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**TIFR Physics-2026**  
**Complete Solution**

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Solution-Mathematical Physics

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## SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q4. Consider a  $2 \times 2$  matrix defined as:

$$A = [a_0 + i\vec{\sigma} \cdot \vec{a}][a_0 - i\vec{\sigma} \cdot \vec{a}]^{-1}$$

where  $a_0$  is a non-zero real number,  $\vec{a}$  is a three-dimensional non-zero real vector, and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ , where  $\sigma_x, \sigma_y, \sigma_z$  are the Pauli spin matrices. The matrix  $A$ :

- (a) Is unitary but not Hermitian                      (b) Is Hermitian but not Unitary  
(c) Is Unitary and Hermitian                      (d) Is neither Unitary nor Hermitian

Ans.:(a)

**Solution.:** Given  $A = [a_0 + i\vec{\sigma} \cdot \vec{a}][a_0 - i\vec{\sigma} \cdot \vec{a}]^{-1}$ . Let  $M = a_0 + i\vec{\sigma} \cdot \vec{a}$ .Since  $a_0$  and  $\vec{a}$  are real, and Pauli matrices are Hermitian ( $\sigma_i^\dagger = \sigma_i$ ), we have

$$M^\dagger = a_0 - i\vec{\sigma} \cdot \vec{a} \Rightarrow A = (M)(M^\dagger)^{-1}$$

**Unitary Test ( $A^\dagger A = I$ ):**  $A^\dagger = [(M)(M^\dagger)^{-1}]^\dagger = [(M^\dagger)^{-1}]^\dagger M^\dagger = M^{-1}M^\dagger$ 

$$A^\dagger A = (M^{-1}M^\dagger)(M)(M^\dagger)^{-1} = M^{-1}(M^\dagger M)(M^\dagger)^{-1} = M^{-1}(MM^\dagger)(M^\dagger)^{-1}$$

Since  $M$  and  $M^\dagger$  commute:  $[M^\dagger, M] = [a_0 - i\vec{\sigma} \cdot \vec{a}, a_0 + i\vec{\sigma} \cdot \vec{a}] = 0$  $\Rightarrow A^\dagger A = M^{-1}M(M^\dagger)(M^\dagger)^{-1} = I$ . Hence,  $A$  is **Unitary**.**Hermitian Test ( $A^\dagger = A$ ):**

$$A = M(M^\dagger)^{-1} \neq M^{-1}M^\dagger = A^\dagger \text{ (unless } M = M^\dagger, \text{ which implies } \vec{a} = 0)$$

Hence,  $A$  is **not Hermitian**Q9. Find:  $\lim_{x \rightarrow +\infty} x \log \left( \frac{x+1}{x-1} \right)$ 

- (a) 0                      (b) 1  
(c) 2                      (d) The limit does not exist

Ans.:(c)

**Solution.:**  $\log \left( \frac{x+1}{x-1} \right) = \log \left( 1 + \frac{2}{x-1} \right) \approx \frac{2}{x-1}$  as  $x \rightarrow \infty$ 

$$\Rightarrow \lim_{x \rightarrow +\infty} x \log \left( \frac{x+1}{x-1} \right) = \lim_{x \rightarrow +\infty} x \cdot \frac{2}{x-1} = \lim_{x \rightarrow +\infty} \frac{2x}{x-1} = \lim_{x \rightarrow +\infty} \frac{2}{1 - 1/x} = 2$$

**Q10.** Scientists are conducting an underground experiment where two interactions,  $A$  and  $B$  can occur. In general, interaction  $A$  is twice as likely to occur compared to interaction  $B$ . Interaction  $A$  would give a signal in the detector with probability  $1/8$ , while interaction  $B$  would give a signal with probability  $1/2$ . If the detector registers a signal, what is the probability that the signal was due to interaction  $A$ ?

- (a)  $\frac{1}{5}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{9}$  (d)  $\frac{2}{3}$

**Ans.:(b)**

**Solution.:** Interaction probabilities:

$$P(A) = 2P(B), P(A) + P(B) = 1 \Rightarrow P(A) = \frac{2}{3}, P(B) = \frac{1}{3}$$

Detector signal probabilities:  $P(S|A) = \frac{1}{8}$ ,  $P(S|B) = \frac{1}{2}$ . Find  $P(A|S) = ?$

Total probability of getting a signal

$$P(S) = P(S|A)P(A) + P(S|B)P(B) = \frac{1}{8} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} \Rightarrow P(S) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$$

$$\text{Using Bayes' Theorem: } P(A|S) = \frac{P(S|A)P(A)}{P(S)} = \frac{\frac{1}{8} \times \frac{2}{3}}{\frac{1}{4}} = \frac{1}{12} \times 4 = \frac{1}{3}$$

**Q20.** Consider the series:

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

Which of the following statements is correct?

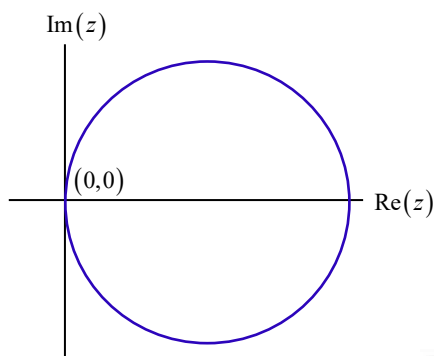
- (a)  $S$  is convergent and is greater than 2 (b)  $S$  is convergent and is less than 2  
(c)  $S$  is convergent and is equal to 2 (d)  $S$  is not convergent

**Ans.:(a)**

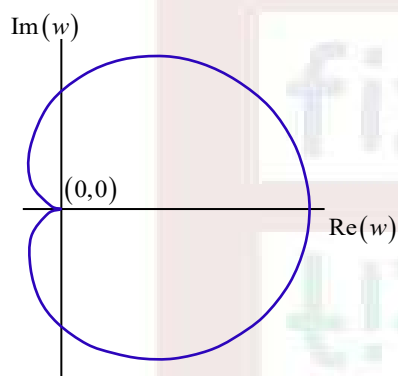
**Solution.:**  $S = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

$S = \zeta(3/2) = 2.612 > 2$  where  $p = \frac{3}{2} > 1$ , , so the series is **convergent**.

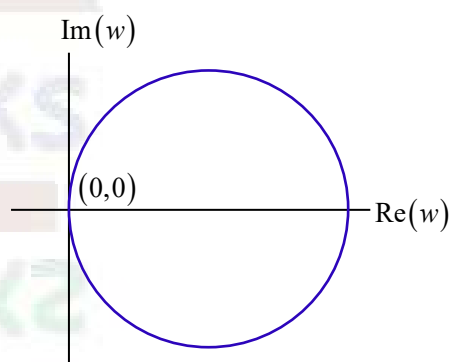
**Q25.** An analytic transformation:  $w = z^2$  is applied on the complex plane. Consider the circle  $C$  in  $z$  as shown on the right. Which of the following represents the image of  $C$  in the  $w$  plane?



