



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2025
Complete Solution

Learn Physics in Right Way

Be Part of Disciplined Learning



Learn Physics in Right Way

JEST Physics-2025

Solution-Mathematical Methods

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q16. Consider the standard notation of discrete finite groups with Z_n corresponding to the rotation by $2\pi/n$ about a given axis, S_n corresponding to the permutation group of the set S having n elements, i.e. $S = \{1, 2, 3, \dots, n\}$, and the Dihedral group D_n corresponding to the reflection and rotation symmetries of a regular polygon with n number of sides. Which of the following is the smallest non-abelian group?

- (a) S_3 (b) Z_3 (c) S_4 (d) D_4

Ans.: (a)

Solution.: (a) S_3 : Symmetric group of 3 elements is of order $= 3! = 6$. It represents symmetries of an equilateral triangle. Doing reflection first then rotation is not equal to rotation first then reflection. It's a non-abelian.

(b) Z_3 : It's a cyclic group of order 3. All cyclic groups are abelian.

(c) S_4 : Symmetric group of 4 elements is of order $= 4! = 24$. It's a non-abelian but larger than S_3 .

(d) D_4 : Dihedral group of square of order $= 2n = 8$. Its non-abelian but order > 6 .

Note: No group of order less than 6 can be non-abelian.

Q17. Given the differential operator: $D \equiv \frac{d^2}{dx^2} + P \frac{d}{dx} + Q$, where P and Q are constants, what

is the eigenvalue corresponding to the eigenfunction $y = e^x$?

- (a) $(P+Q)$ (b) $(1+Q)$ (c) $(P+Q-1)$ (d) $(1+P+Q)$

Ans.: (d)

Solution.: $De^x \equiv \left(\frac{d^2}{dx^2} + P \frac{d}{dx} + Q \right) e^x = (1+P+Q)e^x$

Q24. The number of independent real numbers that parameterize any (3×3) Hermitian matrix is

- (a) 3 (b) 8 (c) 9 (d) 6

Ans.: (c)

Solution.: In Hermitian matrix diagonal elements must be real. So there are 3 real diagonal elements. Upper triangular part has 3 elements and each of diagonal element is complex so there is 2 real parameter per element. Total off diagonal real parameter is 6. Hence the number of independent real numbers that parameterize any (3×3) Hermitian matrix is 9.

Q25. Consider a 2×2 matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ which has eigenvalues $\lambda_1 = \frac{1+\sqrt{5}}{2}$ and $\lambda_2 = \frac{1-\sqrt{5}}{2}$.

For any natural number n which of the following is correct?

- (a) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$ (b) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} - \lambda_2^{n+1} \end{bmatrix}$
- (c) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n + \lambda_2^n \\ \lambda_1^n + \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$ (d) $A^n = \frac{1}{\sqrt{5}} \begin{bmatrix} \lambda_1^{n-1} + \lambda_2^{n-1} & \lambda_1^n - \lambda_2^n \\ \lambda_1^n - \lambda_2^n & \lambda_1^{n+1} + \lambda_2^{n+1} \end{bmatrix}$

Ans.: (a)

Solution.: Since A has distinct eigenvalues, it is diagonalizable:

$$A = PDP^{-1} \Rightarrow A^n = PD^nP^{-1}$$

So entries of A^n must be linear combination of λ_1^n and λ_2^n .

For this specific matrix, $A^n = \begin{bmatrix} F_{n-1} & F_n \\ F_n & F_{n+1} \end{bmatrix}$ where $F_n = \frac{\lambda_1^n - \lambda_2^n}{\sqrt{5}}$.

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q2. Evaluate $\vec{\nabla} \cdot (r^4 \vec{r})$, where $\vec{r}$ represents a three-dimensional position vector.

- (a) $4r^4$ (b) 0 (c) $5r^4$ (d) $7r^4$

Ans.: (d)

Solution.: $\vec{\nabla} \cdot (r^4 \vec{r}) = \vec{\nabla} \cdot (r^5 \hat{r}) = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \times r^5) = \frac{1}{r^2} (7r^6) = 7r^4$

Q6. Consider the group S_4 corresponding to the permutations of the set S having four elements, say $S = \{1, 2, 3, 4\}$. How many non-identity self-inverse (i.e. order 2) elements does S_4 have?

- (a) 8 (b) 12 (c) 9 (d) 6

Ans.: (c)

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q7. What is the value of the integral

$$I = \frac{3}{2\pi} \oint_C \frac{dz}{1+z^2},$$

where the contour C is a circle of radius 2 centered at the origin?

Ans.: 0

Solution.: For singular points $1+z^2=0 \Rightarrow z=\pm i$ is a simple pole and both lies inside the circle of radius 2 centered at the origin.

$$\text{Residue at } z = +i: a_{-1} = \lim_{z \rightarrow i} (z-i) \frac{1}{1+z^2} = \lim_{z \rightarrow i} (z-i) \frac{1}{(z-i)(z+i)} = \frac{1}{2i}$$

$$\text{Residue at } z = -i: a_{-1} = \lim_{z \rightarrow -i} (z+i) \frac{1}{1+z^2} = \lim_{z \rightarrow -i} (z+i) \frac{1}{(z-i)(z+i)} = -\frac{1}{2i}$$

$$\text{Sum of residue} = \frac{1}{2i} - \frac{1}{2i} = 0.$$

$$I = \frac{3}{2\pi} \oint_C \frac{dz}{1+z^2} = \frac{3}{2\pi} \times (2\pi i \times 0) = 0$$

Q9. A 3×3 matrix M satisfies $M^2 - 3M + 2I = 0$. Find the determinant of the matrix M if its trace is 6.

Ans.: 6

Solution.: If λ is any eigenvalue of M , then it satisfies $\lambda^2 - 3\lambda + 2 = 0$ $\lambda = 1, 2$

$$\therefore \lambda_1 + \lambda_2 + \lambda_3 = 6 \Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 2. \text{ Thus } \det M = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 2 \cdot 2 \cdot 2 = 8$$

Learn Physics in Right Way



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2025

Solution- Classical Mechanics

Learn Physics in Right Way
Solution- Classical Mechanics - JAM Physics-2022

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q1. For a relativistic point particle, the momentum is $\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - v^2/c^2}}$, where $\vec{v}$ is its velocity as measured by an inertial observer. Then the acceleration is in the same direction as the applied force

- (a) always
- (b) never
- (c) only when force is neither parallel nor perpendicular to the velocity
- (d) only when force is parallel or perpendicular to the velocity

Ans.: (d)

Solution.: According to Newton's second law in relativistic mechanics, force is the rate of change of relativistic momentum: $\vec{F} = \frac{d\vec{p}}{dt}$, $\vec{p} = \gamma m_0 \vec{v}$ where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

$$\text{So } \vec{F} = \gamma m_0 \frac{d\vec{v}}{dt} + \vec{v} m_0 \frac{d\gamma}{dt} = m_0 \left(\gamma \vec{a} + \vec{v} \frac{d\gamma}{dt} \right)$$

So force contains a component along acceleration and another component along velocity.

