha P_2) = 0 \Rightarrow P_3V_3 - (P_2V_2 - \alpha P_2)\left(1 - \frac{\beta}{V_3}\right) = 0$

and $\because P_3V_3 - P_1V_1 + \frac{\beta P_1V_1}{V_3} = 0 \Rightarrow P_3V_3 + P_1V_1\left(\frac{\beta}{V_3} - 1\right) = 0$

Q39. An ideal mono-atomic gas is expanded adiabatically from A to B. It is then compressed in an isobaric process from B to C. Finally, the pressure is increased in an isochoric process from C to A. The cyclic process is shown in the figure below. For this system, which of the following is (are) correct?



(A) Work done along the path AB is $(P_1V_1 - P_2V_2)$

(B) Total work done during the entire process is $\frac{3}{2}(P_1V_1 - P_2V_2) + P_2(V_1 - V_2)$

(C) Total heat absorbed during the entire process is $\frac{3}{2}(P_1 - P_2)V_1$

(D) Total change in internal energy during the entire process is $\frac{5}{2}P_2(V_2 - V_1)$

Ans.: (B), (C), (D)

Solution.: Process $A \to B$: $W_{AB} = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1) = \frac{1}{1-5/3}(P_2V_2 - P_1V_1) = \frac{3}{2}(P_1V_1 - P_2V_2)$

Process $B \to C$: $W_{BC} = \int_{V_2}^{V_1} P_2 dV = P_2(V_1 - V_2)$; Process $C \to A$: $W_{CA} = 0$

Total work done during the entire process is $W_T = W_{AB} + W_{BC} + W_{CA}$

$\Rightarrow W_T = \frac{3}{2}(P_1V_1 - P_2V_2) + P_2(V_1 - V_2)$

Total heat absorbed refers to the heat added to the system, which occurs only during the isochoric process CA: $Q_{CA} = \mu C_V \Delta T = \frac{3}{2}(P_1 - P_2)V_1$

$\because \mu = 1,\ C_V = \frac{3}{2}R,\ P_1V_1 - P_2V_1 = (P_1 - P_2)V_1 = R(T_1 - T_2) = R\Delta T$

Section C: Q.41 - Q.50 Carry ONE mark each.

Q44. G1 and G2 are two ideal gases at temperatures T_1 and T_2, respectively. The molecular weight of the constituents of G1 is half that of G2. If the average speeds of the molecules of both gases are equal, then assuming Maxwell-Boltzmann distributions for the molecular speeds, the ratio $\frac{T_2}{T_1}$ is ______________. (in integer)

Ans.: 2

Solution.: $\langle v\rangle = \sqrt{\frac{8k_BT}{m\pi}} = \sqrt{\frac{8RT}{M\pi}} \qquad \because k_B = \frac{R}{N_A},\ M = mN_A$

$$\langle v_1\rangle = \langle v_2\rangle \Rightarrow \sqrt{\frac{8RT_1}{M_1\pi}} = \sqrt{\frac{8RT_2}{M_2\pi}} \Rightarrow \frac{T_1}{M_1} = \frac{T_2}{M_2} \Rightarrow \frac{T_2}{T_1} = \frac{M_2}{M_1} = \frac{M_2}{M_2/2} = 2$$

Q46. In a two-level atomic system, the excited state is $0.2eV$ above the ground state. Considering the Maxwell-Boltzmann distribution, the temperature at which 2% of the atoms will be in the excited state is _________K. (up to two decimal places)

(Boltzmann constant $k_B = 8.62\times10^{-5}$ eV/K)