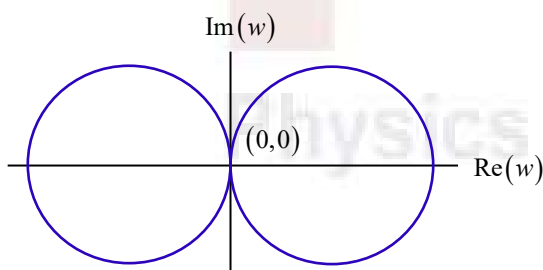
(a)



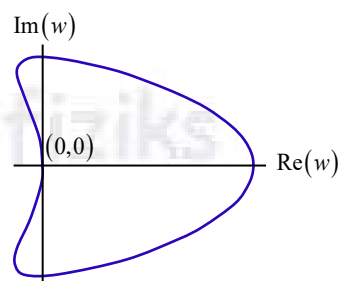
(b)



(c)



(d)



**Ans.:(a)**

**Solution.:** Transformation  $w = z^2$  applied to the circle  $C$  passing through the origin in the  $z$ -plane centered at  $(a, 0)$ .

Equation of circle  $C$  in  $z$ -plane:  $(x - a)^2 + y^2 = a^2 \Rightarrow x^2 + y^2 = 2ax$

In polar coordinates:  $r = 2a \cos \theta$

Under  $w = z^2$ :  $|w| = R = r^2 = 4a^2 \cos^2 \theta$ ;  $\arg(w) = \phi = 2\theta \Rightarrow \theta = \frac{\phi}{2}$

$$|w| = R = 4a^2 \left( \frac{1 + \cos 2\theta}{2} \right) = 4a^2 \left( \frac{1 + \cos \phi}{2} \right) = 2a^2 (1 + \cos \phi)$$

This is the standard equation of a **cardioid** with a cusp at the origin.

## SECTION B- (For only Integrated Ph.D. candidates)

Q7. Consider the function:

$$f(x) = \frac{4x - 11x^3}{(x^2 + 1)^2}$$

In the domain  $x \in [1, 4]$ . What is the difference between the maximum value of the function and its minimum value? (Select the closest answer.)

(a) 1.45

(b) 3.2

(c) 3.96

(d) 0.82

Ans.:(a)

**Solution.:**  $f(x) = \frac{4x - 11x^3}{(x^2 + 1)^2}$ , in the domain  $x \in [1, 4]$ .

Evaluate at boundary points:  $f(1) = \frac{4(1) - 11(1)^3}{(1^2 + 1)^2} = -\frac{7}{4} = -1.75$

$$f(4) = \frac{4(4) - 11(4)^3}{(4^2 + 1)^2} = \frac{16 - 704}{289} = -\frac{688}{289} \approx -2.3806$$

Find critical points:  $f'(x) = \frac{(x^2 + 1)^2(4 - 33x^2) - (4x - 11x^3)2(x^2 + 1) \times 2x}{(x^2 + 1)^4} = 0$

$$\Rightarrow \frac{(x^2 + 1)(4 - 33x^2) - 4(4x - 11x^3)x}{(x^2 + 1)^3} = 0 \Rightarrow (x^2 + 1)(4 - 33x^2) = 4(4x - 11x^3)x$$

$$\Rightarrow 4x^2 - 33x^4 + 4 - 33x^2 = 16x^2 - 44x^4 \Rightarrow 44x^4 - 33x^4 + 4 = 16x^2 - 4x^2 + 33x^2$$

$$\Rightarrow 11x^4 + 4 = 45x^2 \Rightarrow 11x^4 - 45x^2 + 4 = 0 \Rightarrow (11x^2 - 1)(x^2 - 4) = 0$$

$$\Rightarrow x = 2 \text{ in the domain } x \in [1, 4].$$

$$f(2) = \frac{4(2) - 11(2)^3}{(2^2 + 1)^2} = \frac{8 - 88}{225} = -\frac{80}{25} = -3.2$$

The difference between the maximum value of the function and its minimum value

$$= -1.75 - (-3.2) = 1.45$$

**Q10.** The curve that solves  $\frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$  and passes through the point (1,0) is given by:

- (a)  $2y^3 - x^3 + x = 0$  (b)  $y^3 + x^3 - x^2 = 0$   
(c)  $y^3 + x^3 - x^2 - y^2 = 0$  (d)  $2y^3 - x^3 + x + y = 0$

**Ans.:(b)**

**Solution.:**  $\therefore \frac{dy}{dx} = \frac{2y^3 - x^3}{3xy^2}$ . Let  $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} = \frac{2v^3 - 1}{3v^2}$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v^3 - 1}{3v^2} - v = \frac{2v^3 - 1 - 3v^3}{3v^2} = \frac{-1 - v^3}{3v^2} \Rightarrow \frac{3v^2}{1 + v^3} dv = -\frac{dx}{x} \Rightarrow \ln(1 + v^3) = -\frac{x^2}{2} + c$$

$$\Rightarrow \ln\left(1 + \frac{y^3}{x^3}\right) = -\ln x + c$$

$$\text{At } (1,0): \Rightarrow \ln(1+0) = -\ln 1 + c \Rightarrow c = 0$$

$$\Rightarrow \ln\left(1 + \frac{y^3}{x^3}\right) = -\ln x \Rightarrow 1 + \frac{y^3}{x^3} = \frac{1}{x} \Rightarrow \frac{x^3 + y^3}{x^3} = \frac{1}{x} \Rightarrow y^3 + x^3 - x^2 = 0$$

### SECTION C- (For only Ph.D. candidates)

**Q9.** The inner product of two complex functions  $f(x), g(x)$ , over the interval  $x \in [0,1]$  is defined as:

$$\langle f | g \rangle = \int_0^1 dx x(1-x) f^*(x) g(x)$$

The adjoint,  $\hat{O}^\dagger$ , of a linear operator  $\hat{O}$  is defined by the equation:

$$\langle f | \hat{O} g \rangle = \langle \hat{O}^\dagger f | g \rangle$$

What is the adjoint of the operator  $\hat{O} = \frac{d}{dx}$ ? Assume that both  $f(x)$  and  $g(x)$  are differentiable.

- (a)  $-\frac{d}{dx}$  (b)  $\frac{2x-1}{x(1-x)} - \frac{d}{dx}$   
(c)  $\frac{1}{x(1-x)} \frac{d}{dx}$  (d)  $\frac{-1}{x(1-x)} \frac{d}{dx}$

**Ans.:(b)**

**Solution.:** By definition of adjoint:  $\langle f | \hat{O}g \rangle = \int_0^1 dx x(1-x) f^*(x) \frac{dg}{dx}$

$$\Rightarrow \langle f | \hat{O}g \rangle = \left[ x(1-x) f^*(x) g(x) \right]_0^1 - \int_0^1 dx \frac{d}{dx} \left[ x(1-x) f^*(x) \right] g(x)$$

$$\Rightarrow \langle f | \hat{O}g \rangle = 0 - \int_0^1 dx \left[ (1-2x) f^*(x) + x(1-x) \frac{df^*}{dx} \right] g(x)$$

$$\Rightarrow \langle f | \hat{O}g \rangle = \int_0^1 dx x(1-x) \left[ \frac{(2x-1)}{x(1-x)} f^*(x) - \frac{df^*}{dx} \right] g(x) = \langle \hat{O}^\dagger f | g \rangle \Rightarrow \hat{O}^\dagger = \frac{2x-1}{x(1-x)} - \frac{d}{dx}$$

**Q10.** What is the value of the following integral:  $\int_0^\infty dx \frac{\sin x}{x} \frac{1}{x^2 + a^2}$  (Assume  $a > 0$ .)