That is why, in relativistic mechanics, $\vec{F}$ and $\vec{a}$ do not generally point in the same direction.

- If the force is **parallel to velocity**, then the motion is along one straight line. Therefore, $\vec{a} \parallel \vec{v}$ and hence $\vec{F} \parallel \vec{a}$.
- If the force is **perpendicular to velocity** then the speed does not change; only the **direction of velocity** changes. Thus $\vec{a} \perp \vec{v}$, in this case also $\vec{a} \parallel \vec{F}$.

Q2. A boat is floating in a pond with still water. There is a heavy stone on the boat. If the stone is dropped gently into the water, what happens to the water level in the pond after the stone sinks completely?

- (a) The level goes down
- (b) The level goes up
- (c) The level goes up or down depending on the size of the stone
- (d) The level remains the same

Ans.: (a)

Solution.: Initially, the stone is **inside the boat**. Suppose stone mass is m , density is ρ_s , and water density is ρ_w .

When the stone is in the boat

The boat must displace water whose weight equals the weight of the boat + stone.

The additional volume of water displaced because of stone is $V_1 = \frac{mg}{\rho_w g} = \frac{m}{\rho_w}$

After the stone sinks

The stone now displaces water equal to its own volume: $V_2 = \frac{m}{\rho_s}$

Since the stone is described as **heavy**, we take $\rho_s > \rho_w$.

Thus $\frac{m}{\rho_s} < \frac{m}{\rho_w} \Rightarrow V_2 < V_1$. Thus, **less water is displaced**, causing the water level to fall.

Floating object $\rightarrow$ displaces water according to its **weight**.

Sinking object $\rightarrow$ displaces water according to its **volume**.

Q4. Suppose the mass of the Sun is reduced to half of its original value very slowly, e.g., over a billion years, what will be the effect of this on the Earth's orbit?

- (a) The earth flies away
- (b) Orbit remains closed but not elliptical
- (c) Remains elliptical with the same mean radius
- (d) Remains elliptical, but the mean radius changes

Ans.: (d)

Solution.: The Sun's mass is reduced **very slowly** from M to $M/2$.

Because the change is slow, the Earth's orbit adjusts continuously rather than suddenly.

For a slowly changing central mass, $\frac{r}{M} = \text{constant}$, for the orbital scale. Hence, $r \propto \frac{1}{M}$.

$$\therefore M_f = \frac{M_i}{2} \Rightarrow r_f = 2r_i.$$

The orbit therefore expands but remains an ellipse. So the **mean radius changes**.

Q6. A particle is moving under the force field given by $\vec{F} = k\vec{r}$, where k is a positive constant.

The difference in work done (in arbitrary units) if the particle moves from point $A(-1,0,0)$ to point $B(1,0,0)$ following semi-circular paths in the clockwise and anti-clockwise directions on the $X-Y$ plane will be

- (a) $2\pi k$
- (b) 0
- (c) πk
- (d) $\frac{1}{2}\pi k$

Ans.: (b)

Solution.: $\because \vec{F} = k\vec{r} \Rightarrow \vec{\nabla} \times \vec{F} = 0$. The force field is **conservative**.

Work done along any path between two fixed points depends **only on the initial and final points**, not on the path taken.

So the work done on the clockwise path and anticlockwise path is the same. Their difference is zero.

Q20. The Lagrangian of a two-dimensional system is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - k(x^2 + y^2)^{-1.5}$$

Which of the following is/are the constant(s) of motion?

- (a) Energy only (b) None of the others
(c) Angular momentum only (d) Energy and angular momentum

Ans.: (d)

Solution.: $\because L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - k(x^2 + y^2)^{-1.5}$

Notice that L has **no explicit time dependence**. Therefore, energy is conserved.

$$E = \text{constant}$$

Also, the potential depends only on $x^2 + y^2 = r^2$. So it depends only on r , not on the angular coordinate ϕ . Thus, the system has **rotational symmetry**. By conservation of angular momentum,

$$L_z = \text{constant}$$

Therefore both energy and angular momentum are constants of motion.

Q22. A slide calipers instrument has smallest main scale division of 0.4 mm and 40 vernier divisions match with 38 main scale divisions. The vernier constant of this instrument is

- (a) 0.05 mm (b) 0.01 mm (c) 0.02 mm (d) 0.1 mm

Ans.: (c)

Solution.: $1\text{ MSD} = 0.4\text{ mm}$, $40\text{ VSD} = 38\text{ MSD}$

$$\Rightarrow \text{VSD} = \frac{38}{40} \text{ MSD} = \frac{38}{40} \times 0.4\text{ mm} = 0.38\text{ mm}$$

Vernier constant / least count:

$$LC = 1\text{ MSD} - 1\text{ VSD} = 0.4\text{ mm} - 0.38\text{ mm} = 0.02\text{ mm}$$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q7. Consider a circular disk of radius R and mass M in the $X-Y$ plane, with a surface mass density $\sigma(r) = \sigma_0 e^{-r^2/a^2}$, where r is the distance from the center of the disk. What is the moment of inertia around the Z -axis through the center of the disk? [consider $R \gg a$]

- (a) $\frac{1}{2}Ma^2$ (b) $6Ma^2$ (c) Ma^2 (d) $\frac{1}{3}Ma^2$

Ans.: (c)

Solution.: Total mass $M = \int \sigma(r) dA = \int_0^\infty \sigma_0 e^{-r^2/a^2} (2\pi r dr) = 2\pi\sigma_0 \int_0^\infty r e^{-r^2/a^2} dr = 2\pi\sigma_0 \times \frac{a^2}{2} = \pi\sigma_0 a^2$

For a ring, moment of inertia $dI = r^2 dm = r^2 \times \sigma(r) dA = 2\pi\sigma_0 r^3 e^{-r^2/a^2} dr$

$$\Rightarrow I = 2\pi\sigma_0 \int_0^\infty r^3 e^{-r^2/a^2} dr = 2\pi\sigma_0 \times \frac{a^4}{2} = \pi\sigma_0 a^4 \Rightarrow I = Ma^2 \quad \because M = \pi\sigma_0 a^2$$

Q14. A particle is moving with velocity $v_x = v_y = v_z = c/2$ in frame S . The ratio of velocity component v_y to the velocity component v'_y , as measured in frame S' moving with velocity $c/2$ with respect to frame S along the common x direction is

- (a) $\cos(\pi/3)$ (b) $\sin(\pi/6)$ (c) $\cos(\pi/6)$ (d) $\sin(\pi/3)$

Ans.: (c)

Solution.: Given in frame S : $v_x = v_y = v_z = \frac{c}{2}$

Frame S' moves with $u = \frac{c}{2}$ along the x -direction.