Ans.: 591.00 to 597.00

Solution.: $P(\varepsilon) \propto e^{-\varepsilon/k_BT}$ so $\frac{P(\varepsilon_2)}{P(\varepsilon_0)} = e^{-(\varepsilon_2-\varepsilon_0)/k_BT} = \frac{2}{100} \Rightarrow -\frac{\varepsilon_2-\varepsilon_0}{k_BT} = \ln\left(\frac{2}{100}\right) = -\ln 50$

$$\Rightarrow \frac{\varepsilon_2-\varepsilon_0}{k_BT} = \ln 50 \Rightarrow T = \frac{\varepsilon_2-\varepsilon_0}{k_B\times\ln 50} = \frac{0.2}{8.62\times10^{-5}\times3.9} = \frac{20000}{33.72} = 593.12\,K$$

Q48. At a particular temperature T, Planck's energy density of black body radiation in terms of frequency is $\rho_T(\nu) = 8\times10^{-18}\frac{J/m^3}{Hz}$ at $\nu = 3\times10^{14}\,Hz$. Then Planck's energy density $\rho_T(\lambda)$ at the corresponding wavelength (λ) has the value _________ $\times10^2\frac{J/m^3}{m}$. (in integer)

[Speed of light $c = 3\times10^8\,m/s$]

Ans.: 24

Solution.: $\rho_T(\lambda) = \rho_T(\nu)\left|\frac{d\nu}{d\lambda}\right|,\quad \nu = \frac{c}{\lambda} \Rightarrow \left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2} = \frac{\nu^2}{c}$

$$\Rightarrow \rho_T(\lambda) = \rho_T(\nu)\frac{\nu^2}{c} = 8\times10^{-18}\times\frac{\left(3\times10^{14}\right)^2}{3\times10^8} = 8\times10^{-18}\times3\times10^{20} = 24\times10^2\,\frac{J/m^3}{m}$$

Section C: Q.51-Q60 Carry TWO marks each.

Q53. One kg of water at 27°C is brought in contact with a heat reservoir kept at 37°C. Upon reaching thermal equilibrium, this mass of water is brought in contact with another heat reservoir kept at 47°C. The final temperature of water is 47°C. The change in entropy of the whole system in this entire process is ___________cal/K. (up to two decimal places)

[Take specific heat at constant pressure of water as 1 cal/(g K)]

Ans.: 0.90 to 1.10

Solution.: Entropy change of reservoir in step-1:

$$\Delta S_{R1} = -\frac{mc_p\Delta T}{T} = -\frac{1000\times1\times10}{310} = -32.3\,\text{Cal/K}$$

Entropy change of reservoir in step-2:

$$\Delta S_{R2} = -\frac{mc_p\Delta T}{T} = -\frac{1000\times1\times10}{320} = -31.3\,\text{Cal/K}$$

Entropy change of the system

$$\Delta S_{system} = mc_p \ln\frac{T_2}{T_1} = 1000\times1\times\ln\frac{320}{310} = 1000\ln\frac{32}{31} = 1000\ln(1.0666) = 64.45\,\text{Cal/K}$$

The change in entropy of the whole system

$$\Delta S_T = \Delta S_{system} + \Delta S_R = (64.45-63.5)\,\text{Cal/K=0.95 Cal/K}$$

Q60. Consider a chamber at room temperature (27°C) filled with a gas having a molecular diameter of $0.35\,nm$. The pressure (in Pascal) to which the chamber needs to be evacuated so that the molecules have a mean free path of $1\,km$ is _________ $\times10^{-5}\,Pa$. (up to two decimal places) (Boltzmann constant $k_B = 1.38\times10^{-23}\,J/K$)

Ans.: 0.70 to 1.20

Solution.: Mean free path $\lambda = \frac{k_BT}{\sqrt{2}p\sigma} \Rightarrow p = \frac{k_BT}{\sqrt{2}\lambda\sigma}$

$T = 27\,^0C \Rightarrow T = (27+273)K = 300K$, $k_B = 1.38\times10^{-23}\,J/K$, $\lambda = 1km = 1000\,m$

$\sigma = \pi d^2 = 3.14\times(0.35\times10^{-9})^2 = 3.14\times1225\times10^{-22} = 38.45\times10^{-20}\,m^2$

$$\Rightarrow p = \frac{(1.38\times10^{-23})\times300}{\sqrt{2}\times10^3\times38.45\times10^{-20}\,m^2} = \frac{4.14\times10^{-21}}{5.438\times10^{-16}} = 0.76\times10^{-5}\,Pa$$

Section A: Q.1-Q.10 Carry ONE mark each.