(a)  $\frac{\pi \sinh a}{2a^2}$

(b)  $\frac{\pi}{2a^2} (1 + e^{-a})$

(c)  $\frac{\pi}{2a^2} (1 - e^{-a})$

(d)  $\frac{\pi \sin a}{2a^2}$

**Ans.:(c)**

**Solution.:**  $\int_0^\infty dx \frac{\sin x}{x} \frac{1}{x^2 + a^2}, a > 0$

Use contour integration with  $f(z) = \frac{e^{iz}}{z(z^2 + a^2)}$  along a indented semi-circular contour:

Indentation around simple pole at  $z = 0$  yields a contribution of

$$i\pi \text{Res} [f(z)]_{z=0} = i\pi \lim_{z \rightarrow 0} (z-0) \frac{e^{iz}}{z(z^2 + a^2)} = \frac{i\pi}{a^2}$$

Upper half-plane simple pole at  $z = ia$  yield

$$2\pi i \times \text{Res} [f(z)]_{z=ia} = 2\pi i \times \lim_{z \rightarrow ia} (z-ia) \frac{e^{iz}}{z(z+ia)(z-ia)} = 2\pi i \times \frac{e^{-a}}{ia(2ia)} = 2\pi i \left( -\frac{e^{-a}}{2a^2} \right)$$

Applying Cauchy's Residue Theorem:  $PV \int_{-\infty}^\infty \frac{e^{ix}}{x(x^2 + a^2)} dx - \frac{i\pi}{a^2} = 2\pi i \left( -\frac{e^{-a}}{2a^2} \right)$

Taking the imaginary part:  $\int_{-\infty}^\infty \frac{\sin x}{x(x^2 + a^2)} dx = \frac{\pi}{a^2} - \frac{\pi e^{-a}}{a^2} = \frac{\pi}{a^2} (1 - e^{-a})$

$$\int_0^\infty \frac{\sin x}{x(x^2 + a^2)} dx = \frac{\pi}{2a^2} (1 - e^{-a})$$



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Solution-Classical Mechanics

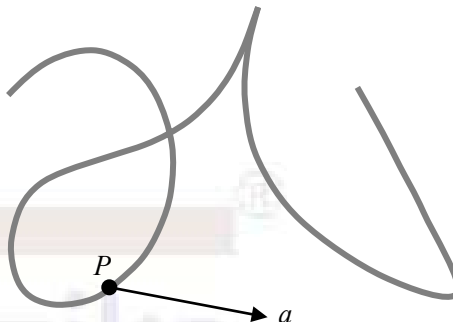
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## SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

Q1.

- (1) Is it possible for the trajectory of a particle in a two-dimensional plane with a continuous velocity and acceleration to have the shape shown in the figure?
- (2) Is it possible for its acceleration vector at  $P$  on the trajectory to point in the direction,  $a$ , as shown?



- (a) (1) Yes (2) No  
(b) (1) Yes (2) Yes  
(c) (1) No (2) No  
(d) (1) No (2) Yes

Ans.:(a)

Solution.:

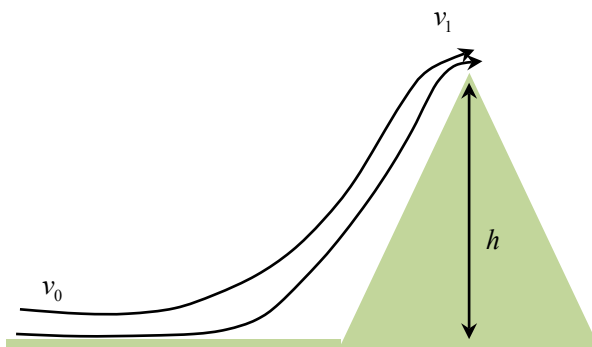
**Trajectory Feasibility:**

A smooth path in 2D space with continuous velocity and acceleration can have self-intersections or smooth curves like the one shown.

**Acceleration Vector at  $P$ :**

For any curved motion, the acceleration vector  $\vec{a}$  must point toward the concave side (inside) of the curve at that point. In the diagram, vector  $\vec{a}$  points toward the convex side (outside) of the curve. Therefore, this acceleration vector direction is physically impossible.

**Q2.** A steady, incompressible air stream of density  $\rho = 1.2 \text{ kg/m}^3$  blows horizontally at  $v_0 = 15 \text{ m/s}$  towards a hill of height  $h = 50 \text{ m}$ . Because the streamlines constrict over the hilltop, the air speed there increases to  $v_1 = 19.5 \text{ m/s}$ . The difference between the pressure at the base and the hilltop is closest to:



- (a)  $93 \text{ N/m}^2$       (b)  $495 \text{ N/m}^2$       (c)  $588 \text{ N/m}^2$       (d)  $681 \text{ N/m}^2$

**Ans.:(d)**

**Solution.:** Using Bernoulli's Principle along a streamline between the base (point 0) and the hilltop (point 1):

$$P_0 + \frac{1}{2}\rho v_0^2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho gh$$

pressure difference  $\Delta P = P_0 - P_1 = \frac{1}{2}\rho(v_1^2 - v_0^2) + \rho gh$

$$\Rightarrow \Delta P = P_0 - P_1 = \frac{1}{2}(1.2)(19.5^2 - 15^2) + 1.2 \times 9.8 \times 50 = (0.6)(380.25 - 225) + 588$$

$$\Rightarrow \Delta P = P_0 - P_1 = 93.15 + 588 \approx 681 \text{ N/m}^2$$

**Q21.** A relativistic particle of mass  $m$  moving under the central force of gravity with angular momentum  $L$  around a massive body of mass  $M$  experiences the following potential:

$$V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2 r^3}$$

where the last term is the relativistic correction to the Newtonian formula. For sufficiently large  $L$ , the particle has:

- (a) Two circular orbits. The smaller one is stable and the larger is unstable
- (b) Three circular orbits. The middle one is unstable and the others stable
- (c) Three circular orbits. The middle one is stable and the others unstable
- (d) Two circular orbits. The smaller one is unstable and the larger is stable

**Ans.:(d)**

**Solution.:**

Given effective potential:  $V(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2 r^3}$

To find circular orbits, set  $\frac{dV}{dr} = 0 \Rightarrow \frac{GMm}{r^2} - \frac{L^2}{mr^3} + \frac{3GML^2}{mc^2 r^4} = 0$

Multiplying by  $r^4$ :  $(GMm)r^2 - \left(\frac{L^2}{m}\right)r + \frac{3GML^2}{mc^2} = 0$

This is a quadratic equation in  $r$ , which yields at most two real positive roots for sufficiently large angular momentum  $L$ .

