



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2026
Complete Solution

Learn Physics in Right Way

Be Part of Disciplined Learning



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2026

Solution-Mathematical Methods

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q2. A detector has a 80% particle detection efficiency. If, in a given time, 10 particles pass through the detector, what is the probability that exactly 9 particles are detected?

- (a) $(0.8)^{10}$ (b) $(0.8)^9$
(c) $0.2 \times (0.8)^9$ (d) $2 \times (0.8)^9$

Ans.: (d)

Solution.:

Given:

Single particle detection probability (success), $p = 0.8$ Non-detection probability (failure), $q = 1 - p = 1 - 0.8 = 0.2$ Total particles passing through, $n = 10$ Number of detected particles, $k = 9$ This follows a Binomial distribution: $P(X = k) = {}^nC_k p^k q^{n-k}$

$$P(X = 9) = {}^{10}C_9 (0.8)^9 (0.2)^{10-9} = 10 \times (0.8)^9 \times 0.2 = 2 \times (0.8)^9$$

Q10. For two Pauli matrices σ_1, σ_2 , what is the value of the expression

$$\text{Determinant} (\sigma_1^{7641}) - \text{Trace} (\sigma_2^{7641})?$$

The Pauli matrices are given as: $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

- (a) -1 (b) 1
(c) -11 (d) 11

Ans.: (a)

Solution.: Given $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$\det(\sigma_1) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1$$

$$\det(\sigma_1^{7641}) = [\det(\sigma_1)]^{7641} = [-1]^{7641} = -1$$

$$\sigma_2^2 = \sigma_2 \cdot \sigma_2 = I \Rightarrow \sigma_2^{7641} = (\sigma_2^2)^{3820} \sigma_2 = I \sigma_2 = \sigma_2 \Rightarrow \text{Trace}(\sigma_2^{7641}) = \text{Trace}(\sigma_2) = 0$$

$$\text{Thus, Determinant} (\sigma_1^{7641}) - \text{Trace} (\sigma_2^{7641}) = -1 - 0 = -1$$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q4. What is the general solution of the equation $\frac{df(t)}{dt} = f(t) + 6 \int f(t) dt$?

- (a) $f(t) = Ae^{2t} + Be^{3t}$ (b) $f(t) = Ae^{2t} + Be^{-3t}$
(c) $f(t) = Ae^{-2t} + Be^{3t}$ (d) $f(t) = Ae^{-2t} + Be^{-3t}$

Ans.: (c)

Solution.: $\because \frac{df(t)}{dt} = f(t) + 6 \int f(t) dt$

Differentiating both sides with respect to t :

$$\frac{d^2 f(t)}{dt^2} = \frac{df(t)}{dt} + 6f(t) \Rightarrow \frac{d^2 f(t)}{dt^2} - \frac{df(t)}{dt} - 6f(t) = 0$$

Auxiliary equation: $\lambda^2 - \lambda - 6 = 0 \Rightarrow (\lambda - 3)(\lambda + 2) = 0 \Rightarrow \lambda = 3, -2$

Therefore, the general solution is: $f(t) = Ae^{-2t} + Be^{3t}$

Q7. What is the value of the integral $\int_0^\infty \frac{k \sin(kr)}{k^2 + m^2} dk$ for $r > 0$?

- (a) $\frac{\pi r}{1 + m^2 r^2}$ (b) $\frac{1}{r \sqrt{1 + m^2 r^2}}$
(c) $\frac{1}{4\pi r} e^{-mr}$ (d) $\frac{\pi}{2} e^{-mr}$

Ans.: (d)

Solution.: $\int_0^\infty \frac{k \sin(kr)}{k^2 + m^2} dk = \frac{\pi}{2} e^{-mr}, r > 0, m > 0$

Q10. Consider the solution of the differential equation $y'' + 2xy' + \lambda y = 0$ around $x = 0$. For which of the following values of λ is the solution a polynomial?

- (a) 1 (b) -3
(c) 2 (d) -2

Ans.: (d)

Solution.: comparing $y'' + 2xy' + \lambda y = 0$ against Hermite form $y'' - 2xy' + 2ny = 0$ then $\lambda = -2n$

So $\lambda = -2$ is valid integer.

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q10. A diagnostic test for a particular disease detects it with an efficiency of 99.99% in infected people. It also detects a false positive result in 0.02% of people who are not infected. If one person in 10000 in the general population is infected, what is the probability (in nearest integer percent) that someone who tests positive is actually infected?

Ans.: 33**Solution.:**Sensitivity: $P(+|D) = 0.9999$, False positive rate: $P(+|\text{no } D) = 0.0002$, $P(D) = 0.0001$

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|\text{no } D)P(\text{no } D)}$$
$$\Rightarrow P(D|+) = \frac{0.9999 \times 0.0001}{0.9999 \times 0.0001 + 0.0002 \times 0.9999} = \frac{0.00009999}{0.00009999 + 0.00019998}$$
$$\Rightarrow P(D|+) = \frac{0.00009999}{0.00029997} \approx 0.3333 \approx 33\%$$



Learn Physics in Right Way

JEST Physics-2026

Solution- Classical Mechanics

Learn Physics in Right Way
Solution-Classical Mechanics - JAM Physics-2022

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q9. The equation of a closed surface in the lab frame is

$$16x^2 + 25y^2 + 25z^2 = 25$$

An observer moving along the x -axis perceives the closed surface as a sphere of unit radius. If c is the speed of light in vacuum, what is the speed of the observer in the lab frame?

- | | |
|----------------------|---------------------|
| (a) $\frac{3}{5}c$ | (b) $\frac{4}{5}c$ |
| (c) $\frac{16}{25}c$ | (d) $\frac{9}{25}c$ |

Ans.: (a)

Solution.: In the lab frame, the ellipsoid $16x^2 + 25y^2 + 25z^2 = 25$ is:

$$\frac{x^2}{(5/4)^2} + \frac{y^2}{1} + \frac{z^2}{1} = 1$$

So semi-axes: along x : $a_x = \frac{5}{4}$, along y, z : $a_y = 1, a_z = 1$.

For the moving observer, length contraction occurs only along the direction of motion (x -axis). The observer sees a sphere of unit radius, so the contracted length along x must equal 1 (since y, z remains 1).

Length contraction formula:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow 1 = \frac{5}{4} \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{16}{25} \Rightarrow \frac{v^2}{c^2} = \frac{9}{25} \Rightarrow v = \frac{3}{5}c$$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q3. A satellite is orbiting in an elliptical orbit of non-zero eccentricity about a massive planet of mass M . If the angular speed of the satellite is ω_* at the point of closest distance a from the planet (where $\dot{r} = 0$ and $\ddot{r} > 0$), which of the following relations is true? G is the Gravitational constant.

- | | |
|-------------------------------------|------------------------------------|
| (a) $a\omega_*^2 = \frac{GM}{2a^2}$ | (b) $a\omega_*^2 = \frac{GM}{a^2}$ |
| (c) $a\omega_*^2 < \frac{GM}{a^2}$ | (d) $a\omega_*^2 > \frac{GM}{a^2}$ |

Ans.: (d)

Solution.: The radial equation of motion in a central gravitational potential is:

$$\ddot{r} - r\omega^2 = -\frac{GM}{r^2} \Rightarrow \ddot{r} = r\omega^2 - \frac{GM}{r^2}$$

$$\text{At closest distance } r = a, \omega = \omega_* \text{ and } \ddot{r} > 0: \Rightarrow \ddot{r} = a\omega_*^2 - \frac{GM}{a^2} > 0 \Rightarrow a\omega_*^2 > \frac{GM}{a^2}$$

Q17. The Lagrangian of a particle of unit mass is given by $L = -\sqrt{1 - (dx/dt)^2} - \alpha x$, where α is a constant. The trajectory of the particle in the phase space is:

- (a) a straight line (b) a hyperbola
(c) an ellipse (d) a circle

Ans.: (b)

Solution.:

Canonical Momentum $p = \frac{\partial L}{\partial \dot{x}} = \frac{\dot{x}}{\sqrt{1-\dot{x}^2}} \quad \because L = -\sqrt{1-\dot{x}^2} - \alpha x$

$$\text{Hamiltonian } H = p\dot{x} - L = \frac{\dot{x}^2}{\sqrt{1-\dot{x}^2}} + \sqrt{1-\dot{x}^2} + \alpha x = \frac{1}{\sqrt{1-\dot{x}^2}} + \alpha x$$

$$\because p = \frac{\dot{x}}{\sqrt{1-\dot{x}^2}} \Rightarrow p^2(1-\dot{x}^2) = \dot{x}^2 \Rightarrow \dot{x}^2(1+p^2) = p^2 \Rightarrow \dot{x}^2 = \frac{p^2}{1+p^2}$$

$$\Rightarrow 1 - \dot{x}^2 = \frac{1}{1 + p^2} \Rightarrow \frac{1}{\sqrt{1 - \dot{x}^2}} = \sqrt{1 + p^2}$$

Thus $H(x, p) = \sqrt{1 + p^2} + \alpha x = E(\text{constant})$

Phase Space Equation p vs x :

$$1+p^2=(E-\alpha x)^2 \Rightarrow (E-\alpha x)^2-p^2=1 \Rightarrow X^2-p^2=1$$

This represents a quadratic relation of the form $X^2 - p^2 = 1$, which is the equation of a **hyperbola**.

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q2. Consider the action $S = \int_0^2 \left[(dx/dt)^2 + 6x \right] dt$. If the boundary conditions are such that $x(0) = 0$, and $x(2) = 6$, what is the value of the action evaluated on the extremal path?

Ans.: 48

Solution.: $\because S = \int_0^2 \left[(dx/dt)^2 + 6x \right] dt \Rightarrow L(x, \dot{x}, t) = \dot{x}^2 + 6x$

Euler-Lagrange Equation: $\frac{\partial L}{\partial x} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = 0 \Rightarrow 6 - \frac{d}{dt} (2\dot{x}) = 0 \Rightarrow \ddot{x} = 3$

$$\Rightarrow \dot{x} = 3t + C_1 \Rightarrow x = \frac{3}{2}t^2 + C_1t + C_2$$

$$\because x(0) = 0 \Rightarrow C_2 = 0 \text{ and } x(2) = 6 \Rightarrow 6 = \frac{3}{2}2^2 + C_1 \times 2 + 0 \Rightarrow C_1 = 0$$

The extremal path is $x = \frac{3}{2}t^2 \Rightarrow \dot{x} = 3t$, so $S = \int_0^2 \left[(3t)^2 + 6 \times \frac{3}{2}t^2 \right] dt$

$$\Rightarrow S = \int_0^2 [9t^2 + 9t^2] dt = \int_0^2 [18t^2] dt = \left[18 \frac{t^3}{3} \right]_0^2 = 6 \times 2^3 = 48$$

Q3. The Lagrangian of a particle in one dimension is $L = 10\dot{x}^2 + 6\dot{x}t^2 + \alpha xt$ where $\dot{x} \equiv dx/dt$ and α is a constant. The particle starts moving at time $t = 0$ from the position $x(0) = 0.5$ with $\dot{x}(0) = 0.5$ in appropriate units. For what value of α , is the position of the particle at time t given by $x(t) = \frac{t+1}{2}$?

Ans.: 12

Solution.: Learn Physics in Right Way

Given $L = 10\dot{x}^2 + 6\dot{x}t^2 + \alpha xt$

Euler-Lagrange Equation:

$$\frac{\partial L}{\partial x} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = 0 \Rightarrow \alpha t - \frac{d}{dt} (20\dot{x} + 6t^2) = 0 \Rightarrow \alpha t = 20\ddot{x} + 12t$$

$$\text{Put } x(t) = \frac{t+1}{2} : \dot{x} = \frac{1}{2}, \ddot{x} = 0 \Rightarrow \alpha t = 20 \times 0 + 12t \Rightarrow \alpha = 12$$

Q5. Three planets are orbiting a star S_* in coplanar circular orbits of radii $R, 4R$ and $9R$. The time period of the planet closest to the star is 1 year. At some instant the star and the planets are collinear. Ignoring the motion of the star, after how many years will the same aligned configuration of the planets occur again?

Ans.: 216

Solution.: Kepler's Third Law ($T^2 \propto R^3$):

$$R_1 = R \Rightarrow T_1 = 1 \text{ year}; R_2 = 4R \Rightarrow T_2 = T_1 \times (4)^{3/2} = 8 \text{ years}; R_3 = 9R \Rightarrow T_3 = T_1 \times (9)^{3/2} = 27 \text{ years}$$

$$\text{Angular Velocities } \left(\omega = \frac{2\pi}{T} \right): \quad \omega_1 = 2\pi, \quad \omega_2 = \frac{2\pi}{8}, \quad \omega_3 = \frac{2\pi}{27}$$

Condition for Re-alignment:

For all planets to be on the same line again after time t , the relative angular displacements relative to Planet 1 must be integer multiples of 2π .

$$(\omega_1 - \omega_2)t = 2\pi k_1 \Rightarrow 2\pi \left(1 - \frac{1}{8} \right) t = 2\pi k_1 \Rightarrow t = \frac{8}{7} k_1$$

$$(\omega_1 - \omega_3)t = 2\pi k_2 \Rightarrow 2\pi \left(1 - \frac{1}{27} \right) t = 2\pi k_2 \Rightarrow t = \frac{27}{26} k_2$$

$$\text{For smallest common Time } (t > 0): t = \frac{8}{7} k_1 = \frac{27}{26} k_2 \Rightarrow \frac{k_1}{k_2} = \frac{27 \times 7}{26 \times 8} = \frac{189}{208}$$

$$\text{Smallest integers are } k_1 = 189, k_2 = 208: \quad t = \frac{8}{7} \times 189 = 216 \text{ years}$$

Q9. A particle of mass m_0 is initially at rest in an inertial frame and is subjected to a constant force $m_0 g$ along the y -axis. Its motion is governed by the equation:

$$\frac{dp}{dt} = m_0 g,$$

where p is the relativistic momentum. If $y(t)$ depicts the distance covered by the particle in

time t as $y\left(t = \frac{\sqrt{8}c}{g}\right) = n \frac{c^2}{g}$, where c is the speed of light, what is n ?

Ans.: 2

$$\text{Solution.:. Relativistic Momentum: } \frac{dp}{dt} = m_0 g \Rightarrow p(t) = m_0 g t \Rightarrow \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = m_0 g t$$

$$\Rightarrow \frac{v^2}{1 - v^2/c^2} = g^2 t^2 \Rightarrow v^2 = g^2 t^2 (1 - v^2/c^2) \Rightarrow v^2 \left(1 + \frac{g^2 t^2}{c^2} \right) = g^2 t^2$$

$$\Rightarrow v = \frac{dy}{dt} = \frac{gt}{\sqrt{1 + \frac{g^2 t^2}{c^2}}} \Rightarrow y(t) = \int_0^t \frac{gt'}{\sqrt{1 + \frac{g^2 t'^2}{c^2}}} dt'$$

$$\text{Let } u = 1 + \frac{g^2 t'^2}{c^2} \Rightarrow du = \frac{2g^2 t'}{c^2} dt' \Rightarrow t' dt' = \frac{c^2}{2g^2} du$$

$$\Rightarrow y(t) = \frac{c^2}{2g} \int_1^{1 + \frac{g^2 t^2}{c^2}} u^{-1/2} du = \frac{c^2}{g} \left[\sqrt{1 + \frac{g^2 t^2}{c^2}} - 1 \right]$$

$$\text{At } t = \frac{\sqrt{8}c}{g}: \Rightarrow y(t) = \frac{c^2}{g} \left[\sqrt{1 + \frac{g^2}{c^2} \times \frac{8c^2}{g^2}} - 1 \right] = \frac{c^2}{g} [\sqrt{1+8} - 1] = \frac{2c^2}{g} = n \frac{c^2}{g}$$

$$\Rightarrow n = 2$$

fiziks
Physics by fiziks

Learn Physics in Right Way



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2026

Solution- Electromagnetic Theory

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q5. Two charges $+q$ and $-q$ are placed at positions $a\hat{i}$ and $a\hat{j}$. What is the dipole moment of the system with respect to a point with position vector $\vec{r}_0$?