For the transverse velocity component, $v'_y = \frac{v_y}{\gamma \left(1 - \frac{uv_x}{c^2}\right)}$, $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}}$

$$v'_y = \frac{v_y}{\gamma \left(1 - \frac{uv_x}{c^2}\right)} = \frac{c/2}{\left(\frac{2}{\sqrt{3}}\right) \left(1 - \frac{(c/2)(c/2)}{c^2}\right)} = \frac{\sqrt{3} c/2}{2 \cdot 3/4} = \frac{c}{\sqrt{3}}$$

$$\Rightarrow \frac{v_y}{v'_y} = \frac{c/2}{c/\sqrt{3}} = \frac{\sqrt{3}}{2} = \cos(\pi/6)$$



Learn Physics in Right Way

JEST Physics-2025

Solution- Electromagnetic Theory

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q7. For a plane electromagnetic wave propagating with wave vector $\vec{k}$ in a homogeneous and isotropic medium, which of the following holds?

- (a) $\vec{E} \times \vec{B} = \vec{0}$ (b) None of the others (c) $\vec{E} \cdot \vec{B} = 0$ (d) $\vec{k} \cdot (\vec{E} \times \vec{B}) = 0$

Ans.: (c)

Q11. Consider two identical charged balls, each of mass m and charge q . One of them is initially held fixed on a frictionless insulating horizontal surface and the other is carefully placed above the first one at a height h from the surface, such that the gravitational force on it is balanced by the Coulomb repulsion. The upper ball is now shifted horizontally by a distance d ($d \ll h$) to the right and then both the balls are released. Which way will the balls move immediately after this?

- (a) ball on the surface moves towards right and ball above moves upwards
(b) ball on the surface moves towards left, ball above moves downwards
(c) both balls oscillate around their original positions
(d) both balls remain static at their new positions

Ans.: (b)

Solution.: Initially Coulomb repulsion balances gravity on the upper ball i.e. $mg = \frac{1}{4\pi\epsilon_0} \frac{q^2}{h^2}$. So, there is perfect vertical equilibrium.

When upper ball is shifted slightly to the right then line joining the two charges is tilted and distance increases. Now Coulomb force on upper ball is towards upwards and rightward but upward coulomb force is less than mg so ball above moves downwards.

Coulomb force on lower ball acts along the tilted line but away from the upper ball so force has horizontal component towards left. So, ball on the surface moves towards left.

Q13. The electric dipole moment of a charge distribution is independent of the choice of the origin of coordinates only if

- (a) the charge distribution is discrete (b) the total charge adds up to zero
(c) there is no magnetic field present (d) the charge distribution is not time-dependent

Ans.: (b)

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q1. A circular loop of radius a , carrying a current I in an anticlockwise direction (when seen downwards from the positive Z axis), is placed on the $X - Y$ plane centered at the origin. What is the magnetic field on the $X - Y$ plane at $r \gg a$?

- (a) $\frac{\mu_0 I}{4} \frac{a^2}{r^3}$ in the negative Z direction (b) $\frac{\mu_0 I}{4} \frac{a^2}{r^3} \hat{r}$
- (c) 0 (d) $\frac{\mu_0 I}{4\pi} \frac{a^2}{r^3}$ in the positive Z direction

Ans.: (a)

Solution.: $\vec{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$

On the $X - Y$ plane at $r \gg a$, $\theta = \frac{\pi}{2}$: $\vec{B} = \frac{\mu_0 m}{4\pi r^3} \hat{\theta} = \frac{\mu_0 (I \times \pi a^2)}{4\pi r^3} (-\hat{z}) = \frac{\mu_0 I a^2}{4r^3} (-\hat{z})$

Q11. The time averaged electrostatic potential of a neutral H-atom is given by

$$\Phi(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2} \right)$$

The classical charge distribution corresponding to this is

- (a) $-\frac{q}{8\pi} \alpha^3 e^{-\alpha r}$ (b) $-\frac{q}{8\pi} \alpha^3 e^{-\alpha r} + q\delta^3(\vec{r})$
- (c) $\frac{q}{8\pi} \alpha^3 e^{-\alpha r} \left(1 + \frac{\alpha r}{2} \right) - q\delta^3(\vec{r})$ (d) $q e^{-\alpha r} \left(1 + \frac{\alpha r}{2} \right)$

Ans.: (b)

Solution.:

$$\vec{E} = -\vec{\nabla}\phi = -\frac{\partial\phi}{\partial r} \hat{r} = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial r} \left[\frac{1}{r} \left(1 + \frac{\alpha r}{2} \right) \exp(-\alpha r) \right] \hat{r} = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial r} \left[\left(\frac{1}{r} + \frac{\alpha}{2} \right) \exp(-\alpha r) \right] \hat{r}$$

$$\Rightarrow \vec{E} = -\frac{q}{4\pi\epsilon_0} \left[\left(\frac{1}{r} + \frac{\alpha}{2} \right) (-\alpha) \exp(-\alpha r) + \exp(-\alpha r) \left(-\frac{1}{r^2} \right) \right] \hat{r}$$

$$\Rightarrow \vec{E} = \frac{q}{4\pi\epsilon_0} \left[\exp(-\alpha r) \left(\frac{\alpha}{r} + \frac{\alpha^2}{2} \right) \hat{r} + \exp(-\alpha r) \frac{\hat{r}}{r^2} \right]$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left\{ r^2 \times \exp(-\alpha r) \left(\frac{\alpha}{r} + \frac{\alpha^2}{2} \right) \right\} + \vec{\nabla} \cdot \left\{ \exp(-\alpha r) \frac{\hat{r}}{r^2} \right\} \right]$$

$$\begin{aligned}
\Rightarrow \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left\{ \exp(-\alpha r) \left(\alpha r + \frac{\alpha^2 r^2}{2} \right) \right\} + \exp(-\alpha r) \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) + \frac{\hat{r}}{r^2} \cdot \vec{\nabla} \{ \exp(-\alpha r) \} \right] \\
\Rightarrow \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \left[\frac{\exp(-\alpha r)}{r^2} \left\{ (\alpha + \alpha^2 r) - \alpha \left(\alpha r + \frac{\alpha^2 r^2}{2} \right) \right\} + \exp(-\alpha r) \times 4\pi\delta(r) + \frac{\hat{r}}{r^2} \cdot \{ -\alpha \exp(-\alpha r) \hat{r} \} \right] \\
\Rightarrow \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \exp(-\alpha r) \left[\left(\frac{\alpha}{r^2} + \frac{\alpha^2}{r} \right) - \left(\frac{\alpha^2}{r} + \frac{\alpha^3}{2} \right) + 4\pi\delta(r) - \frac{\alpha}{r^2} \right] \\
\Rightarrow \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \exp(-\alpha r) \left[-\frac{\alpha^3}{2} + 4\pi\delta(r) \right] = \frac{1}{\epsilon_0} \exp(-\alpha r) \left[q\delta(r) - \frac{q}{4\pi} \frac{\alpha^3}{2} \right] \\
\Rightarrow \rho &= \epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \left[q\delta(r) - \frac{q}{4\pi} \frac{\alpha^3}{2} \right] \exp(-\alpha r) \\
\Rightarrow \rho &= q\delta(r) - \frac{q}{8\pi} \alpha^3 \exp(-\alpha r)
\end{aligned}$$