The smaller root  $r_1$  corresponds to a local maximum of  $V(r) \rightarrow$  **Unstable circular orbit**.

The larger root  $r_2$  corresponds to a local minimum of  $V(r) \rightarrow$  **Stable circular orbit**.

**Q24.** The longitudinal disturbance generated by an earthquake, travels through the earth's crust and reaches  $1000\text{ km}$  in 3 mins from the epicentre of the earthquake. Assuming the density of the Earth's crust is  $2700\text{ kg/m}^3$ , the Bulk's modulus of the crust is closest to:

(Ignore the shear modulus and local variations in the density and the Bulk modulus.)

- (a)  $8.3 \times 10^8 \text{ Nm}^{-2}$
- (b)  $2.1 \times 10^{11} \text{ Nm}^{-2}$
- (c)  $8.3 \times 10^{10} \text{ Nm}^{-2}$
- (d)  $2.1 \times 10^9 \text{ Nm}^{-2}$

**Ans.:(c)**

**Solution.:**

The longitudinal wave (P-wave) speed in a solid medium is given by:

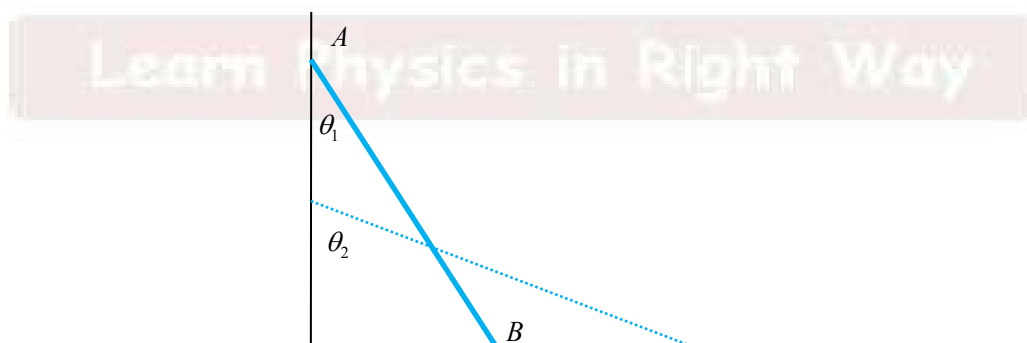
$$v = \sqrt{\frac{B}{\rho}}$$

$$\text{Bulk modulus } B = \rho v^2 = 2700 \times \left[ \frac{1000 \times 10^3}{3 \times 60} \text{ m/s} \right]^2 = 2700 \times [5555.56]^2 \approx 8.3 \times 10^{10} \text{ Nm}^{-2}$$

### SECTION B- (For only Integrated Ph.D. candidates)

**Q1.** Consider a uniform rod with length  $L$  and mass  $m$  with one end  $A$  on the wall and the other end  $B$  on the ground. Initially it is at rest at an angle  $\theta_1$  with the wall and starts slipping. What is the speed of  $A$  when the ladder's angle with the wall is  $\theta_2$ ? Ignore friction.

(FYI: The moment of inertia of a uniform rod about its centre is  $I = \frac{1}{12}mL^2$ .)



- (a)  $\sqrt{3gL(\cos \theta_1 - \cos \theta_2)}$
- (b)  $\sin \theta_2 \sqrt{3gL(\cos \theta_1 - \cos \theta_2)}$
- (c)  $\sqrt{12gL(\cos \theta_1 - \cos \theta_2)}$
- (d)  $\sin \theta_2 \sqrt{6gL(\cos \theta_1 - \cos \theta_2)}$

**Ans.:(b)**

### Solution.: Position of the Center of Mass

Because the rod is uniform, its center of mass is located at its midpoint, at a distance of  $\frac{L}{2}$  from either end. When the rod makes an angle  $\theta$  with the vertical wall:

- The vertical position of point  $A$  on the wall is  $y_A = L \cos \theta$ .
- The vertical position of the center of mass is:  $y_{cm} = \frac{L}{2} \cos \theta$

### Potential Energy of the Center of Mass

The potential energy depends purely on the height of the center of mass  $y_{cm}$ :

$$U = mgy_{cm} = mg \frac{L}{2} \cos \theta$$

As the rod slips from angle  $\theta_1$  to  $\theta_2$ , the decrease in potential energy is:

$$\Delta U = mg \frac{L}{2} (\cos \theta_1 - \cos \theta_2)$$

### Velocity of the Center of Mass ( $v_{cm}$ )

The coordinates of the center of mass as a function of angle  $\theta$  are:

$$x_{cm} = \frac{L}{2} \sin \theta, \quad y_{cm} = \frac{L}{2} \cos \theta$$

Differentiating both components with respect to time  $\theta$ :

$$v_{x,cm} = \frac{dx_{cm}}{dt} = \left( \frac{L}{2} \cos \theta \right) \dot{\theta} = \frac{\omega L}{2} \cos \theta \quad \text{and} \quad v_{y,cm} = \frac{dy_{cm}}{dt} = \left( -\frac{L}{2} \sin \theta \right) \dot{\theta} = -\frac{\omega L}{2} \sin \theta \quad \therefore \dot{\theta} = \omega$$

The magnitude of the linear velocity of the center of mass ( $v_{cm}$ ) is:

$$v_{cm} = \sqrt{v_{x,cm}^2 + v_{y,cm}^2} = \frac{\omega L}{2} \sqrt{\cos^2 \theta + \sin^2 \theta} = \frac{\omega L}{2}$$

Total kinetic energy:

$$K_{Total} = K_{trans} + K_{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 = \frac{1}{2} m \left( \frac{\omega L}{2} \right)^2 + \frac{1}{2} \left( \frac{mL^2}{12} \right) \omega^2 = \frac{1}{6} mL^2 \omega^2$$

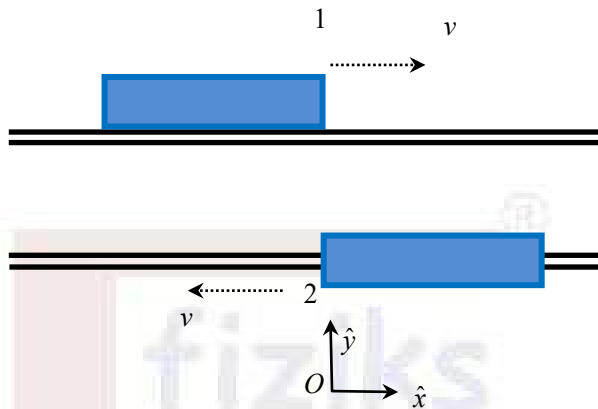
$$\therefore \Delta U = K_{Total} \Rightarrow mg \frac{L}{2} (\cos \theta_1 - \cos \theta_2) = \frac{1}{6} mL^2 \omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L} (\cos \theta_1 - \cos \theta_2)}$$

Finally, the downward velocity of top end  $A$  is:

$$v_A = \left| \frac{dy_A}{dt} \right| = L \sin \theta_2 \omega = L \sin \theta_2 \sqrt{\frac{3g}{L} (\cos \theta_1 - \cos \theta_2)}$$

$$\Rightarrow v_A = \sin \theta_2 \sqrt{3gL (\cos \theta_1 - \cos \theta_2)}$$

**Q3.** Two trains of proper length  $L$  each move on neighbouring tracks. An observer at rest with the tracks,  $O$ , notes the speed of train 1 to be  $v\hat{x}$  and train 2 to be  $-v\hat{x}$ .  $O$  also notes that at  $t=0$  the front ends of the two trains have the same  $x$  position,  $x=0$ . According to a train engineer sitting at the front of train 1, what is the time interval between the passing by of the front and the back of train 2?