- (a) $\sqrt{2}aq(\hat{i} - \hat{j})$ (b) $qa(\hat{i} - \hat{j})$
(c) $q[a(\hat{i} - \hat{j}) - \vec{r}_0]$ (d) $q[a(\hat{i} - \hat{j}) + \vec{r}_0]$

Ans.: (b)

Solution.:

For a system with net charge zero the dipole moment is **independent** of the choice of origin (or reference point $\vec{r}_0$).

$$\vec{p} = +q(a\hat{i}) + (-q)(a\hat{j}) = qa(\hat{i} - \hat{j})$$

Q7. An electromagnetic wave of angular frequency ω is propagating in a dispersive medium with refractive index $n \equiv n(\omega)$. If the speed of light in vacuum is c , the group velocity of the electromagnetic wave in the dispersive medium is

- (a) $\frac{cn}{1 + \frac{\omega}{n} \frac{dn}{d\omega}}$ (b) $\frac{c}{1 + \frac{n}{\omega} \frac{d\omega}{dn}}$
(c) $\frac{c}{n + \omega \frac{dn}{d\omega}}$ (d) $\frac{c\omega \frac{dn}{d\omega}}{1 + \frac{\omega}{n} \frac{dn}{d\omega}}$

Ans.: (c)

Solution.:

The dispersion relation in a medium with refractive index $n(\omega)$ is:

$$k = \frac{\omega n(\omega)}{c}$$

Taking the derivative with respect to ω : $\frac{dk}{d\omega} = \frac{\omega n(\omega)}{c} = \frac{1}{c} \left[n + \omega \frac{dn}{d\omega} \right]$

$$\text{Group velocity } v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{d\omega}}$$

Q8. The vector $\vec{B}$ is defined as $\vec{B} = B_1 f(x) (\sin(ay)\hat{i} + \cos(ay)\hat{j}) + 2B_2 a^2 xy\hat{k}$ where a , B_1 and B_2 are constants with appropriate dimensions. For which of the following forms of the function $f(x)$ will $\vec{B}$ be a valid magnetic field?

- (a) $\ln(ax)$ (b) $\sin(ax)$
(c) ax (d) e^{ax}

Ans.: (d)

Solution.: For $\vec{B}$ to be a valid magnetic field, it must satisfy $\vec{\nabla} \cdot \vec{B} = 0$.

$$\therefore \vec{B} = B_1 f(x) (\sin(ay)\hat{i} + \cos(ay)\hat{j}) + 2B_2 a^2 xy\hat{k}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow B_1 f'(x) \sin(ay) + B_1 f(x) (-a \sin(ay)) + 0 = 0$$

$$\Rightarrow B_1 \sin(ay) [f'(x) - af(x)] = 0 \Rightarrow f'(x) = af(x) \Rightarrow \ln f(x) = ax + c \Rightarrow f(x) \propto e^{ax}$$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q5. In an electrodynamical set-up, a gauge transformation takes the vector potential $\vec{A}$ to $\vec{A} + \vec{\nabla}f$ where $f = \alpha x^2 t^2$ with a positive constant α . Which of the following statements is correct about the electric field ($\vec{E}$) and scalar potential ϕ at $(t, x, y, z) = (-1, 1, 0, 0)$?

- (a) $\vec{E}$ remains same and ϕ decreases (b) Both $\vec{E}$ and ϕ increase
(c) $\vec{E}$ remains same and ϕ increases (d) Both $\vec{E}$ and ϕ remain same

Ans.: (c)

Solution.: Gauge transformation: $\vec{A} + \vec{\nabla}f$ where $f = \alpha x^2 t^2$ with a positive constant α .

The electric field is unchanged under a gauge transformation if the scalar potential is also transformed as $\phi \rightarrow \phi - \frac{\partial f}{\partial t}$

So $\vec{E}$ remains the same.

$$\text{Now, } \frac{\partial f}{\partial t} = 2\alpha x^2 t$$

$$\text{At } (t, x, y, z) = (-1, 1, 0, 0): \frac{\partial f}{\partial t} = 2\alpha (1)^2 (-1) = -2\alpha, \text{ thus } \phi \rightarrow \phi - \frac{\partial f}{\partial t} = \phi + 2\alpha$$

Since $\alpha > 0$, ϕ increases. Therefore $\vec{E}$ remains same and ϕ increases

Q11. Consider a polarized dielectric sphere with the polarization given as $\vec{P}(r) = P_0 r \hat{r}$, where r is the distance from the center of the sphere and P_0 is a constant. The bound charge density at a point (r, θ, ϕ) inside the space is

- (a) $-P_0 \cos \theta \sin \phi$ (b) $P_0 \sin \theta \cos \phi$
(c) $P_0 \cos \theta \cos \phi$ (d) $-P_0 \sin \theta \cos \phi$

Ans.: (d)

Solution.:

Given as $\vec{P}(r) = P_0 r \hat{r}$.

Bound volume charge density is

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -\frac{\partial}{\partial x}(P_0 r) = -P_0 \frac{\partial}{\partial x}(\sqrt{x^2 + y^2 + z^2}) = -P_0 \frac{1}{2}(x^2 + y^2 + z^2)^{-1/2} 2x = -P_0 \frac{x}{r}$$

$$\Rightarrow \rho_b = -P_0 \frac{r \sin \theta \cos \phi}{r} = -P_0 \sin \theta \cos \phi$$

Q15. The potential on the surface of a spherical region of unit radius in vacuum is $V(\theta) = [2 \cos^2(\theta/2) + \cos^2 \theta] V_0$, where V_0 is a constant. The electrostatic potential in the region, $r \leq 1$ unit is

- (a) $\left(\frac{2+r^2}{3} + r \cos \theta + r^2 \cos^2 \theta \right) V_0$ (b) $\left(\frac{4-r^2}{3} + r \cos \theta + r^2 \cos^2 \theta \right) V_0$
(c) $\left(\frac{3+r^2}{4} + r \cos \theta + r^2 \cos^2 \theta \right) V_0$ (d) $\left(\frac{3-r^2}{2} + r \cos \theta + r^2 \cos^2 \theta \right) V_0$

Ans.: (b)

Solution.:

First, rewrite the boundary condition $V(1, \theta)$ in terms of Legendre polynomials $P_l(\cos \theta)$:

Using standard trigonometric identities:

$$2 \cos^2(\theta/2) = 1 + \cos \theta = P_0(\cos \theta) + P_1(\cos \theta)$$

$$\cos^2 \theta = \frac{2P_2(\cos \theta) + 1}{3} = \frac{1}{3} P_0(\cos \theta) + \frac{2}{3} P_2(\cos \theta) \quad \because P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$$

$$V(1, \theta) = [2 \cos^2(\theta/2) + \cos^2 \theta] V_0 = V_0 \left[\frac{4}{3} P_0(\cos \theta) + P_1(\cos \theta) + \frac{2}{3} P_2(\cos \theta) \right]$$

The general interior solution for Laplace's equation inside a sphere ($r \leq 1$) is:

$$V(r, \theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos \theta)$$

Matching coefficients at $r=1$: $A_0 = \frac{4}{3}V_0$, $A_1 = V_0$, $A_2 = \frac{2}{3}V_0$

Substituting back into the interior potential equation:

$$V(r, \theta) = V_0 \left[\frac{4}{3} + r P_1(\cos \theta) + \frac{2}{3} r^2 P_2(\cos \theta) \right] = V_0 \left[\frac{4}{3} + r \cos \theta + \frac{2}{3} r^2 \left(\frac{3 \cos^2 \theta - 1}{2} \right) \right]$$

$$\Rightarrow V(r, \theta) = \left(\frac{4-r^2}{3} + r \cos \theta + r^2 \cos^2 \theta \right) V_0$$

Q18. A circular ring of radius R with a line charge density $\lambda = \lambda_0 \cos \phi$ lies on the $x-y$ plane centered at the origin. What is the approximate work done in bringing a unit positive test charge from infinity to the point (r, θ, ϕ) when $r/R \ll 1$?