Q8. Consider radioactive decays $A \rightarrow B$ with half-life $\left(T_{1/2}\right)_A$ and $B \rightarrow C$ with half-life $\left(T_{1/2}\right)_B$.

At any time t, the number of nuclides of B is given by

$$\left(N_B\right)_t = \frac{\lambda_A}{\lambda_B - \lambda_A}\left(N_A\right)_0\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right),$$

where $\left(N_A\right)_0$ is the number of nuclides of A at $t = 0$. The decay constants of A and B are λ_A and λ_B, respectively. If $\left(T_{1/2}\right)_B < \left(T_{1/2}\right)_A$, then the ratio $\frac{\left(N_B\right)_t}{\left(N_A\right)_t}$ at time $t \gg \left(T_{1/2}\right)_A$ is

[$\left(N_A\right)_t$ is the number of nuclides of A at time t]

(A) $\frac{\lambda_A}{\lambda_B - \lambda_A}$ (B) $\frac{\lambda_B}{\lambda_A}$ (C) $\frac{\lambda_A}{\lambda_B}$ (D) $\frac{\lambda_B}{\lambda_B - \lambda_A}$

Ans.: (A)

Solution.:

This is the case of transient radioactive equilibrium,

$\left(T_{1/2}\right)_B < \left(T_{1/2}\right)_A \Rightarrow \lambda_B > \lambda_A$

As t becomes very large; $e^{-\lambda_A t} \gg e^{-\lambda_B t}$

$$\left(N_B\right)_t \approx \frac{\lambda_A}{\lambda_B - \lambda_A}\left(N_A\right)_0 e^{-\lambda_A t} \Rightarrow \left(N_B\right)_t \approx \frac{\lambda_A}{\lambda_B - \lambda_A}\left(N_A\right)_t \Rightarrow \frac{\left(N_B\right)_t}{\left(N_A\right)_t} \approx \frac{\lambda_A}{\lambda_B - \lambda_A}$$

Q9. For a non-relativistic free particle, the ratio of phase velocity to group velocity is

(A) 2 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{4}$

Ans.: (B)

For a non-relativistic particle

$$v_p = \frac{E}{p} = \frac{p^2/2m}{p} = \frac{p}{2m} = \frac{mv}{2m} = \frac{v}{2}; \quad v_g = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m} = \frac{mv}{m} = v$$

$$\Rightarrow \frac{v_p}{v_g} = \frac{v/2}{v} = \frac{1}{2}$$

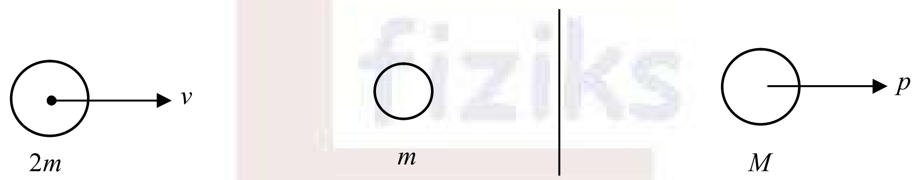
Section A: Q.11-Q.30 Carry TWO marks each.