(a)  $\frac{L}{2v}$

(b)  $\frac{L}{2v}\sqrt{1-v^2/c^2}$

(c)  $\frac{L}{2v}\sqrt{\frac{1-v^2/c^2}{1+v^2/c^2}}$

(d)  $\frac{L}{2v}(1-v^2/c^2)$

**Ans.:(d)**

**Solution.:**

Both trains move in opposite directions at speed  $v$  relative to observer  $O$ .

Relative velocity of Train 2 with respect to Train 1:

$$v_{rel} = \frac{v - (-v)}{1 - \frac{(v)(-v)}{c^2}} = \frac{2v}{1 + v^2/c^2}$$

Proper length of Train 2 is  $L$ . Due to Lorentz contraction in Train 1's frame:

$$L'_2 = L\sqrt{1 - \frac{v_{rel}^2}{c^2}} = L\frac{1 - v^2/c^2}{1 + v^2/c^2}$$

The time taken for Train 2 to pass the engineer at the front of Train 1 is:

$$\Delta t' = \frac{L'_2}{v_{rel}} = \frac{L(1 - v^2/c^2)(1 + v^2/c^2)}{\frac{2v}{1 + v^2/c^2}}$$

$$\Rightarrow \Delta t' = \frac{L}{2v}(1 - v^2/c^2)$$

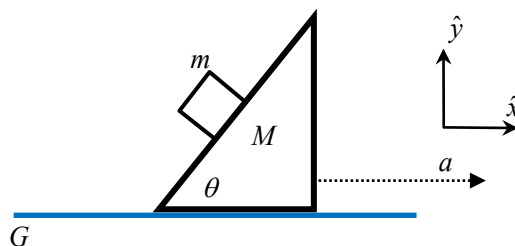
**Q4.** A mass  $m$  is placed on a wedge of mass  $M$  and angle  $\theta$  as shown, which in turn is placed on the horizontal ground. Initially,  $m$  and  $M$  are at rest with respect to the ground and are released from this position. What is the acceleration  $a$  of  $M$ ? Assume that all surfaces are frictionless.

(a)  $a = \frac{mg \sin \theta}{M + m}$

(b)  $a = \frac{mg \cos \theta}{M + m}$

(c)  $a = \frac{mg \cos \theta \sin \theta}{M + m \sin^2 \theta}$

(d)  $a = 0$



**Ans.:(c)**

**Solution.: Kinematic Constraint (Relative Acceleration)**

Let  $a$  be the magnitude of the horizontal acceleration of the wedge to the right ( $\vec{a}_M = a\hat{x}$ )

Let  $a_r$  be the acceleration of mass  $m$  relative to the wedge, pointing down along the inclined plane at angle  $\theta$ .

The components of  $m$ 's acceleration relative to the wedge are:

- Relative horizontal component:  $a_r \cos \theta$  (leftward)
- Relative vertical component:  $a_r \sin \theta$  (downward)

Therefore, the absolute acceleration components of  $m$  relative to the ground are:

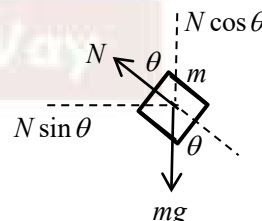
$$a_{m,x} = a - a_r \cos \theta, \quad a_{m,y} = -a_r \sin \theta$$

Since there are no external forces acting on the  $(m + M)$  system in the horizontal direction (all surfaces are frictionless):  $\sum F_{x, \text{system}} = 0$

$$Ma + ma_{m,x} = 0 \Rightarrow Ma + m(a - a_r \cos \theta) = 0 \Rightarrow (M + m)a = ma_r \cos \theta \Rightarrow a_r = \frac{(M + m)a}{m \cos \theta}$$

**Free Body Diagram for Mass  $m$ :** Forces acting on mass  $m$ :

- Gravity:  $mg$  acting vertically downward.
- Normal force from wedge:  $N$  perpendicular to the incline (pointing up and right at angle  $\theta$  relative to vertical).



**Vertical equation for  $m$ :**

$$N \cos \theta - mg = ma_{m,y} = m(-a_r \sin \theta) \Rightarrow N \cos \theta = mg - ma_r \sin \theta$$

**Horizontal force equation for wedge  $M$ :**

The normal force  $N$  exerts an equal and opposite force on wedge  $M$  pushing it to the right:

$$N \sin \theta = Ma \Rightarrow N = \frac{Ma}{\sin \theta}$$

$$\therefore N \cos \theta = mg - ma_r \sin \theta \text{ and } N = \frac{Ma}{\sin \theta}. \text{ So } \left( \frac{Ma}{\sin \theta} \right) \cos \theta = mg - ma_r \sin \theta$$

$$\Rightarrow Ma \cot \theta = mg - m \frac{(M+m)a}{m \cos \theta} \sin \theta \quad \therefore a_r = \frac{(M+m)a}{m \cos \theta}$$

$$\Rightarrow Ma \cot \theta = mg - (M+m)a \frac{\sin \theta}{\cos \theta} \Rightarrow Ma \cos \theta = mg \sin \theta - (M+m)a \frac{\sin^2 \theta}{\cos \theta}$$

$$\Rightarrow Ma \cos^2 \theta = mg \sin \theta \cos \theta - (M+m)a \sin^2 \theta$$

$$\Rightarrow a [M \cos^2 \theta + (M+m) \sin^2 \theta] = mg \sin \theta \cos \theta$$

$$\Rightarrow a [M (\cos^2 \theta + \sin^2 \theta) + m \sin^2 \theta] = mg \sin \theta \cos \theta$$

$$\Rightarrow a [M + m \sin^2 \theta] = mg \sin \theta \cos \theta \Rightarrow a = \frac{mg \sin \theta \cos \theta}{M + m \sin^2 \theta}$$

**Q8.** An earth imaging satellite is in a near earth circular orbit (see figure) with an orbital period of 100 minutes and observes earth with a light of wavelength  $550 \text{ nm}$ . If it can resolve the ground with  $0.5 \text{ m}$ , the minimum diameter of the mirror on the satellite should be closest to:

(Assume the system is diffraction limited. The mass and radius of Earth are  $5.97 \times 10^{24} \text{ kg}$  and  $6.4 \times 10^3 \text{ km}$ , respectively.) (Figure not to scale)

(a)  $10 \text{ m}$

(b)  $5 \text{ m}$

(c)  $0.5 \text{ m}$

(d)  $1 \text{ m}$

**Ans.:(d)**

**Solution.:** Using Kepler's Third Law for a circular orbit:  $T^2 = \frac{4\pi^2 r^3}{GM_e} \Rightarrow r^3 = \frac{GM_e T^2}{4\pi^2}$

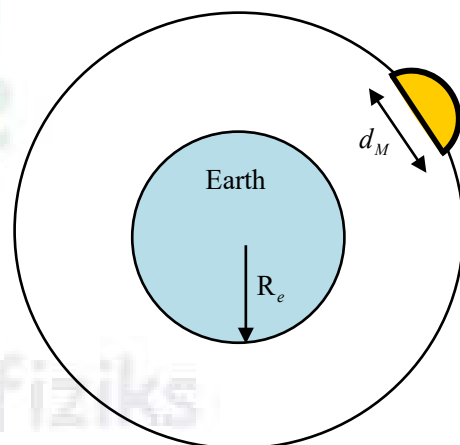
$$\Rightarrow r^3 = \frac{(6.674 \times 10^{-11}) \times (5.97 \times 10^{24}) (6000)^2}{4\pi^2} \approx 3.633 \times 10^{20} \text{ m}^3 \Rightarrow r \approx 7.135 \times 10^6 \text{ m} = 7135 \text{ km}$$

$$\text{Altitude } h = r - R_e = 7135 \text{ km} - 6400 \text{ km} = 735 \text{ km} = 7.35 \times 10^5 \text{ m}$$

### Rayleigh's Criterion for Diffraction-Limited Resolution

The linear spatial resolution  $\Delta x$  on the ground is given by:  $\Delta x = 1.22 \frac{\lambda h}{d}$

$$\text{Aperture diameter } d = 1.22 \frac{\lambda h}{\Delta x} = \frac{1.22 \times (5.5 \times 10^{-7} \text{ m}) (7.35 \times 10^5 \text{ m})}{0.5 \text{ m}} \approx 0.986 \text{ m} \approx 1 \text{ m}$$



SECTION C- (For only Ph.D. candidates)

**Q5.** A classical particle of mass  $m$  moving in the  $x-y$  plane is constrained to move along the curve  $x^4 + y^4 = 1$ . There are no other external forces acting on it. If  $(r, \theta)$  are the polar coordinates, which of the following is true?

(a)  $\ddot{\theta} \propto r^2 \sin 4\theta$

(b)  $\ddot{\theta} + 2\frac{\dot{r}\dot{\theta}}{r} \propto r^2 \sin 4\theta$

(c)  $\ddot{\theta} \propto r^4 \sin^2 2\theta$

(d)  $\ddot{\theta} + 2\frac{\dot{r}\dot{\theta}}{r} \propto r^4 \sin^2 2\theta$

**Ans.:(b)**

**Solution.:** In polar coordinates  $x = r \cos \theta$ ,  $y = r \sin \theta$ .

$$\text{So } x^4 + y^4 = 1 \Rightarrow r^4 (\cos^4 \theta + \sin^4 \theta) = 1 \Rightarrow F(r, \theta) = r^4 (\cos^4 \theta + \sin^4 \theta) - 1 = 0$$

**Direction of the constraint force**

Since there are no other external forces, the only force on the particle is the constraint force.

For a particle constrained to a curve  $F(x, y) = 0$ , the constraint force is along  $\vec{\nabla} F = 0$ . Thus

$$\vec{a} \propto \vec{\nabla} F.$$

We only need the  $\theta$ -component.

$$\text{So } a_\theta \propto (\vec{\nabla} F)_\theta = \frac{1}{r} \frac{\partial F}{\partial \theta}$$

$$\frac{\partial F}{\partial \theta} = \frac{\partial}{\partial \theta} [r^4 (\cos^4 \theta + \sin^4 \theta) - 1] = r^4 (-4 \cos^3 \theta \sin \theta + 4 \sin^3 \theta \cos \theta)$$

$$\Rightarrow \frac{\partial F}{\partial \theta} = 4r^4 \sin \theta \cos \theta (\sin^2 \theta - \cos^2 \theta) = 2r^4 \sin 2\theta (-\cos 2\theta) = -r^4 \sin 4\theta \Rightarrow \frac{1}{r} \frac{\partial F}{\partial \theta} = -r^3 \sin 4\theta$$

$$\Rightarrow a_\theta \propto -r^3 \sin 4\theta \Rightarrow (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \propto -r^3 \sin 4\theta$$

$$\Rightarrow \ddot{\theta} + \frac{2\dot{r}\dot{\theta}}{r} \propto r^2 \sin 4\theta$$

**Q6.** A particle of rest mass  $m$  moving at a relativistic speed  $v$  collides with an identical particle at rest and merges with it to form a composite of rest mass  $M = \frac{4}{\sqrt{3}}m$ . What is the ratio of the speed of  $M$  to the initial speed  $v$ ?

- (a)  $\frac{2}{3}$  (b)  $\frac{1}{2}$  (c)  $\frac{3}{5}$  (d)  $\frac{5}{8}$

**Ans.:(d)**

**Solution.:** Initial energy:

For the moving particle,  $E_1 = \gamma mc^2$  and for the particle at rest,  $E_2 = mc^2$  where  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

Therefore, total energy is  $E = E_1 + E_2 = (\gamma + 1)mc^2$

**Initial momentum**

Only the first particle has momentum:  $p = \gamma mv$

**Use the invariant relation:** For the final composite particle,  $E^2 - p^2c^2 = M^2c^4$

$$\Rightarrow (\gamma + 1)^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = M^2 c^4 \Rightarrow \gamma^2 m^2 c^4 + m^2 c^4 + 2\gamma m^2 c^4 - \gamma^2 m^2 v^2 c^2 = M^2 c^4$$

$$\Rightarrow \gamma^2 m^2 c^4 (1 - v^2/c^2) + m^2 c^4 + 2\gamma m^2 c^4 = M^2 c^4 \Rightarrow m^2 c^4 + m^2 c^4 + 2\gamma m^2 c^4 = M^2 c^4$$

$$\Rightarrow 2m^2 c^4 (1 + \gamma) = M^2 c^4 \Rightarrow M^2 = 2m^2 (1 + \gamma) \Rightarrow \left( \frac{4}{\sqrt{3}} m \right)^2 = 2m^2 (1 + \gamma)$$

$$\Rightarrow \frac{16}{3} = 2(1 + \gamma) \Rightarrow 1 + \gamma = \frac{8}{3} \Rightarrow \gamma = \left( \frac{8}{3} - 1 \right) \Rightarrow \gamma = \frac{5}{3} \Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = \frac{5}{3} \Rightarrow 1 - v^2/c^2 = \frac{9}{25}$$

$$\Rightarrow \frac{v^2}{c^2} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow v = \frac{4}{5} c$$