(a) $\frac{\lambda_0 R^2}{4 \epsilon_0 r^2} \cos \phi$

(b) $\frac{\lambda_0 R^2}{4 \epsilon_0 r^2} \sin \theta \cos \phi$

(c) $\frac{\lambda_0 R^2}{4 \epsilon_0 r^2}$

(d) $\frac{\lambda_0 R^2}{4 \epsilon_0 r^2} \cos \theta \cos \phi$

Ans.: (b)

Solution.: The work done W to bring a unit positive charge from infinity to point r equals the electric potential $V(r)$ at that point: $W = 1 \cdot V(r)$

For $r \ll R$, we can use the multipole expansion:

Monopole moment $q_T = \int_0^{2\pi} \lambda R d\phi' = R \lambda_0 \int_0^{2\pi} \cos \phi' d\phi' = 0$

Dipole moment $\vec{p} = \int \vec{r}' dq = \int_0^{2\pi} (R \cos \phi' \hat{x} + R \sin \phi' \hat{y}) (\lambda_0 \cos \phi' R d\phi') = \lambda_0 R^2 \pi \hat{x}$

The potential due to a dipole $\vec{p}$ at position r is $V(r) = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} = \frac{(\lambda_0 R^2 \pi \hat{x}) \cdot \hat{r}}{4\pi\epsilon_0 r^2} = \frac{\lambda_0 R^2}{4\epsilon_0 r^2} \sin \theta \cos \phi$

$$\Rightarrow W = 1 \cdot V(r) = \frac{\lambda_0 R^2}{4\epsilon_0 r^2} \sin \theta \cos \phi$$

Q20. An infinite material has electrons moving freely in the background of fixed positive charges. Initially the average charge density of electrons (ρ^-) and that of the positive charges (ρ^+), satisfy $\rho^+ = -\rho^- = \rho_0$. What is the average net charge density of the material if an electric field $\vec{E}$ is turned on resulting in a steady velocity $\vec{u}$ of the electrons throughout the material? c is the speed of light.

- (a) 0
- (b) $\rho_0 \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1 \right)$
- (c) $\rho_0 \left(1 - \frac{1}{\sqrt{1-u^2/c^2}} \right)$
- (d) $\rho_0 \left(1 - \frac{1}{(1-u^2/c^2)^{3/2}} \right)$

Ans.: (c)

Solution.:

Initially: $\rho^+ = \rho_0$, $\rho^- = -\rho_0$

Electrons move with steady velocity u . Due to relativistic length contraction, the electron density increases by the factor $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$.

Thus the moving electron charge density becomes $\rho_{moving}^- = -\rho_0 \gamma$

Positive charge density remains $\rho^+ = \rho_0$

Net charge density:

$$\rho_{net} = \rho_0 - \rho_0 \gamma = \rho_0 (1 - \gamma) = \rho_0 \left(1 - \frac{1}{\sqrt{1-u^2/c^2}} \right)$$

Learn Physics in Right Way

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q1. Two concentric conducting spheres of radii $4m$ and $5m$ respectively, enclose a medium of conductivity σ (in units of $m^{-1}\Omega^{-1}$) between their surfaces. The potential difference between the surfaces of the spheres is maintained at V (in volts). In this setup, there will be a current I (in amperes) between the spheres. What is the characteristic length scale $I/(4\pi\sigma V)$ of the system in metres?

Ans.: 20**Solution.:**

The resistance R between two concentric spherical shells of radii r_1 and r_2 filled with a conducting medium is:

$$R = \frac{1}{4\pi\sigma} \left(\frac{1}{r_1} - \frac{1}{r_2} \right).$$

Using Ohm's law:

$$V = IR = \frac{I}{4\pi\sigma} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{I}{4\pi\sigma} \left(\frac{r_2 - r_1}{r_1 r_2} \right) \Rightarrow \frac{I}{4\pi\sigma V} = \frac{r_1 r_2}{r_2 - r_1}$$

$$\Rightarrow \frac{I}{4\pi\sigma V} = \frac{4 \times 5}{5 - 4} = 20m \quad \because r_1 = 4m, r_2 = 5m$$

Learn Physics in Right Way



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2026

Solution- Quantum Mechanics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q1. Let $\hat{A}$ and $\hat{B}$ be two operators. Consider the operator $\hat{C} = [\hat{A} + \hat{A}^\dagger, \hat{B}\hat{B}^\dagger]$, where $[\hat{x}, \hat{y}]$ represents the commutator between the operators $\hat{x}$ and $\hat{y}$. Which of the following statements about $\hat{C}$ is true?

- (a) $\hat{C}$ is always Hermitian
- (b) $\hat{C}$ is anti-Hermitian
- (c) $\hat{C}$ is Hermitian if $\hat{A}$ is Hermitian and $\hat{B}$ is anti-Hermitian
- (d) $\hat{C}$ is Hermitian if both $\hat{A}$ and $\hat{B}$ are Hermitian

Ans.: (b)**Solution.:**

Let the two terms inside the commutator be: $\hat{X} = \hat{A} + \hat{A}^\dagger$ and $\hat{Y} = \hat{B}\hat{B}^\dagger$

Check Hermiticity of $\hat{X}$ and $\hat{Y}$:

$\hat{X}^\dagger = (\hat{A} + \hat{A}^\dagger)^\dagger = \hat{A}^\dagger + (\hat{A}^\dagger)^\dagger = \hat{A}^\dagger + \hat{A} = \hat{X}$, so $\hat{X}$ is Hermitian.

$\hat{Y}^\dagger = (\hat{B}\hat{B}^\dagger)^\dagger = (\hat{B}^\dagger)^\dagger \hat{B} = \hat{B}\hat{B}^\dagger = \hat{Y}$, so $\hat{Y}$ is Hermitian.

Check Hermiticity of $\hat{C}$:

The operator $\hat{C} = [\hat{X}, \hat{Y}] = \hat{X}\hat{Y} - \hat{Y}\hat{X} \Rightarrow \hat{C}^\dagger = [\hat{X}, \hat{Y}]^\dagger = (\hat{X}\hat{Y} - \hat{Y}\hat{X})^\dagger = (\hat{X}\hat{Y})^\dagger - (\hat{Y}\hat{X})^\dagger$

$\Rightarrow \hat{C}^\dagger = \hat{Y}^\dagger \hat{X}^\dagger - \hat{X}^\dagger \hat{Y}^\dagger = \hat{Y}\hat{X} - \hat{X}\hat{Y} = -(\hat{X}\hat{Y} - \hat{Y}\hat{X}) = -\hat{C} \Rightarrow \hat{C}^\dagger = -\hat{C}$

Thus $\hat{C}$ is anti-Hermitian.

Learn Physics in Right Way

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q6. Consider a state $|\lambda\rangle = e^{\lambda a^\dagger} |0\rangle$ defined in a one-dimensional harmonic oscillator such that $a|\lambda\rangle = \lambda|\lambda\rangle$. Here a and $a^\dagger$ are the annihilation and creation operators respectively and $|0\rangle$ is the ground state. What is $\langle\lambda_1|\lambda_2\rangle$?