Section C (NAT)**Correct answer: +3, wrong answer: 0.**

(No Question)



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2025

Solution- Quantum Mechanics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q3. Consider a quantum system that is evolved sequentially with a finite sequence of Hermitian Hamiltonians $\{H_0, H_1, \dots, H_n\}$. The full evolution operator is written as:

$$O = U_n U_{n-1} \dots U_1 U_0 = e^{-iH}, \text{ with } U_j = e^{-iH_j} \text{ and, } j = 0, 1, \dots, n$$

Then H is

- (a) undefined (b) a Hermitian operator
(c) a unitary operator (d) None of the others

Ans.: (b)

Solution.: Check Unitary Nature of U_j :

Since each H_j is a Hermitian operator ($H_j^\dagger = H_j$), each individual time-evolution operator

$$U_j = e^{-iH_j} \text{ is unitary: } U_j^\dagger U_j = e^{iH_j} e^{-iH_j} = I$$

Check Unitary Nature of Full Operator O :

$$O^\dagger O = (U_n \dots U_0)^\dagger (U_n \dots U_0) = U_0^\dagger \dots U_n^\dagger U_n \dots U_0 = I$$

Therefore, O is a **unitary operator**.

Since $O = e^{-iH}$ is a unitary operator so H is Hermitian.

Q8. A quantum mechanical system is spanned by the eigenstates $|a_1\rangle$ and $|a_2\rangle$ of a Hermitian operator A with eigenvalues a_1 and a_2 respectively. If there is no degeneracy, what is the expectation value of the operator $(A - a_1)(A - a_2)$ in the state $\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}$?

- (a) 1 (b) 0 (c) $(a_2 - a_1)(a_1 - a_2)$ (d) $\frac{(a_2 - a_1)(a_1 - a_2)}{2}$

Ans.: (b)

Solution.: By definition of eigenstates:

$$A|a_1\rangle = a_1|a_1\rangle \Rightarrow (A - a_1)|a_1\rangle = 0 \text{ and } A|a_2\rangle = a_2|a_2\rangle \Rightarrow (A - a_2)|a_2\rangle = 0$$

$$(A - a_1)(A - a_2)|\psi\rangle = (A - a_1)(A - a_2)\left(\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}\right) = 0 \quad \text{where } |\psi\rangle = \frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}}$$

$$\therefore (A - a_1)(A - a_2)|a_1\rangle = (A - a_2)(A - a_1)|a_1\rangle = 0 \text{ and}$$

$$(A - a_1)(A - a_2)|a_2\rangle = (A - a_1)(A - a_2)|a_2\rangle = 0$$

$$\text{So } \langle (A - a_1)(A - a_2) \rangle = \langle \psi | (A - a_1)(A - a_2) | \psi \rangle = \langle \psi | 0 \rangle = 0$$

Q9. Three observers successively measure the spin of a given proton along z -axis, x -axis and again z -axis, respectively. The first observer finds the projection to be $+\frac{1}{2}$. Assuming no other factors, what is the probability that the third observer finds the spin projection to be $-\frac{1}{2}$?

- (a) 0 (b) 1 (c) 0.5 (d) None of the others

Ans.: (c)

Solution.: Initial Measurement (1st Observer along z -axis):

Finds spin projection $+\frac{1}{2}$, placing the proton in state $|+z\rangle$.

Second Measurement (2nd Observer along x -axis):

Express $|+z\rangle$ in terms of x -axis spin eigenstates $|+x\rangle$ and $|-x\rangle$:

$$|+z\rangle = \frac{1}{\sqrt{2}}|+x\rangle + \frac{1}{\sqrt{2}}|-x\rangle$$

Regardless of whether the second measurement yields $+x$ or $-x$, the system collapses into one of these states with a probability of 50%.

Third Measurement (3rd Observer along z -axis):

Both x -eigenstates are equal-weight superpositions of z -eigenstates:

$$|+x\rangle = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle \quad \text{and} \quad |-x\rangle = \frac{1}{\sqrt{2}}|+z\rangle - \frac{1}{\sqrt{2}}|-z\rangle$$

The probability of measuring spin projection $-\frac{1}{2}$ (i.e., state $|-z\rangle$) from either $|+x\rangle$ or $|-x\rangle$ is:

$$P(-z) = \left| \frac{1}{\sqrt{2}} \right|^2 = \frac{1}{2} = 0.5$$

Q14. Consider the time-independent Schrodinger equation with a real potential and suppose $\psi(x)$ is a solution of this equation. Which of the following is true?

- (a) ψ^* is a solution of the same equation
(b) ψ^* is never a solution of the same equation
(c) is a solution of the same equation only if the potential is symmetric about $x = 0$
(d) is a solution of the same equation only if the potential vanishes at infinity

Ans.: (a)

Q19. Match the following statements

A1	Photoelectric effect	B1	Involves loosely bound or free electrons
A2	Compton scattering	B2	Inverse photoelectric effect
A3	Pair production	B3	Needs a minimum of 1.02 MeV of energy for the incident radiation
A4	Bremsstrahlung	B4	Involves bound electrons and depends on the specifics of the material

(a) A1-B3, A2-B4, A3-B2, A4-B1

(b) A1-B3, A2-B2, A3-B1, A4-B4

(c) A1-B4, A2-B1, A3-B3, A4-B2

(d) A1-B4, A2-B3, A3-B2, A4-B1

Ans.: (c)

Solution.:

A1	Photoelectric effect	B4	Involves bound electrons and depends on the specifics of the material
A2	Compton scattering	B1	Involves loosely bound or free electrons
A3	Pair production	B3	Needs a minimum of 1.02 MeV of energy for the incident radiation
A4	Bremsstrahlung	B2	Inverse photoelectric effect

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q5. For a particle in a one-dimensional box of width L , the uncertainty Δp in momentum in the n -th eigenstate of energy for large n is

(a) $\frac{n\pi\hbar}{L}$

(b) $\frac{\hbar}{n\pi L}$

(c) $\frac{2n\hbar}{L}$

(d) $\frac{2n\pi\hbar}{L}$

Ans.: (a)

Solution.: Wavefunction for 1D Box: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$. By symmetry $\langle p \rangle = 0$.

$$E_n = \frac{\langle p^2 \rangle}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \Rightarrow \langle p^2 \rangle = \frac{n^2 \pi^2 \hbar^2}{L^2}$$

Uncertainty Δp in momentum

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{n^2 \pi^2 \hbar^2}{L^2} - 0} = \frac{n\pi\hbar}{L}$$

Q15. For a one-dimensional simple harmonic oscillator with mass m and angular frequency ω , consider a perturbation λx^4 in the Hamiltonian ($\lambda > 0$). What is the lowest order correction to the ground state energy?