Q26. Consider a relativistic particle of rest mass $2m$ moving with a speed v along the x direction. It collides with another relativistic particle of rest mass m moving with same speed but in the opposite direction. These two particles coalesce to form one particle whose rest mass M is ($\beta = \frac{v}{c}$, where c is the speed of light)

(A) $m\sqrt{\frac{9-\beta^2}{1-\beta^2}}$ (B) $2m\sqrt{\frac{3-\beta^2}{1-\beta^2}}$ (C) $\frac{m}{2}\sqrt{\frac{9-\beta^2}{2-\beta^2}}$ (D) $\frac{m}{4}\sqrt{\frac{1-\beta^2}{2-\beta^2}}$

Ans.: (A)

Solution.:



$$p = \frac{(2m)v}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{mv}{\sqrt{1-\beta^2}}$$

Conservation of energy: $\frac{(2m)c^2}{\sqrt{1-\beta^2}} + \frac{mc^2}{\sqrt{1-\beta^2}} = \sqrt{M^2c^4 + p^2c^2}$

$$\Rightarrow \frac{9m^2c^4}{1-\beta^2} = M^2c^4 + \frac{m^2v^2c^2}{1-\beta^2} \Rightarrow \frac{9m^2}{1-\beta^2} = M^2 + \frac{m^2\beta^2}{1-\beta^2} \Rightarrow M^2 = \frac{9-\beta^2}{1-\beta^2}m^2 \Rightarrow M = m\sqrt{\frac{9-\beta^2}{1-\beta^2}}$$

Q27. A particle of mass m is subjected to a potential $V(x)$. If its wavefunction is given by

$$\psi(x,t) = \alpha x^2 e^{-\beta x} e^{i\gamma t/\hbar},\ x > 0 \quad \text{and} \quad \psi(x,t) = 0,\ x \le 0$$

then $V(x)$ is (α, β and γ are constants of appropriate dimensions)

(A) $-\gamma + \frac{\hbar^2}{2m}\left(\frac{2}{x^2} - \frac{4\beta}{x} + \beta^2\right)$ (B) $-\gamma + \frac{\hbar^2}{2m}\left(\frac{2}{x^2} + \frac{4\beta}{x} + \beta^2\right)$

(C) $-\gamma + \frac{\hbar^2}{2m}\left(\frac{2}{x^2} - \frac{4\beta}{x} - \beta^2\right)$ (D) $-\gamma + \frac{\hbar^2}{2m}\left(-\frac{2}{x^2} - \frac{4\beta}{x} + \beta^2\right)$

Ans.: (A)

Solution.: Time-independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$.

$\Rightarrow V(x) = E + \frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}$. Here $e^{i\gamma t/\hbar} = e^{-iEt/\hbar}$, $E = -\gamma$

$\because \psi(x) = \alpha x^2 e^{-\beta x} \Rightarrow \psi' = \alpha e^{-\beta x}\left(2x - \beta x^2\right) \Rightarrow \psi'' = \alpha e^{-\beta x}\left(\beta^2 x^2 - 4\beta x + 2\right)$

$$\Rightarrow \frac{\psi''}{\psi} = \frac{\alpha e^{-\beta x}\left(\beta^2 x^2 - 4\beta x + 2\right)}{\alpha x^2 e^{-\beta x}} = \beta^2 - \frac{4\beta}{x} + \frac{2}{x^2}$$

Thus $V(x) = -\gamma + \frac{\hbar^2}{2m}\left(\beta^2 - \frac{4\beta}{x} + \frac{2}{x^2}\right) = -\gamma + \frac{\hbar^2}{2m}\left(\frac{2}{x^2} - \frac{4\beta}{x} + \beta^2\right)$

Q28. Two non-relativistic particles with masses m_1 and m_2 move with momenta $\vec{p}_1$ and $\vec{p}_2$, respectively, in an inertial frame S. In another inertial frame S', moving with a constant speed with respect to S, the same particles are observed to have momenta $\vec{p_1'}$ and $\vec{p_2'}$, respectively. Galilean invariance implies that

(A) $m_2\vec{p_1'} - m_1\vec{p_2'} = m_2\vec{p}_1 - m_1\vec{p}_2$ (B) $m_2\vec{p_1'} + m_1\vec{p_2'} = m_2\vec{p}_1 + m_1\vec{p}_2$