- (a) $e^{\lambda_1^* \lambda_2}$ (b) $e^{\lambda_1^* + \lambda_2}$
(c) $\delta(\lambda_1 - \lambda_2)$ (complex δ -function) (d) 0

Ans.: (a)

Solution.: Using the Taylor series expansion for the exponential function $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$|\lambda_2\rangle = e^{\lambda_2 a^\dagger} |0\rangle = \sum_{n=0}^{\infty} \frac{(\lambda_2 a^\dagger)^n}{n!} |0\rangle = \sum_{n=0}^{\infty} \frac{\lambda_2^n}{n!} (a^\dagger)^n |0\rangle = \sum_{n=0}^{\infty} \frac{\lambda_2^n}{n!} \sqrt{n!} |n\rangle \quad \because (a^\dagger)^n |0\rangle = \sqrt{n!} |n\rangle$$

$$\Rightarrow |\lambda_2\rangle = \sum_{n=0}^{\infty} \frac{\lambda_2^n}{\sqrt{n!}} |n\rangle \quad \because (a^\dagger)^n |0\rangle = \sqrt{n!} |n\rangle$$

Similarly, for the bra state $\langle\lambda_1|$ (taking the Hermitian conjugate):

$$\langle\lambda_1| = \left(\sum_{m=0}^{\infty} \frac{\lambda_1^m}{\sqrt{m!}} |m\rangle \right)^\dagger = \sum_{m=0}^{\infty} \frac{(\lambda_1^*)^m}{\sqrt{m!}} \langle m|$$

$$\text{So } \Rightarrow \langle\lambda_1|\lambda_2\rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\lambda_1^*)^m \lambda_2^n}{\sqrt{m!} n!} \langle m|n\rangle = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\lambda_1^*)^m \lambda_2^n}{\sqrt{m!} n!} \delta_{mn}$$

$$\Rightarrow \langle\lambda_1|\lambda_2\rangle = \sum_{n=0}^{\infty} \frac{(\lambda_1^*)^n \lambda_2^n}{n!} = \sum_{n=0}^{\infty} \frac{(\lambda_1^* \lambda_2)^n}{n!} = e^{\lambda_1^* \lambda_2}$$

Q8. What is $\left\langle \frac{1}{r} \right\rangle$ in the state $|n, l, m\rangle$ of a hydrogen atom in terms of the Bohr radius a_0 ?

- (a) $\frac{1}{2n^2 a_0}$ (b) $\frac{1}{2n a_0}$
(c) $\frac{1}{n^2 a_0}$ (d) $\frac{1}{4n^2 a_0}$

Ans.: (c)

Solution.: Using standard quantum mechanics expectation value formulas for a Hydrogen-like

$$\text{atom } (Z=1): \left\langle \frac{1}{r} \right\rangle = \frac{1}{n^2 a_0}$$

Q9. An operator A is expressed in terms of angular momentum operators as:

$$A = L^2 + aL_z^2 + b(L_+L_- + L_-L_+),$$

where $L_{\pm} = L_x \pm iL_y$ and a, b are arbitrary constants. Under what condition does A commute with L_x^2 ?

- (a) $a = -b$ (b) $2a = b$
(c) $a = b$ (d) $a = 2b$

Ans.: (d)

Solution.: Given $A = L^2 + aL_z^2 + b(L_+L_- + L_-L_+)$ where $L_{\pm} = L_x \pm iL_y$

$$L_+L_- = (L_x + iL_y)(L_x - iL_y) = L_x^2 + L_y^2 - i[L_x, L_y] = L_x^2 + L_y^2 + \hbar L_z$$

$$L_-L_+ = (L_x - iL_y)(L_x + iL_y) = L_x^2 + L_y^2 + i[L_x, L_y] = L_x^2 + L_y^2 - \hbar L_z$$

$$L_+L_- + L_-L_+ = 2(L_x^2 + L_y^2) = 2(L^2 - L_z^2) \quad \because L^2 = L_x^2 + L_y^2 + L_z^2$$

$$\Rightarrow A = L^2 + aL_z^2 + 2b(L^2 - L_z^2) = (1+2b)L^2 + (a-2b)L_z^2$$

$$\Rightarrow [A, L_x^2] = [(1+2b)L^2 + (a-2b)L_z^2, L_x^2] = (1+2b)[L^2, L_x^2] + (a-2b)[L_z^2, L_x^2]$$

$$\because [L^2, L_x^2] = 0, [L_z^2, L_x^2] = i\hbar L_z \neq 0 \therefore [A, L_x^2] = 0 \Rightarrow (a-2b)[L_z^2, L_x^2] = 0 \Rightarrow a = 2b$$

Q14. Which of the following Hamiltonians correctly represents a one-dimensional simple harmonic oscillator of mass m , frequency ω and minimum of the potential at $x = a$?

(a) $H \equiv -\frac{\hbar^2}{2}m\omega^2 \frac{\partial^2}{\partial p^2} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2 a^2$

(b) $H \equiv -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial p^2} + \frac{1}{2}m\omega^2 (p-a)^2$

(c) $H = -\frac{\hbar^2}{2}m\omega^2 \frac{\partial^2}{\partial p^2} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2 a^2 - i\hbar m\omega^2 a \frac{\partial}{\partial p}$

(d) $H = -\frac{\hbar^2}{2}m\omega^2 \frac{\partial^2}{\partial p^2} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2 a^2 + i\hbar m\omega^2 a \frac{\partial}{\partial p}$

Ans.: (c)

Solution.: In momentum representation, $\hat{x} = i\hbar \frac{\partial}{\partial p}$ and $\hat{p} = p$

The Hamiltonian is: $H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 (\hat{x} - a)^2$

$$\therefore (\hat{x} - a)^2 = \left(i\hbar \frac{\partial}{\partial p} - a \right)^2 = -\hbar^2 \frac{\partial^2}{\partial p^2} - 2ia\hbar \frac{\partial}{\partial p} + a^2$$

$$\Rightarrow H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 (\hat{x} - a)^2 = -\frac{\hbar^2}{2}m\omega^2 \frac{\partial^2}{\partial p^2} + \frac{p^2}{2m} + \frac{1}{2}m\omega^2 a^2 - i\hbar m\omega^2 a \frac{\partial}{\partial p}$$

Q16. A particle subject to the one-dimensional infinite square well potential

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

is initially in the state $\psi(x) = \begin{cases} Ax(a-x), & \text{if } 0 \leq x \leq a, \\ 0 & \text{otherwise} \end{cases}$ where A and a are constants. Let

$P_1(t)$ be the probability that a measurement of the energy of the particle at time t yields the

outcome $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$. Which of the following is true for time $t > 0$?

- (a) $P_1(t) \propto t^2$ (b) $P_1(t) \propto \exp\left\{-\frac{\pi^2 \hbar}{2ma^2} t\right\}$
(c) $P_1(t) \propto 1 - \exp\left\{-\frac{\pi^2 \hbar}{2ma^2} t\right\}$ (d) $P_1(t)$ is independent of time

Ans.: (d)

Solution.:

Any general state can be expanded in terms of energy eigenstates:

$$\psi(x, t) = \sum c_n \phi_n(x) e^{-iE_n t / \hbar}$$

The probability $P_1(t)$ of measuring energy E_1 is given by $|c_1|^2 = \left| \langle \phi_1 | \psi(0) \rangle \right|^2$.

Since energy eigenstates are stationary and Hamiltonian is time-independent, $|c_1|^2$ is independent of time.

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q4. In a two-dimensional Hilbert space, two Hermitian non-commuting operators A and B have non-degenerate normalized eigenstates, which satisfy $A|\psi_A^i\rangle = a_i|\psi_A^i\rangle$ and $B|\psi_B^i\rangle = b_i|\psi_B^i\rangle$ where $i = 1, 2$. The relations:

$$|\psi_A^1\rangle = \frac{1}{\sqrt{5}}|\psi_B^1\rangle + \frac{2}{\sqrt{5}}|\psi_B^2\rangle; |\psi_B^1\rangle = \frac{1}{\sqrt{5}}|\psi_A^1\rangle - \frac{2}{\sqrt{5}}|\psi_A^2\rangle$$

are given. After a measurement of B is done on the state $|\psi_A^2\rangle$, what is the percentage probability of finding the system in the state $|\psi_A^1\rangle$?