[The position operator expressed in terms of the raising and lowering operators is

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger)$$

- (a) $\frac{3\lambda}{2} \left(\frac{\hbar}{m\omega} \right)^2$ (b) $\frac{5\lambda}{4} \left(\frac{\hbar}{m\omega} \right)^2$ (c) $\frac{3\lambda}{4} \left(\frac{\hbar}{m\omega} \right)^2$ (d) $\frac{5\lambda}{2} \left(\frac{\hbar}{m\omega} \right)^2$

Ans.: (c)

Solution.: First-Order Energy Correction Formula:

$$E_0^{(1)} = \langle 0 | H' | 0 \rangle = \lambda \langle 0 | x^4 | 0 \rangle \quad \because H' = \lambda x^4$$

$$\because \hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a} + \hat{a}^\dagger) = \alpha (\hat{a} + \hat{a}^\dagger) \quad \text{where } \alpha = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\Rightarrow E_0^{(1)} = \lambda \langle 0 | x^4 | 0 \rangle = \alpha^4 \lambda \langle 0 | (\hat{a} + \hat{a}^\dagger)^4 | 0 \rangle$$

The annihilation operator $\hat{a}$ and creation operator $\hat{a}^\dagger$ act on a state $|n\rangle$ according to:

- $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
- $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$
- $\hat{a}|0\rangle = 0$ (annihilating the ground state yields zero)

Because we are taking the inner product with the ground state $\langle 0 | \dots | 0 \rangle$, any product of 4 operators acting on $|0\rangle$ **must result in the state $|0\rangle$** to give a non-zero overlap. If it results in $|1\rangle, |2\rangle$ etc., the inner product with $\langle 0 |$ will be zero due to orthogonality ($\langle 0 | n \rangle = 0$ for $n \neq 0$).

For 4 operators, the valid sequence acting on $|0\rangle$ is:

- $\hat{a}\hat{a}\hat{a}^\dagger\hat{a}^\dagger|0\rangle = \hat{a}\hat{a}(\sqrt{2}|2\rangle) = 2|0\rangle$
- $\hat{a}\hat{a}^\dagger\hat{a}\hat{a}^\dagger|0\rangle = \hat{a}\hat{a}^\dagger|0\rangle = |0\rangle$

$$\text{Thus } E_0^{(1)} = \lambda \langle 0 | x^4 | 0 \rangle = \alpha^4 \lambda \langle 0 | (\hat{a} + \hat{a}^\dagger)^4 | 0 \rangle = \alpha^4 \lambda (2+1) = 3 \left(\sqrt{\frac{\hbar}{2m\omega}} \right)^4 \lambda = \frac{3\lambda}{4} \left(\frac{\hbar}{m\omega} \right)^2$$

$$\Rightarrow E_0^{(1)} = \frac{3\lambda}{4} \left(\frac{\hbar}{m\omega} \right)^2$$

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q1. Given the mass of the proton $m_p \approx 1836 m_e$ and mass of the deuteron $m_d \approx 3670 m_e$, where m_e is the electron mass, find the fractional shift (in parts per million, to the nearest integer) of the ground state energy of the deuterium atom as compared to H-atom.

Ans.: 272

Solution.: Ground state energy of hydrogen-like atom is proportional to reduced mass:

$$E \propto \mu = \frac{m_e M}{m_e + M}$$

$$\text{For Hydrogen: } \mu_H = \frac{m_e m_p}{m_e + m_p} = \frac{1836}{1837} m_e \quad \because m_p \approx 1836 m_e$$

$$\text{For Deuterium: } \mu_D = \frac{m_e m_d}{m_e + m_d} = \frac{3670}{3671} m_e \quad \because m_d \approx 3670 m_e$$

Fractional shift:

$$\frac{\Delta E}{E_H} = \frac{E_D - E_H}{E_H} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{3670}{3671} \times \frac{1837}{1836} - 1 \approx 2.721 \times 10^{-4}$$

$$\text{In parts per million: } 2.721 \times 10^{-4} \times 10^6 = 272 \text{ ppm}$$

Q3. The average lifetime of a muon in its rest frame is 2200 ns . What will be the average distance (in meters, to the nearest integer) travelled by it, which created with a velocity of $\frac{1}{3}c$, before it decays? Here c is the speed of light.

Ans.: 233

Solution.: Muon proper lifetime: $\tau_0 = 2200 \text{ ns} = 2.2 \times 10^{-6} \text{ s}$, velocity $v = \frac{c}{3}$

$$\text{Lorentz factor: } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 1/9}} = \frac{3}{\sqrt{8}} \approx 1.06066$$

Lab lifetime:

$$\tau = \gamma \tau_0 = 1.06066 \times 2.2 \times 10^{-6} \text{ s} = 2.333 \times 10^{-6} \text{ s}$$

Distance:

$$d = v \tau = \frac{c}{3} \times 2.333 \times 10^{-6} = \frac{3 \times 10^8}{3} \times 2.333 \times 10^{-6} \approx 233.3 \text{ m}$$

Q8. Suppose the wave function of a free particle in one dimension obeys $\frac{d^2\psi}{dx^2} = -4\psi$ in units where $\hbar = 1$. What is the magnitude of the momentum of the particle?

Ans.: 2

Solution.: Given $\frac{d^2\psi}{dx^2} = -4\psi$

For Plane Wave $\psi = e^{ikx}$: $\frac{d^2\psi}{dx^2} = -k^2\psi = -4\psi \Rightarrow k^2 = 4$

$$\Rightarrow k = 2$$



Learn Physics in Right Way



Learn Physics in Right Way

JEST Physics-2025

Solution- Thermodynamics and Statistical Mechanics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q5. If the lattice contribution to the C_V of a solid crystal at temperature $2K$ is found to be $0.5 mJ \text{ mole}^{-1} K^{-1}$, what will be the corresponding contribution at temperature $4K$?

- (a) $2 mJ \text{ mole}^{-1} K^{-1}$ (b) $1 mJ \text{ mole}^{-1} K^{-1}$ (c) $4 mJ \text{ mole}^{-1} K^{-1}$ (d) $8 mJ \text{ mole}^{-1} K^{-1}$

Ans.: (c)

Solution.: At low temperatures, the lattice specific heat of a solid follows the Debye Law i.e.

$$C_V \propto T^3. \quad \frac{C_V(4K)}{C_V(2K)} = \left(\frac{4}{2}\right)^3 = 8 \Rightarrow C_V(4K) = 8 \times C_V(2K) = 8 \times 0.5 = 4 mJ \text{ mole}^{-1} K^{-1}$$

Q10. One mole of ideal gas with a constant C_V undergoes a reversible adiabatic expansion.