(C) $m_1\vec{p_1'} - m_2\vec{p_2'} = m_1\vec{p}_1 - m_2\vec{p}_2$ (D) $m_1\vec{p_1'} + m_2\vec{p_2'} = m_1\vec{p}_1 + m_2\vec{p}_2$

Ans.: (A)

Solution.:

In both the inertial frames, their relative velocities will be same: $v_1^1 - v_2^1 = v_1 - v_2$

$$\Rightarrow \frac{p_1^1}{m_1} - \frac{p_2^1}{m_2} = \frac{p_1}{m_1} - \frac{p_2}{m_2} \qquad \Rightarrow m_2 p_1^1 - m_1 p_2^1 = m_2 p_1 - m_1 p_2$$

Q29. The binding energy $B(A,Z)$ of an atomic nucleus of mass number A, atomic number Z, and number of neutrons $N = A - Z$, can be expressed as

$$B(A,Z) = a_1 A - a_2 A^{\frac{2}{3}} - a_3\frac{Z^2}{A^{\frac{1}{3}}} - a_4\frac{(A-2Z)^2}{A},$$

where a_1, a_2, a_3, and a_4 are constants of appropriate dimensions. Let $B(A,Z')$ be the binding energy of a mirror nucleus (which has the same A, but the number of protons and neutrons are interchanged). Then, at constant A, $\left[B(A,Z) - B(A,Z')\right]$ is

(A) proportional to Z^2 (B) proportional to $\left(Z^2 - N^2\right)$

(C) proportional to N^2 (D) constant

Ans.: (B)

Solution.: $\because N = A - Z \Rightarrow A - 2Z = A - 2(A - N) = 2N - A$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

In ${}^{A}_{Z}x$ nuclei: $B(A,Z) = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(2N-A)^2}{A}$

In mirror nuclei, ${}^{A}_{Z'}y$: No of protons $= Z'$ = No. of neutrons in ${}^{A}_{Z}x$; $Z' = A - Z = N$

$$B(A,Z') = a_1 A - a_2 A^{2/3} - a_3 \frac{N^2}{A^{1/3}} - a_4 \frac{(2N-A)^2}{A}$$

$$B(A,Z) - B(A,Z') = a_3 \frac{N^2}{A^{1/3}} - a_3 \frac{Z^2}{A^{1/3}} = -\frac{a_3}{A^{1/3}}\left(Z^2 - N^2\right)$$

$B(A,Z) - B(A,Z') \propto \left(Z^2 - N^2\right)$ for constant A.

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Section C: Q.41 - Q.50 Carry ONE mark each.

Q47. Neutrons of energy 8 MeV are incident on a potential step of height 48 MeV. As they penetrate the classically forbidden region, the distance at which the probability density of finding neutrons decreases by a factor of 100 is __________fm. (up to two decimal places)

(Take $\hbar c = 200\, MeVfm$, and the rest mass energy of neutron = $1 GeV$)

Ans.: 1.55 to 1.70

Solution.: Wave function decays as $\psi(x) \propto e^{-kx}$, $k = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{2(mc^2)(V_0 - E)}}{\hbar c}$

Here, $\hbar c = 200\, MeVfm, mc^2 = 1000 MeV, V_0 = 48 MeV$, $E = 8 MeV$

$$k = \frac{\sqrt{2 \times 1000 \times 40}}{200} = 1.414\, fm^{-1}$$

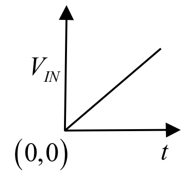
Probability density decrease by factor 100. $|\psi(x)|^2 \propto e^{-2kx}$

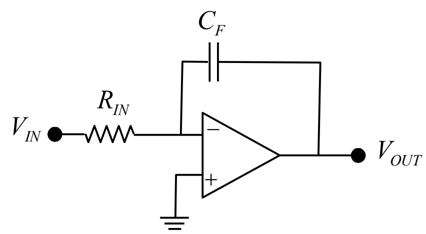
So $\frac{|\psi(x)|^2}{|\psi(0)|^2} = \frac{1}{100} \Rightarrow e^{-2kx} = 10^{-2} \Rightarrow 2kx = \ln 100 = 4.605 \Rightarrow x = \frac{4.605}{2 \times 1.414} = 1.63\, fm$

Section C: Q.51-Q60 Carry TWO marks each.