Ans.: 32

Solution.: $|\psi_A^1\rangle = \frac{1}{\sqrt{5}}|\psi_B^1\rangle + \frac{2}{\sqrt{5}}|\psi_B^2\rangle; |\psi_B^1\rangle = \frac{1}{\sqrt{5}}|\psi_A^1\rangle - \frac{2}{\sqrt{5}}|\psi_A^2\rangle$

$$\frac{2}{\sqrt{5}}|\psi_A^2\rangle = \frac{1}{\sqrt{5}}|\psi_A^1\rangle - |\psi_B^1\rangle = \frac{1}{\sqrt{5}}\left(\frac{1}{\sqrt{5}}|\psi_B^1\rangle + \frac{2}{\sqrt{5}}|\psi_B^2\rangle\right) - |\psi_B^1\rangle \Rightarrow \frac{2}{\sqrt{5}}|\psi_A^2\rangle = -\frac{4}{5}|\psi_B^1\rangle + \frac{2}{5}|\psi_B^2\rangle$$

$$\Rightarrow |\psi_A^2\rangle = -\frac{2}{\sqrt{5}}|\psi_B^1\rangle + \frac{1}{\sqrt{5}}|\psi_B^2\rangle$$

Initially, the particle is in $|\psi_A^2\rangle$ and we measure B . $\therefore |\psi_A^2\rangle = -\frac{2}{\sqrt{5}}|\psi_B^1\rangle + \frac{1}{\sqrt{5}}|\psi_B^2\rangle$

$$P(B=b_1) = \frac{4}{5} \text{ and } P(B=b_2) = \frac{1}{5}$$

$$\therefore |\psi_A^1\rangle = \frac{1}{\sqrt{5}}|\psi_B^1\rangle + \frac{2}{\sqrt{5}}|\psi_B^2\rangle.$$

If the measurement gave $|\psi_B^1\rangle$: $P(\psi_A^1 | b_1) = \frac{1}{5}$

If the measurement gave $|\psi_B^2\rangle$: $P(\psi_A^1 | b_2) = \frac{4}{5}$

Combine the two possibilities $P(\psi_A^1) = P(\psi_A^1 | b_1)P(B=b_1) + P(\psi_A^1 | b_2)P(B=b_2)$

$$\Rightarrow P(\psi_A^1) = \frac{1}{5} \cdot \frac{4}{5} + \frac{4}{5} \cdot \frac{1}{5} = \frac{8}{25} \approx 32\%$$

Q8. A system of two spin-1/2 particles is in the state $|\uparrow\downarrow\rangle$ for time $t < 0$. At $t = 0$, an interaction Hamiltonian $\hat{H} = C\hat{S}^{(1)} \cdot \hat{S}^{(2)}$ is turned on, where C is a constant of the appropriate dimension and $\hat{S}^{(1)}$ and $\hat{S}^{(2)}$ are the spin angular momentum operators of the first and second particle respectively. What is the probability in percentage to find the system in the state $|\downarrow\uparrow\rangle$ at time $t = \frac{2\pi}{3Ch}$?

Ans.: 75

Solution.: The triplet and singlet states: $|T_0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ and $|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\Rightarrow |\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}(|T_0\rangle + |S\rangle) \quad \text{and} \quad |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}(|T_0\rangle - |S\rangle)$$

$$\hat{H} = C\hat{S}^{(1)} \cdot \hat{S}^{(2)} = C\frac{1}{2}(S^2 - S_1^2 - S_2^2); \quad S_1^2 = S_1(S_1+1)\hbar^2 = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2 = \frac{3}{4}\hbar^2 = S_2^2$$

$$\Rightarrow \hat{H} = \frac{C}{2}\left(S(S+1) - \frac{3}{2}\right)\hbar^2$$

The energy eigenvalues:

$$\text{Singlet}(S=0): E_s = \frac{C}{2}\left(0 - \frac{3}{2}\right)\hbar^2 = -\frac{3Ch^2}{4}$$

$$\text{Singlet}(S=1): E_t = \frac{C}{2}\left(2 - \frac{3}{2}\right)\hbar^2 = \frac{Ch^2}{4}$$

$$\text{Time evolution: } |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_t t/\hbar}|T_0\rangle + e^{-iE_s t/\hbar}|S\rangle\right)$$

$$\langle\downarrow\uparrow|\psi(t)\rangle = \frac{1}{2}\left(e^{-iE_t t/\hbar} - e^{-iE_s t/\hbar}\right) \quad \because |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}(|T_0\rangle - |S\rangle)$$

$$e^{-iE_t t/\hbar} = e^{-i\left(\frac{Ch^2}{4}\right)\left(\frac{2\pi}{3Ch}\right)/\hbar} = e^{-i\frac{\pi}{6}} = \frac{\sqrt{3}}{2} - i\frac{1}{2} \quad \text{and} \quad e^{-iE_s t/\hbar} = e^{-i\left(-\frac{3Ch^2}{4}\right)\left(\frac{2\pi}{3Ch}\right)/\hbar} = e^{i\frac{\pi}{2}} = i$$

$$\langle\downarrow\uparrow|\psi(t)\rangle = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - i\frac{1}{2} - i\right) = \frac{1}{2}\left(\frac{\sqrt{3}}{2} - i\frac{3}{2}\right) \Rightarrow |\langle\downarrow\uparrow|\psi(t)\rangle|^2 = \frac{1}{4}\left[\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2\right]$$

$$\Rightarrow |\langle\downarrow\uparrow|\psi(t)\rangle|^2 = \frac{1}{4}\left(\frac{3}{4} + \frac{9}{4}\right) = \frac{3}{4} = 75\%$$



Learn Physics in Right Way

JEST Physics-2026

Solution- Thermodynamics and Statistical Mechanics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

Q3. The critical volume, pressure and temperature of a van der Waals gas are V_c, P_c , and T_c , respectively. By some means the strength of the attraction between the molecules of the gas is changed. The change in V_c, P_c and T_c are

- (a) $\Delta V_c \neq 0, \Delta P_c \neq 0$ and $\Delta T_c \neq 0$ (b) $\Delta V_c = 0, \Delta P_c \neq 0$ and $T_c \neq 0$
(c) $\Delta V_c = 0, \Delta P_c = 0$ and $\Delta T_c = 0$ (d) $\Delta V_c \neq 0, \Delta P_c = 0$ and $\Delta T_c = 0$

Ans.: (b)

Solution.: The van der Waals critical constants expressed in terms of parameters a (attraction strength) and b (co-volume/size of molecules) are:

$$V_c = 3b, P_c = \frac{a}{27b^2}, T_c = \frac{8a}{27Rb}$$

Changing the strength of molecular attraction modifies parameter a while keeping parameter b constant. So, $\Delta V_c = 0, \Delta P_c \neq 0$ and $T_c \neq 0$

Q6. The internal energy of a gas in a cubical container of length L is proportional to L^{-2} . Which power law does the pressure follow?