Which one of the following equations is valid? $\left[\gamma = \frac{C_P}{C_V} \text{ for the gas} \right]$

- (a) $P^{1-\gamma} T^\gamma = \text{constant}$ (b) $V^\gamma T = \text{constant}$
(c) $P^{\gamma-1} T^\gamma = \text{constant}$ (d) $VT^\gamma = \text{constant}$

Ans.: (a)

Solution.: $PV^\gamma = C$ and $PV = RT$. Then $P\left(\frac{RT}{P}\right)^\gamma = C \Rightarrow P^{1-\gamma} T^\gamma = \text{constant}$.

Q15. Calculate the partition function for two indistinguishable bosonic particles at a temperature

T , which can be distributed in two single-particle energy levels ϵ_1 and ϵ_2 . Consider $\beta = \frac{1}{k_B T}$.

- (a) $(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2})^2$ (b) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$
(c) $e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-2\beta(\epsilon_1+\epsilon_2)}$ (d) $\frac{1}{2!}(e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2})^2$

Ans.: (b)

Solution.: For two identical bosons, the allowed many-body states are:

Both particles in the energy levels ϵ_1 , Both particles in the energy levels ϵ_2 .

One particle in each energy levels ϵ_1 and ϵ_2 .

The partition function is $Z = e^{-2\beta\epsilon_1} + e^{-2\beta\epsilon_2} + e^{-\beta(\epsilon_1+\epsilon_2)}$.

Q21. Which of the following thermodynamic variables is not a function of state for an ideal gas?

- (a) Entropy (b) None of the others (c) Temperature (d) Pressure

Ans.: (b)

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q3. The volume of a nucleus, treated as a Fermi gas in three-dimensional space, is proportional to the number of fermions presents in it. If the total number of fermions is changed from N to $2N$, the total energy of the system will

- (a) be doubled (b) be half of its original value
(c) remain the same (d) be 4 times its original value

Ans.: (a)

Solution.:

Since volume $V \propto N$ so $\frac{V}{N} = \text{constant}$.

Total energy of the system $E = \frac{3}{5} N \varepsilon_F$ and $\varepsilon_F \propto \left(\frac{N}{V}\right)^{2/3} = \text{constant}$.

Thus $E \propto N$ so $\frac{E(2N)}{E(N)} = 2$.

Q4. Consider a two-dimensional Fermi gas at $0K$ with Fermi energy ε_F . The average energy per particle of this gas is

- (a) $\frac{3\varepsilon_F}{5}$ (b) $\frac{\varepsilon_F}{4}$ (c) $\frac{\varepsilon_F}{2}$ (d) $\frac{\varepsilon_F}{3}$

Ans.: (c)

Solution.:

For a two-dimensional non-relativistic Fermi gas (with spin degeneracy $g = 2$) the average

energy per particle is $\frac{E}{N} = \frac{1}{2} \varepsilon_F$.

Q12. Given an isolated thermodynamic system with a total energy E , total volume V and total number of particles N , then condition for stable thermal equilibrium, in terms of its entropy S under small changes ΔE and ΔV , is given by

- (a) $S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) + 2S(E, V, N) < 0$
(b) $S(E + \Delta E, V + \Delta V, N) - S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$
(c) $-S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$
(d) $S(E + \Delta E, V + \Delta V, N) + S(E - \Delta E, V - \Delta V, N) - 2S(E, V, N) < 0$

Ans.: (d)

Solution.:

Taylor Expansion

$$S(E + \Delta E, V + \Delta V, N) \approx S(E, V) + \frac{\partial S}{\partial E} \Delta E + \frac{\partial S}{\partial V} \Delta V \\ + \frac{1}{2} \left[\frac{\partial^2 S}{\partial E^2} (\Delta E)^2 + \frac{\partial^2 S}{\partial E \partial V} (\Delta V)^2 + 2 \frac{\partial^2 S}{\partial E \partial V} \Delta E \Delta V \right] + \dots$$

$$S(E - \Delta E, V - \Delta V, N) \approx S(E, V) - \frac{\partial S}{\partial E} \Delta E - \frac{\partial S}{\partial V} \Delta V \\ + \frac{1}{2} \left[\frac{\partial^2 S}{\partial E^2} (\Delta E)^2 + \frac{\partial^2 S}{\partial E \partial V} (\Delta V)^2 + 2 \frac{\partial^2 S}{\partial E \partial V} \Delta E \Delta V \right] + \dots$$

Adding above two expression gives

$$S(E + \Delta E, V + \Delta V) + S(E - \Delta E, V - \Delta V) = 2S(E, V) + \left[\frac{\partial^2 S}{\partial E^2} (\Delta E)^2 + \frac{\partial^2 S}{\partial E \partial V} (\Delta V)^2 + 2 \frac{\partial^2 S}{\partial E \partial V} \Delta E \Delta V \right]$$

$$S(E + \Delta E, V + \Delta V) + S(E - \Delta E, V - \Delta V) - 2S(E, V) = (\text{quadratic form in } \Delta E, \Delta V)$$

For S to be a maximum, this quadratic form must be negative for all non-zero $(\Delta E, \Delta V)$. Thus

$$S(E + \Delta E, V + \Delta V) + S(E - \Delta E, V - \Delta V) - 2S(E, V) < 0$$

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q5. A current of $10A$ is maintained for $1s$ in a resistor of resistance 25Ω , which is thermally insulated. The initial temperature of the resistor is $23^\circ C$. The resistor has a mass of $10gm$ and a specific heat of $836Jkg^{-1}K^{-1}$. What is the entropy change of the resistor, rounding off to the nearest whole number in units of JK^{-1} ?

Ans.: 6

Solution.:

Energy dissipated in the resistor $E = I^2 R t = (10)^2 \times 25 \times 1 = 2500J$. Due to this energy resistor is heated.

$$\text{So } Q = mc\Delta T = 2500J \Rightarrow \Delta T = \frac{2500}{0.010 \times 836} = 299.043K.$$

$$\text{So, final temperature } T_f = T_i + \Delta T = 296.15 + 299.043 = 595.193K$$

$$\text{So, } \Delta S = \int_{T_i}^{T_f} \frac{mc dT}{T} = mc \ln \frac{T_f}{T_i} = 8.36 \ln \frac{595.193}{296.15} = 8.36 \ln(2.0097) = 8.36 \times 0.698 = 5.8J/K$$

$$\Rightarrow \Delta S \approx 6J/K$$

Q10. A heat engine works between a high temperature source and a sink at $27^\circ C$. If the maximum efficiency possible for it to achieve is 50%, what is the temperature of the source in $^\circ C$?