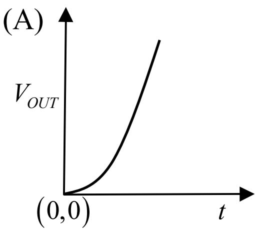
Section A: Q.1-Q.10 Carry ONE mark each.

Q10. If the input voltage waveform V_{IN} is a ramp function (as shown in the V_{IN}-t plot below), then the output wave form (V_{OUT}) for the given circuit diagram having an ideal operational amplifier (Op-Amp) is

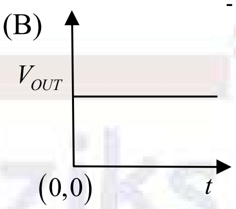




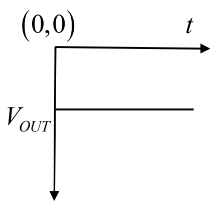
(A)



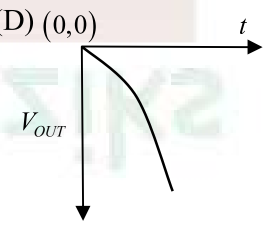
(B)



(C)



(D)



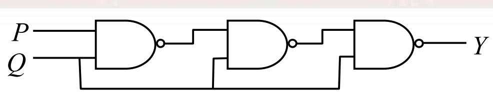
Ans.: (D)

Solution.: $V_{OUT} = -\frac{1}{R_{IN}C_F}\int_0^t V_{IN}dt = -\frac{1}{R_{IN}C_F}\int_0^t tdt = -kt^2$ where k is some constant.

Section A: Q.11-Q.30 Carry TWO marks each.

Section B: Q.31 - Q.40 Carry TWO marks arks each.

Q31. In the logic circuit shown below, for which of the following combination(s) of inputs P and Q, the output Y will be 0?

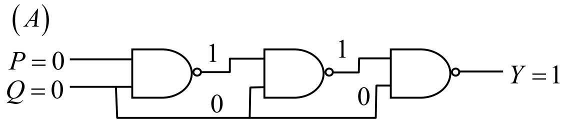


(A) $P=0, Q=0$ (B) $P=0, Q=1$ (C) $P=1, Q=0$ (D) $P=1, Q=1$

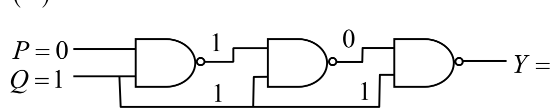
Ans.: (D)

Solution.:

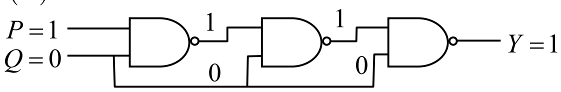
(A)



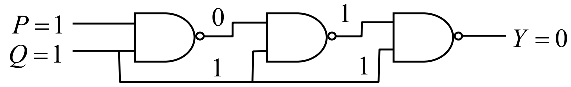
(B)



(C)



(D)



Q33. Which of these cubic lattice plane pairs is(are) perpendicular to each other?

(A) $(100),(010)$ (B) $(220),(001)$

(C) $(110),(010)$ (D) $(112),(220)$

Ans.: (A), (B)

Solution.: Two planes $(h_1k_1l_1)$ and $(h_2k_2l_2)$ are perpendicular to each other if the dot product of their normal vectors is zero: $h_1h_2 + k_1k_2 + l_1l_2 = 0$.