**The ratio of the speed of  $M$  to the initial speed  $v$ :**

$$\because E = (\gamma + 1)mc^2 = \gamma_v Mc^2 \Rightarrow \left( \frac{5}{3} + 1 \right) mc^2 = \frac{1}{\sqrt{1-V^2/c^2}} \left( \frac{4}{\sqrt{3}} m \right) c^2 \quad \because \gamma = \frac{5}{3}, M = \frac{4}{\sqrt{3}} m$$

$$\Rightarrow \frac{8}{3} mc^2 = \frac{1}{\sqrt{1-V^2/c^2}} \frac{4}{\sqrt{3}} mc^2 \Rightarrow \frac{1}{\sqrt{1-V^2/c^2}} = \frac{2}{\sqrt{3}} \Rightarrow 1 - \frac{V^2}{c^2} = \frac{3}{4} \Rightarrow \frac{V^2}{c^2} = \frac{1}{4} \Rightarrow V = \frac{c}{2}$$

$$\text{So } \frac{V}{v} = \frac{c/2}{4c/5} = \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$$



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Solution-Electromagnetic Theory

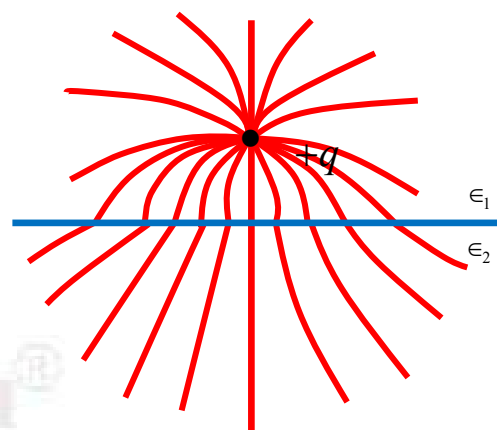
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**SECTION A-(For both Int. Ph.D. and Ph.D. candidates)**

**Q5.** The figure on the right shows a positive point charge  $(+q)$  placed above an interface (horizontal line in the figure) between two dielectrics with permittivities,  $\epsilon_1$  and  $\epsilon_2$ . The curves shown are field lines of the electric displacement,  $\vec{D}$ . Which of the following statements is true?

- (a)  $\epsilon_1 > \epsilon_2$  and the sign of the net surface charge at the interface is positive
- (b)  $\epsilon_2 > \epsilon_1$  and the sign of the net surface charge at the interface is positive
- (c)  $\epsilon_2 > \epsilon_1$  and the sign of the net surface charge at the interface is negative
- (d)  $\epsilon_1 > \epsilon_2$  and the sign of the net surface charge at the interface is negative



**Ans.:(c)**

**Solution.:**

The refraction of field lines follows:  $\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}$

**Observation from Figure:** In medium 1 (top), the field lines are more vertical (closer to the normal, smaller  $\theta_1$ ). In medium 2 (bottom), the lines spread out further from the normal (larger  $\theta_2$ ). Since  $\theta_1 < \theta_2$ , we have  $\tan \theta_1 < \tan \theta_2$  which implies:  $\epsilon_2 > \epsilon_1$ .

**Sign of Net Surface Charge at Interface**

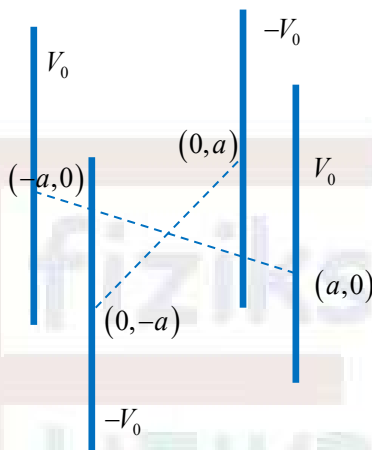
The positive point charge  $+q$  in region 1 induces polarization in both dielectric media.

Because  $\epsilon_2 > \epsilon_1$ , dielectric 2 is more polarizable than dielectric 1

The bound polarization charge density at the interface is:  $\sigma_b = (\vec{P}_1 - \vec{P}_2) \cdot \hat{n}$

Since medium 2 polarizes more strongly away from  $+q$ , negative bound charge accumulates on the upper face of medium 2 while positive bound charge accumulates on the lower face of medium 1. The net total bound surface charge at the interface attracted toward the  $+q$  charge is **negative**.

**Q7.** Four infinitely long electrodes of negligible thickness are placed parallel to the  $z$  axis and pass through the points at the edges of a square in the  $(x,y)$  plane at  $(\pm a,0)$  and  $(0,\pm a)$ . The potential at  $(0,0)$  is 0. If the electrodes at  $(\pm a,0)$  are maintained at a potential  $V_0$ , while the electrodes at  $(0,\pm a)$  are maintained at a potential  $-V_0$ , the electric field in the vicinity of the electrodes is given by:



(a)  $\vec{E} = -\frac{V_0}{a}(\hat{x} - \hat{y})$

(b)  $\vec{E} = -\frac{2V_0}{a^2}(x\hat{x} - y\hat{y})$

(c)  $\vec{E} = \frac{2\pi V_0}{a^2} \left( \sin\left(\frac{2\pi x}{a}\right)\hat{x} - \sin\left(\frac{2\pi y}{a}\right)\hat{y} \right)$

(d)  $\vec{E} = -\frac{V_0}{x^2 + y^2}(\hat{x} - \hat{y})$

**Ans.:(b)**

**Solution.:**

Near the origin  $(x, y \ll a)$ , the potential can be modeled via a quadrupole expansion satisfying

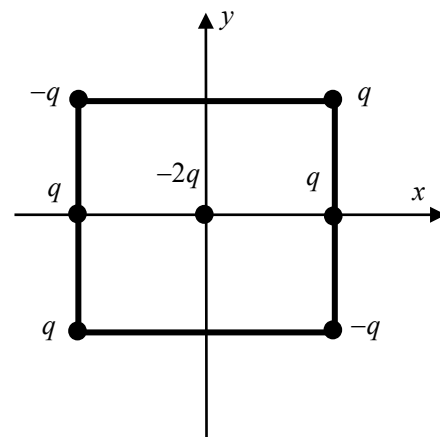
Laplace's equation  $\nabla^2 V = 0$ :

$$V(x, y) = C(x^2 - y^2).$$

Using the boundary condition:  $V(a, 0) = V_0$ , we find  $C = \frac{V_0}{a^2}$

$$\Rightarrow V(x, y) = \frac{V_0}{a^2}(x^2 - y^2). \text{ So } \vec{E} = -\vec{\nabla}V = -\frac{2V_0}{a^2}(x\hat{x} - y\hat{y})$$

**Q15.** The figure on the right, shows point charges arranged in the  $x, y$  plane on the centre, vertices, and some edges of a square with vertices at  $(\pm 1, \pm 1, 0)$  in standard units. Given the charge distribution in the figure, the electric field far away from the charges ( $r = \sqrt{x^2 + y^2 + z^2}$  denotes the distance from the origin) falls off as:



- (a)  $\frac{1}{r^3}$       (b)  $\frac{1}{r^2}$       (c)  $\frac{1}{r^4}$       (d)  $\frac{1}{r^5}$

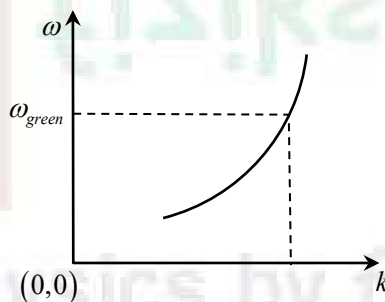
**Ans.:(c)**

**Solution.:**  $Q_{Total} = q - q + q - q + q - 2q + q = 0$ . So there is no monopole term.

Because of the symmetry of the arrangement, the dipole contributions cancel:  $\vec{p} = 0$ .