Ans.: 327

$$\text{Solution.: } \eta = 1 - \frac{T_l}{T_h} \Rightarrow 0.5 = 1 - \frac{300}{T_h} \Rightarrow \frac{300}{T_h} = 0.5 \Rightarrow T_h = 600K \Rightarrow T_h = 600 - 273 = 327^\circ C$$

Learn Physics in Right Way



Learn Physics in Right Way

JEST Physics-2025
Solution- Electronics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q23. A capacitor with capacitance C is connected in series with a resistor of resistance R and an ideal DC source with voltage V_s . At one instant during the charging of the capacitor if the resistor is replaced by a wire of zero resistance, which of the following statements is true?

- (a) None of the others is true
- (b) The voltage across the capacitor will increase slowly
- (c) The voltage across the capacitor will drop immediately to zero
- (d) The capacitor immediately attains the source voltage V_s

Ans.: (d)

Solution.: $v(t) = V_s (1 - e^{-t/RC})$ so time constant $\tau = RC$.

When $R = 0$, $\tau = 0$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q10. A silicon $p-n$ junction diode operates at 27°C . The current I is doubled when the forward bias is increased. The increase in the forward bias is closest to:

[Assume $I \gg I_s$, where I_s is the reverse saturation current and the emission coefficient $\eta_{Si} = 2$]

- (a) 18 mV
- (b) 36 mV
- (c) 54 mV
- (d) 72 mV

Ans.: (b)

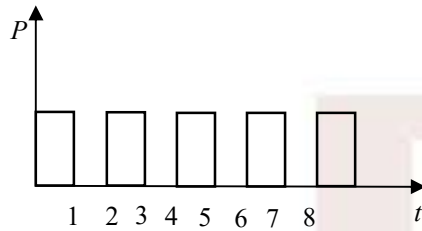
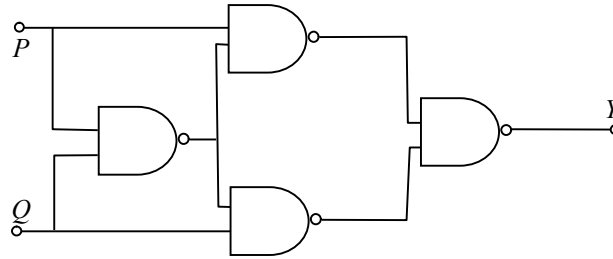
Solution.:

$$\text{Diode current } I = I_s \left(e^{\frac{qV}{\eta k_B T}} - 1 \right) \approx I_s e^{\frac{qV}{\eta k_B T}} \quad \because I \gg I_s$$

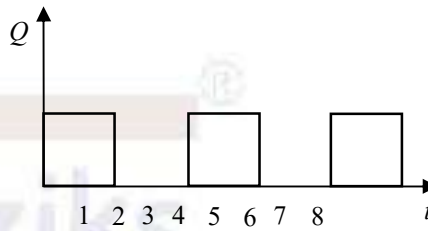
$$I_1 = I_s e^{\frac{qV_1}{\eta k_B T}}, \quad I_2 = I_s e^{\frac{q(V_1 + \Delta V)}{\eta k_B T}} = 2I_1 \Rightarrow I_s e^{\frac{q(V_1 + \Delta V)}{\eta k_B T}} = 2I_s e^{\frac{qV_1}{\eta k_B T}} \Rightarrow e^{\frac{q\Delta V}{\eta k_B T}} = 2 \Rightarrow \Delta V = \frac{\eta k_B T}{q} \ln 2$$

$$\Rightarrow \Delta V = 2 \times 26\text{ mV} \times 0.693 \approx 36\text{ mV}$$

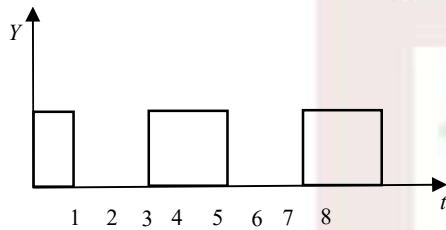
Q13. For the circuit and the inputs P and Q shown, which of the following is the correct output Y ?



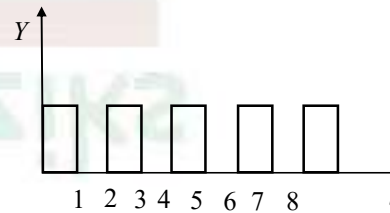
(a)



(b)



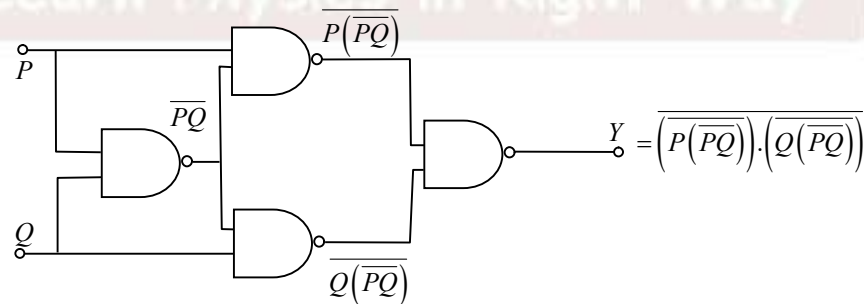
(c)



(d)

Ans.: (d)

Solution.:



$$\overline{(P(PQ))} . \overline{(Q(PQ))} = \overline{(P(PQ))} + \overline{(Q(PQ))} = P(\bar{P} + \bar{Q}) + Q(\bar{P} + \bar{Q}) = P\bar{Q} + \bar{P}Q = P \oplus Q$$

So, output is ex-OR i.e. inequality comparator.

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q2. If a resistor of $10\text{ k}\Omega$ and a capacitor of $0.5\text{ }\mu\text{F}$ are connected in series across an AC supply of 220 V (*rms*) at 50 Hz , what is the average power (in *mW*, to the nearest integer) dissipated in the circuit?

Ans.: 3443

Solution.: Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 0.5 \times 10^{-6}} \approx 6366\text{ }\Omega$

Impedance $Z = \sqrt{R^2 + X_C^2} = \sqrt{(10000)^2 + (6366)^2} \approx 11855\text{ }\Omega$

RMS current $I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{11855}\text{ A} = 0.01855\text{ A}$

Average power $P_{avg} = I_{rms}^2 R = (0.01855)^2 \times 10^4 \approx 3443\text{ mW}$



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2025

Solution- Waves and Optics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q12. A wire of length 0.01 mm is placed perpendicular to the axis of a thin convex lens, 30 cm from its center. If the focal length of the lens is 20 cm , what is the length of the image of the wire?