(A) $(100),(010)$: $1.0 + 0.1 + 0.0 = 0$

(B) $(220),(001)$: $2.0 + 2.0 + 0.1 = 0$

(C) $(110),(010)$: $1.0 + 1.1 + 0.0 = 1 \neq 0$

(D) $(112),(220)$: $1.2 + 1.2 + 2.0 = 4 \neq 0$

Q40. For a body centered cubic (bcc) system, the X-ray diffraction peaks are observed for the following $h^2 + k^2 + l^2$ value(s) [h, k and l are Miller indices]

(A) 3 (B) 4 (C) 5 (D) 7

Ans.: (B)

Solution.: For a body centered cubic (bcc) crystal structure, the condition for observing an X-ray diffraction peak is that the sum of Miller indices $h + k + l = even$.

The possible values of $h^2 + k^2 + l^2$ for allowed reflections are 2,4,6,8,10,12,12,16, and so on.

Section C: Q.41 - Q.50 Carry ONE mark each.

Q45. An ideal p-n junction diode (ideality factor $\eta = 1$) is operating in forward bias at room temperature (thermal energy = $26\,meV$). If the diode current is $26\,mA$ for an applied bias of $1.0V$, the dynamic resistance (r_{ac}) is ____________ Ω. (up to two decimal places)

Ans.: 0.95 to 1.05

Solution.: dynamic resistance $r_{ac} = \dfrac{26\eta}{I(mA)}\Omega = \dfrac{26\times 1}{26}\Omega = 1.00\Omega$

Q49. The ratio of the density of atoms between the (111) and (110) planes in a simple cubic (sc) lattice is ___________. (up to two decimal places)

Ans.: 0.82

Solution.: Planar density $(\text{PD}) = \dfrac{\text{Number of atoms lying in the plane}}{\text{Area of the plane within the unit cell}}$

Let lattice constant $= a$

Planar density of (110) plane in SC:

Atoms lying in (110) plane $=$ 4 corner atoms $\times$ contribution $1/4 = 1$ atom

Area $A_{110} = a \times a\sqrt{2} = a^2\sqrt{2} \Rightarrow PD_{110} = \dfrac{1}{a^2\sqrt{2}}$

Planar density of (111) plane in SC:

Atoms lying in (111) plane $=$ 3 corner atoms $\times$ contribution $1/6 = 1/2$ atom

Area $A_{111} = \dfrac{\sqrt{3}}{2}a^2 \Rightarrow PD_{111} = \dfrac{1/2}{\sqrt{3}/2a^2} = \dfrac{1}{\sqrt{3}a^2}$

Thus $\dfrac{PD_{111}}{PD_{110}} = \dfrac{1/\sqrt{3}a^2}{1/\sqrt{2}a^2} = \dfrac{\sqrt{2}}{\sqrt{3}} \approx 0.82$

Q50. The packing fraction for a two-dimensional hexagonal lattice having sides $2r$ with atoms of radii r placed at each vertex and at the center is ______________. (up to two decimal places)

Ans.: 0.91

Solution.:

Packing fraction for a two-dimensional hexagonal lattice (with a center atom)

Given: Regular hexagon of side $a = 2r$ and atoms of radii r placed at each vertex and at the center.

Number of atoms per unit cell

6 corner atoms, each shared by 3 hexagons $6 \times \dfrac{1}{3} = 2$ and 1 atom fully inside at the center.

$N = 2 + 1 = 3$ atoms.

Area occupied by atoms

Area of one atom (circle): $= \pi r^2$; Total atomic Area: $= 3\pi r^2$

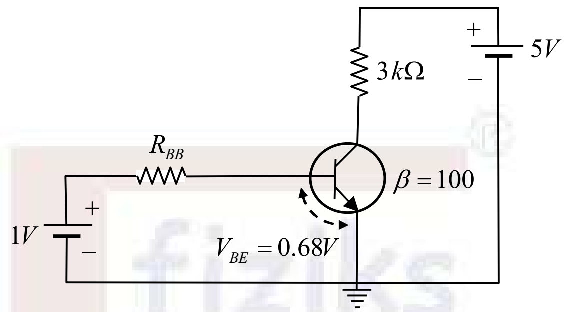
Area of hexagonal unit cell

Area of a regular hexagon of side a is $A_{hex} = \dfrac{3\sqrt{3}}{2}a^2 = \dfrac{3\sqrt{3}}{2}(2r)^2 = 6\sqrt{3}\,r^2$