- (a) L^{-4} (b) L^{-5}
(c) L^{-2} (d) L^{-3}

Ans.: (b)

Solution.: From thermodynamics, the relationship between pressure p , internal energy U , and

volume V is given by: $p = -\left(\frac{\partial U}{\partial V}\right)_s$

For a cubical container of side L , the volume is $V = L^3 \Rightarrow dV = 3L^2 dL$

Given $U \propto L^{-2}$

$$p \propto \frac{dU}{dV} = \frac{dU/dL}{dV/dL} \propto \frac{L^{-3}}{L^2} \propto L^{-5}$$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q2. There are N non-interacting magnetic moments of unit strength, fixed at certain positions in free space. These moments are subjected to a constant magnetic field $\vec{B} = B\hat{k}$ and the temperature of the system is T . If k_B is the Boltzmann constant, the Helmholtz free energy F of the system is

- (a) $-Nk_B T \left[\ln \left(2 \sinh \frac{B}{k_B T} \right) + \ln \left(\frac{B}{k_B T} \right) \right]$ (b) $Nk_B T \left[\ln \left(2 \sinh \frac{B}{k_B T} \right) + \ln \left(\frac{B}{k_B T} \right) \right]$
(c) $-Nk_B T \left[\ln \left(2 \sinh \frac{B}{k_B T} \right) - \ln \left(\frac{B}{k_B T} \right) \right]$ (d) $Nk_B T \left[\ln \left(2 \sinh \frac{B}{k_B T} \right) - \ln \left(\frac{B}{k_B T} \right) \right]$

Ans.: (c)

Solution.: For a classical magnetic moment $\vec{\mu}$ oriented at an angle θ relative to the magnetic field direction ($\vec{B} = B\hat{k}$), its energy $E(\theta)$ is:

$$E(\theta) = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta = -B \cos \theta \quad \because \mu = 1$$

Since the magnetic moments are continuous classical vectors in 3D space, we integrate over all possible spatial directions (the solid angle $d\Omega = \sin \theta d\theta d\phi$)

Single-Particle Partition Function $Z_1 = \int_0^\pi \int_0^{2\pi} e^{-\beta E(\theta)} \sin \theta d\theta d\phi$ where $\beta = \frac{1}{k_B T}$.

$$\Rightarrow Z_1 = 2\pi \int_0^\pi e^{\beta B \cos \theta} \sin \theta d\theta. \text{ Let } u = \beta B \cos \theta \Rightarrow du = -\beta B \sin \theta d\theta \Rightarrow d\theta = \frac{du}{-\beta B \sin \theta}$$

$$\Rightarrow Z_1 = -2\pi \int_{\beta B}^{-\beta B} \frac{e^u}{\beta B} du = 2\pi \int_{-\beta B}^{\beta B} \frac{e^u}{\beta B} du = \frac{2\pi}{\beta B} [e^{\beta B} - e^{-\beta B}] \Rightarrow Z_1 = \frac{4\pi}{\beta B} \sinh \beta B$$

Since the N magnetic moments are localized at fixed positions in space, they are **distinguishable**. The total partition function for N non-interacting identical systems is simply:

$$Z_N = (Z_1)^N = \left[4\pi \frac{\sinh \beta B}{\beta B} \right]^N$$

The thermodynamic relation for Helmholtz free energy F is:

$$F = -k_B T \ln Z_N = -k_B T \ln \left[4\pi \frac{\sinh \beta B}{\beta B} \right]^N = -Nk_B T \ln \left[4\pi \frac{\sinh \beta B}{\beta B} \right]$$

$$F = -Nk_B T \left[\ln \left(2 \sinh \frac{B}{k_B T} \right) - \ln \left(\frac{B}{k_B T} \right) + \ln 2\pi \right]. \text{ Drop the additive constant.}$$

Q12. Two identical bosons in a one-dimensional harmonic oscillator potential (with angular frequency ω) have total energy $E = 100\hbar\omega$. What is the entropy of this state if k_B is the Boltzmann constant?

- (a) $k_B \ln(99)$ (b) $k_B \ln(100)$
 (c) $k_B \ln(50)$ (d) $k_B \ln(198)$

Ans.: (c)

Solution.: For two identical bosons in a 1D harmonic oscillator, the total energy is:

$$E = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)\hbar\omega = (n_1 + n_2 + 1)\hbar\omega = 100\hbar\omega$$

$$\Rightarrow n_1 + n_2 + 1 = 100 \Rightarrow n_1 + n_2 = 99$$

Since the particles are indistinguishable bosons, pairs (n_1, n_2) and (n_2, n_1) count as the same state with $n_1 \leq n_2$. So n_1 goes from 0, 1...49 which gives 50 possible states.

Each state is equally likely (microcanonical ensemble), so entropy: $S = k_B \ln(50)$

Q19. Consider a system of a large even number of hard identical particles confined within a length L in one dimension which is bounded by two reflecting hard boundaries. At time $t = 0$, half of these particles have velocity $+v$ and the other half has velocity $-v$ distributed randomly among them. Assuming that all collisions are elastic, which one of the following statements is correct?

- (a) The velocity distribution of the system remains unchanged in all future times
 (b) The velocity distribution of the system becomes the Maxwell-distribution at the time $t \sim L/v$
 (c) The velocity distribution of system becomes the Maxwell-distribution within the time $t < L/v$
 (d) The velocity distribution changes later from the initial distribution but it never becomes the Maxwell-distribution

Ans.: (a)

Solution.: In one dimension, when two identical hard particles undergo a 1D elastic collision, they simply exchange velocities. Visually and statistically, this is indistinguishable from the particles passing right through each other without interacting. As a result, the overall set of speeds in the system never changes, and the initial velocity distribution remains conserved for all $t > 0$.

Section C (NAT)

Correct answer: +3, wrong answer: 0.

No question



Physics by fiziks

Learn Physics in Right Way

JEST Physics-2026

Solution- Electronics

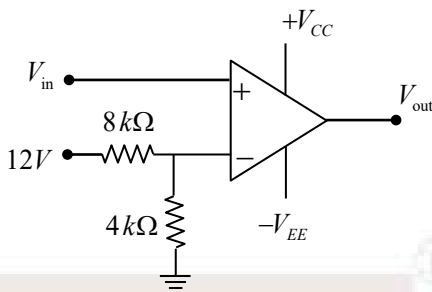
Learn Physics in Right Way

Be Part of Disciplined Learning

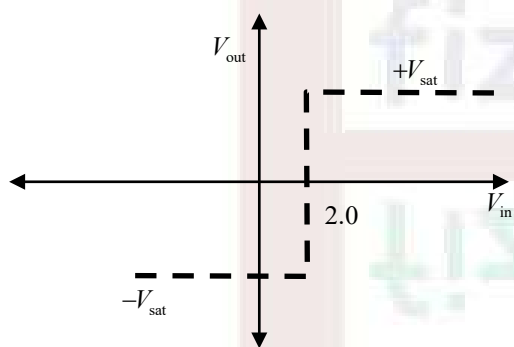
Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

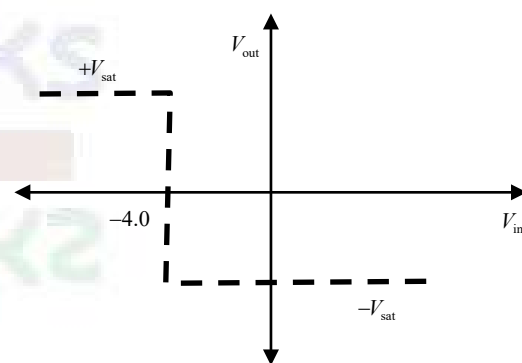
Q4. For the given Op-Amp circuit, which of the following correctly represents the plot of V_{out} vs V_{in} ?