- (a) 0.02 mm (b) 0.01 mm (c) 1 mm (d) 0.03 mm

Ans.: (a)

Solution.:

Object length (height), $h_o = 0.01\text{ mm}$ Object distance, $u = -30\text{ cm}$ (using standard Cartesian sign convention)Focal length of convex lens, $f = +20\text{ cm}$

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1}{v} - \frac{1}{(-30)} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60} \Rightarrow v = +60\text{ cm}$$

Thus, image distance $v = +60\text{ cm}$

$$\text{Linear magnification } m = \frac{v}{u} = \frac{+60}{-30} = -2$$

$$\therefore |m| = \frac{h_i}{h_o} \Rightarrow h_i = |m| \cdot h_o = 2 \times 0.01\text{ mm} = 0.02\text{ mm}$$

Length of the image $h_i = 0.02\text{ mm}$

Q18. The Fraunhofer diffraction pattern formed by an elliptical aperture will be

- (a) circular
(b) elliptical with the semi-major axis parallel to that of the aperture
(c) hyperbolic
(d) elliptical with the semi-major axis perpendicular to that of the aperture

Ans.: (d)

Solution.:

Diffraction pattern dimensions are inversely proportional to the dimensions of the aperture.

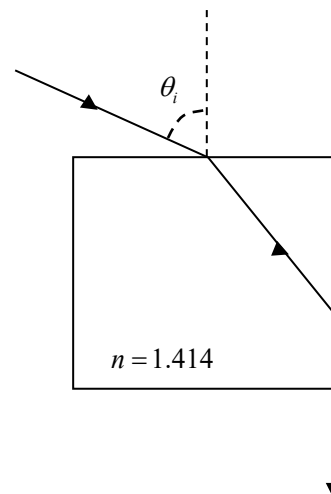
- Along the direction of the **semi-major axis** (larger aperture dimension), diffraction spreads less.
- Along the direction of the **semi-minor axis** (smaller aperture dimension), diffraction spreads more.

Therefore, an elliptical aperture produces an elliptical diffraction pattern, but rotated by 90° -meaning its semi-major axis lies **perpendicular** to the semi-major axis of the aperture.

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q8. A ray of light is incident on a glass cube of refractive index 1.414 as shown in the figure. Find the angle of incidence θ_i , such that the ray grazes down the side of the glass cube.



- (a) $\pi/3$
- (b) $\pi/4$
- (c) 0
- (d) $\pi/2$

Ans.: (d)

Solution.:

Refractive index of cube, $n = 1.414 = \sqrt{2}$

The light ray grazes the vertical side of the cube, so the angle of incidence at the vertical face is equal to the critical angle θ_c .

$$\text{Critical angle at the vertical face: } \sin \theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45^\circ = \frac{\pi}{4}$$

Refraction angle inside the top surface (r): From geometry, the angle of refraction r at the top surface and the angle at the vertical face θ_c form a right triangle with the surface normal:

$$r + \theta_c = 90^\circ \Rightarrow r + 45^\circ = 90^\circ \Rightarrow r = 45^\circ = \frac{\pi}{4}$$

Apply Snell's Law at the top surface:

$$\sin \theta_i = n \sin r \Rightarrow \sin \theta_i = \sqrt{2} \sin 45^\circ = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1 \Rightarrow \theta_i = \frac{\pi}{2}$$

Q9. A block, suspended from a massless spring, is fully immersed in a liquid contained in a reservoir. What is the time period of small oscillations of the block?

[Given: Mass of the block m , density of the block ρ_b , natural length of the spring L , spring constant k , acceleration due to gravity g , density of the liquid ρ_l , damping coefficient of the liquid i.e., damping per unit mass per unit velocity γ]

- (a) $2\pi\sqrt{\frac{m}{k}}$
- (b) $2\pi\sqrt{\frac{L}{(1 - \rho_l/\rho_b)g}}$
- (c) $2\pi\sqrt{\frac{1}{k/m + \gamma^2/4}}$
- (d) $2\pi\sqrt{\frac{1}{k/m - \gamma^2/4}}$

Ans.: (d)

Solution.: The equation of motion for a damped simple harmonic oscillator is given by:

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the undamped natural angular frequency and γ is the damping coefficient per unit mass.

The damped angular frequency ω_d is: $\omega_d = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4}}$

The time period of small damped oscillations is: $T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{\gamma^2}{4}}} = 2\pi \sqrt{\frac{1}{\frac{k}{m} - \frac{\gamma^2}{4}}}$

(**Note:** Buoyancy changes the equilibrium position, but it does not alter the spring constant k or the effective oscillation frequency).

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q4. The Fraunhofer diffraction intensity pattern for light of wavelength λ by a single slit of width a is given by

$$I = A_0^2 \left(\frac{\sin \beta}{\beta} \right)^2$$

where A_0 is the intensity of the central maximum and $\beta = \frac{\pi a \sin \theta}{\lambda}$, where θ is the angle with the incident beam. What is the angular separation in milli-radians, between the two first minima on two sides of the central beam, if $a = 1\text{ mm}$ and $\lambda = 5000\text{ Å}$?

Ans.: 1

Solution.: Slit width $a = 1\text{ mm} = 10^{-3}\text{ m}$; Wavelength, $\lambda = 5000\text{ Å} = 5 \times 10^{-7}\text{ m}$

Condition for single-slit diffraction minima: $a \sin \theta = n\lambda$

For the **first minimum** ($n = 1$): $\sin \theta \approx \theta = \frac{\lambda}{a}$

Angular distance between the first minima on either side: $\Delta\theta = 2\theta = \frac{2\lambda}{a}$

$$\Rightarrow \Delta\theta = \frac{2 \times 5 \times 10^{-7}}{10^{-3}} = 10^{-3} \text{ radians} = 1 \text{ milli radians}$$

Q6. A simple pendulum with effective length l and a bob of mass m has a time period T_1 . Suppose now that the bob is given an electric charge $+Q$. It is made to oscillate just above a two dimensional infinite sheet with surface charge density $+\sigma$, where $\frac{Q\sigma}{mg\epsilon_0} = \frac{3}{2}$, ϵ_0 being the permittivity of free space and g being the acceleration due to gravity. If the period of oscillation in this case is T_2 , determine $\frac{T_2}{T_1}$. [Neglect radiation from the charge]

Ans.: 2

Solution.: Initial time period $T_1 = 2\pi\sqrt{\frac{l}{g}}$

Time period with electric charge (T_2):

An infinite positively charged sheet with surface charge density $+\sigma$ creates a uniform vertical electric field pointing upward: $E = \frac{\sigma}{2\epsilon_0}$

The upward electrostatic force on the charge $+Q$ is:

$$F_e = QE = \frac{\sigma Q}{2\epsilon_0} = \frac{3}{4}mg \quad \because \frac{Q\sigma}{mg\epsilon_0} = \frac{3}{2} \Rightarrow \frac{Q\sigma}{2\epsilon_0} = \frac{3}{4}mg$$

Since gravity acts downward and F_e acts upward:

$$mg_{eff} = mg - F_e = mg - \frac{3}{4}mg = \frac{1}{4}mg \Rightarrow g_{eff} = \frac{g}{4}$$

$$\text{So } T_2 = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g/4}} = 2 \times 2\pi\sqrt{\frac{l}{g}} = 2T_1 \Rightarrow \frac{T_2}{T_1} = 2$$

Learn Physics in Right Way