Packing fraction $\eta = \dfrac{A_{atom}}{A_{hex}} = \dfrac{3\pi r^2}{6\sqrt{3}\,r^2} = \dfrac{\pi}{2\sqrt{3}} \approx \dfrac{3.141}{3.464} \approx 0.91$

Section C: Q.51-Q60 Carry TWO marks each.

Q51. A NPN bipolar junction transistor (BJT) is connected in common emitter (CE) configuration as shown in the circuit diagram below. The amplifier is operating in the saturation region. The collector-emitter saturation voltage $\left(V_{CE}^{sat}\right)$ is $0.2V$. The current gain $\beta = 100$. The maximum value of base resistance R_{BB} is ______________ $k\Omega$. (in integer)

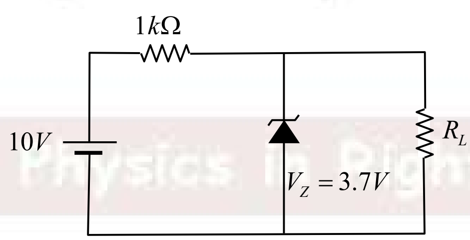


Ans.: 20

Solution.: $\because V_{CE}^{sat} = V_{CC} - I_C^{sat} R_C \Rightarrow I_C^{sat} = \dfrac{V_{CC} - V_{CE}^{sat}}{R_C} = \dfrac{5V - 0.2V}{3k\Omega} = 1.6\,mA$

$$R_{BB} = \frac{1V - 0.68V}{I_B} = \frac{0.32V}{I_C^{sat}/\beta} = \frac{0.32V}{1.6\,mA} \times 100 = 20\,k\Omega$$

Q52. For a Zener diode as shown in the circuit diagram below, the Zener voltage V_Z is $3.7V$. For a load resistance (R_L) of $1k\Omega$, a current I_1 flows through the load. If R_L is decreased to 500Ω, the current changes to I_2.



The ratio $\dfrac{I_2}{I_1}$ is ____________. (up to two decimal places)

Ans.: 1.78 to 1.82

Solution.: If $R_L = 1k\Omega$: open circuit voltage $= \dfrac{1k\Omega}{1k\Omega + 1k\Omega} \times 10V = 5V > 3.7V$, Zener ON.

So $I_1 = \dfrac{3.7\,V}{1k\Omega} = 3.7\,mA$.

If $R_L = 500\Omega = 0.5k\Omega$: open circuit voltage $= \frac{0.5k\Omega}{1k\Omega + 0.5k\Omega} \times 10V = 3.33V < 3.7V$, Zener OFF.

So $I_2 = \frac{3.33V}{0.5k\Omega} = 6.66mA \Rightarrow \frac{I_2}{I_1} = \frac{6.66mA}{3.7mA} = 1.8$

Q59. In an orthorhombic crystal, the lattice constants are 3.0Å, 3.2Å, and 4.0Å. The distance d_{101} between the successive (101) planes is ___________ Å . (up to one decimal place)

Ans.: 2.40

Solution.: For orthorhombic crystal $\frac{1}{d_{hkl}^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}$

$$\Rightarrow \frac{1}{d_{101}^2} = \frac{1^2}{3.0^2} + \frac{0^2}{3.2^2} + \frac{1^2}{4.0^2} = \frac{1}{9} + 0 + \frac{1}{16} = 0.1111 + 0.0625 = 0.1736$$

$$\Rightarrow d_{101} = \frac{1}{\sqrt{0.1736}} \approx 2.40\,\text{Å}$$

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com