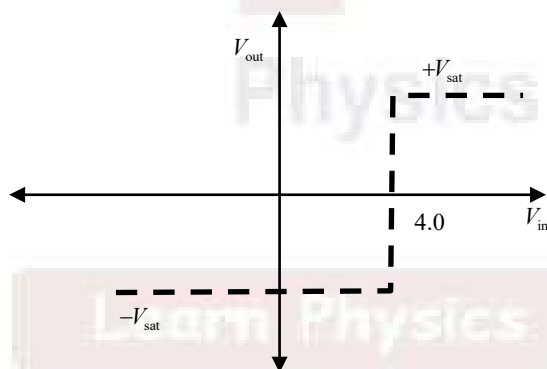
(a)



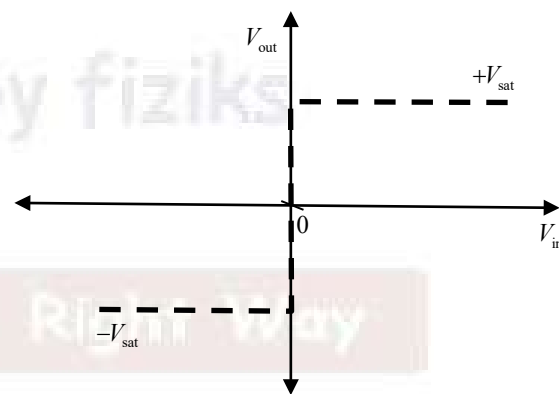
(b)



(c)



(d)



Ans.: (c)

Solution.: The voltage at the inverting terminal $V_- = \frac{4}{4+8} \times 12V = 4V$

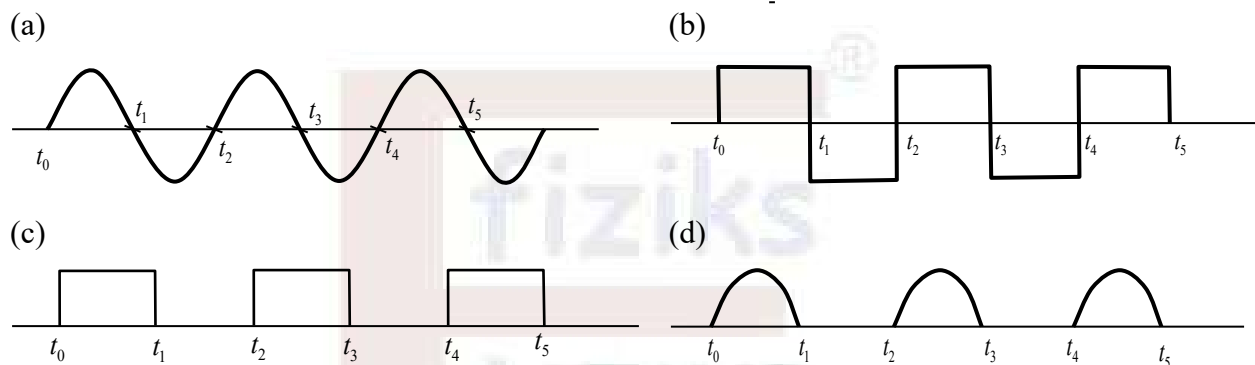
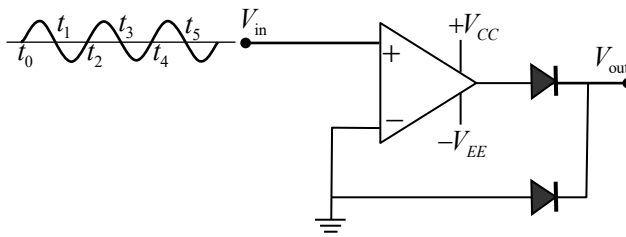
Since the op-amp is configured as an open-loop comparator:

When $V_{in} > V_-$, $V_{out} = +V_{sat}$ and When $V_{in} < V_-$, $V_{out} = -V_{sat}$

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q1. Consider the Op Amp circuit with a sinusoidal voltage input. What is the correct output waveform of this circuit?



Ans.: (c)

Solution.: Non-Inverting Comparator Stage:

The input V_{in} is fed to the non-inverting terminal (+), and the inverting terminal (-) is connected to ground (0 V).

During positive half-cycles ($V_{in} > 0$), the op-amp output goes to $+V_{sat}$.

During negative half-cycles ($V_{in} < 0$), the op-amp output goes to $-V_{sat}$.

Diode Network Analysis:

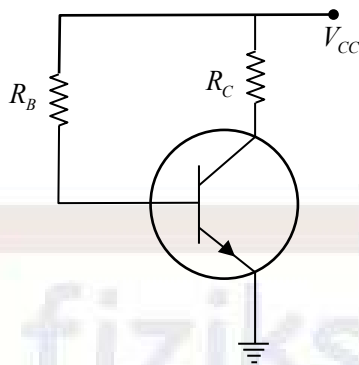
Positive Half-Cycle ($+V_{sat}$): The top diode (connected between the op-amp output and V_{out}) is forward-biased, conducting the positive voltage to V_{out} . The bottom diode (connected to ground) is reverse-biased. Thus, $V_{out} > 0$.

Negative Half-Cycle ($-V_{sat}$): The top diode is reverse-biased, isolating the output terminal. The bottom diode becomes forward-biased and clamps $+V_{sat}$ to 0 V (ground).

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q7. Consider the given circuit with $V_{CC} = 10V$, $R_B = 1M\Omega$, and $R_C = 10k\Omega$. It is given that $V_{CE} = V_{BE}$, V_{CE} is the collector-emitter voltage and V_{BE} is the base-emitter junction voltage. What is the gain β of the transistor?

**Ans.: 100**

Solution.: $I_B = \frac{V_{CC} - V_{BE}}{R_B}$ and $I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{V_{CC} - V_{BE}}{R_C} \because V_{CE} = V_{BE}$

$$\Rightarrow \beta = \frac{I_C}{I_B} = \frac{R_B}{R_C} = \frac{10^6 \Omega}{10^4 \Omega} = 100$$



Learn Physics in Right Way

JEST Physics-2026
Solution- Waves and Optics

Learn Physics in Right Way

Be Part of Disciplined Learning

Section A (MCQ)

Correct answer: +1, wrong answer: -1/3.

No question.

Section B (MCQ)

Correct answer: +3, wrong answer: -1.

Q13. In Young's double slit set-up, the screen is placed at a distance $D = 150\text{ cm}$ from the slits. At a point P on the screen, the second maximum is being observed. What is the minimum amount by which D needs to be changed such that the point P is now at a minimum?

- (a) 80 cm (b) 50 cm
(c) 30 cm (d) 20 cm

Ans.: (c)

Solution.: Path difference for maxima: $\frac{yd}{D} = n\lambda$

Path difference for minima: $\frac{yd}{D} = \left(m + \frac{1}{2}\right)\lambda$

Initially, P is at the 2nd maximum: $\frac{yd}{D} = 2\lambda \Rightarrow yd = 2\lambda D$

New D for minimum at the same y :

Let the new distance be D' . We want P to be at a minimum.

Since $yd = 2\lambda D$ still holds for the same point y , we need:

$$\frac{yd}{D'} = \left(m + \frac{1}{2}\right)\lambda \Rightarrow \frac{2\lambda D}{D'} = \left(m + \frac{1}{2}\right)\lambda \Rightarrow D' = \frac{2D}{\left(m + \frac{1}{2}\right)}$$

$$\text{For } m = 1, D' = \frac{4D}{3} = \frac{4 \times 150}{3} = 200\text{ cm} \Rightarrow \text{change} = 200\text{ cm} - 150\text{ cm} = 50\text{ cm}$$

$$\text{For } m = 2, D' = \frac{4D}{5} = \frac{4 \times 150}{5} = 120\text{ cm} \Rightarrow \text{change} = 150\text{ cm} - 120\text{ cm} = 30\text{ cm}$$

$$\text{For } m = 3, D' = \frac{4D}{7} = \frac{4 \times 150}{7} = 85.7\text{ cm} \Rightarrow \text{change} = 150\text{ cm} - 85.7\text{ cm} = 64.3\text{ cm}$$

The smallest change is 30 cm (option c)

Section C (NAT)

Correct answer: +3, wrong answer: 0.

Q6. Consider a Newton's rings experiment with a biconvex lens of diameter 20 cm and maximum thickness $60\text{ }\mu\text{m}$ placed on a flat reflecting surface, illuminated by normal incident light of wavelength 600 nm . When we change the medium of the experiment from air to a liquid medium, the fringe separation reduces by 20%. What the refractive index of the liquid medium?

Ans.: 1.25**Solution.:** Fringe separation $\propto \frac{1}{\mu}$.

$$\frac{\text{new separation}}{\text{original separation}} = \frac{1/\mu}{1/1} = 0.8 \Rightarrow \frac{1}{\mu} = 0.8 \Rightarrow \mu = \frac{1}{0.8} = 1.25$$

Learn Physics in Right Way