For a quadrupole,  $V \propto \frac{1}{r^3} \Rightarrow E \propto \frac{1}{r^4}$

**Q17.** For a medium, the dispersion relation for the propagation of light is shown below on a linear scale:



Here,  $\omega_{green}$  is the frequency of the green light. Which of the following statements is correct?

- (a) The refractive index of the medium for the red light is smaller than that for the blue light  
(b) The phase velocity of the green light is larger than the group velocity at that frequency  
(c) The speed of light in this medium for green light is same as that in vacuum  
(d) The phase velocity for the green light is smaller than the group velocity at that frequency

**Ans.:(d)**

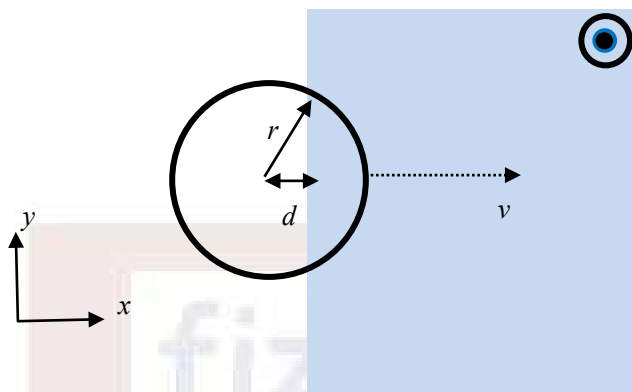
**Solution.:** Phase velocity  $v_p = \frac{\omega}{k}$ . Geometrically, this is the slope of the line joining the origin to the point on the dispersion curve.

Group velocity  $v_g = \frac{d\omega}{dk}$ . This is the **slope of the tangent** to the dispersion curve.

Therefore, at the green-light point, the tangent is steeper than the line joining the origin to that point:  $v_g > v_p$  or  $v_p < v_g$ .

**SECTION B- (For only Integrated Ph.D. candidates)**

**Q2.** A metallic ring of radius  $r$  is pushed into a region of uniform magnetic field  $B\hat{z}$  (shaded region) with a constant speed  $v$  as shown. The magnetic field is 0 outside. The electrical resistance of the loop is  $R$ . What is the external force one needs to apply on the loop to maintain its speed, when the distance of the region's edge to the center is  $d (> 0)$ ?



(a)  $\frac{4r^2 B^2 v}{R} \left(1 - \frac{d^2}{r^2}\right) \hat{x}$

(b)  $\frac{\pi r^2 B^2 v}{R} \left(1 + \frac{d^2}{r^2}\right) \hat{x}$

(c)  $\frac{r^2 B^2 v}{R} \left(1 + \frac{d}{r}\right) \hat{x}$

(d)  $\frac{\pi r^2 B^2 v}{R} \left(1 - \frac{d}{r}\right) \hat{x}$

**Ans.:(a)**

**Solution.:**

When the distance from the centre to the boundary is  $d$ , the chord length of the circle is

$$L = 2\sqrt{r^2 - d^2}$$

As the ring moves with speed  $v$ , the area inside the field changes at the rate

$$\left| \frac{dA}{dt} \right| = Lv = 2v\sqrt{r^2 - d^2}$$

Induced emf:  $|\mathcal{E}| = \left| \frac{d\phi_m}{dt} \right| = B \left| \frac{dA}{dt} \right| = 2Bv\sqrt{r^2 - d^2}$ , Induced current:  $I = \frac{|\mathcal{E}|}{R} = \frac{2Bv\sqrt{r^2 - d^2}}{R}$

Magnetic force  $\vec{F}_m = ILB(-\hat{x})$  since induced current in chord is downwards.

$$\vec{F}_m = \left[ \frac{2Bv\sqrt{r^2 - d^2}}{R} (2\sqrt{r^2 - d^2}) B \right] (-\hat{x}) = \frac{4B^2 v (r^2 - d^2)}{R} (-\hat{x})$$

By Lenz's law, this magnetic force opposes the motion. Thus, the external force must be in  $+\hat{x}$ :

$$\vec{F}_{ext} = \frac{4B^2 v (r^2 - d^2)}{R} \hat{x} = \frac{4r^2 B^2 v}{R} \left(1 - \frac{d^2}{r^2}\right) \hat{x}$$

**Q11.** The vector field,  $\vec{E} = (2xy^2 + z^3, 2x^2y, 3xz^2)V/m$ , gives the electric field in a region where  $(x, y, z)$  are the cartesian coordinates, given in metres. The difference in electric potential between the two points with coordinates  $(x, y, z) = (1, 1, 1)$  and  $(x, y, z) = (1, 2, 3)$ , is given by:

- (a)  $-29V$   
 (b)  $-16V$   
 (c)  $0V$   
 (d) An electric potential cannot be defined for this electric field configuration

**Ans.:(a)**

**Solution.:**

Given  $\vec{E} = (2xy^2 + z^3)\hat{x} + 2x^2y\hat{y} + 3xz^2\hat{z}$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2xy^2 + z^3) & 2x^2y & 3xz^2 \end{vmatrix} = \hat{x}(0-0) - \hat{y}(3z^2-3z^2) + \hat{z}(4xy-4xy) = 0$$

So, an electrostatic potential exists.

$$\vec{E} = -\vec{\nabla}V \Rightarrow -\frac{\partial V}{\partial x} = 2xy^2 + z^3 \Rightarrow V = -(x^2y^2 + z^3x) + C$$

$$V(1,1,1) = V_1 = -((1)^2(1)^2 + (1)^3(1)) + C = -2 + C$$

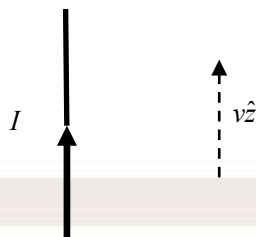
$$V(1,2,3) = V_2 = -((1)^2(2)^2 + (3)^3(1)) + C = -31 + C$$

$$\text{Thus } V_2 - V_1 = (-31 + C) - (-2 + C) = -29 V$$

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**SECTION C- (For only Ph.D. candidates)**

**Q7.** For a static observer, an infinite wire of negligible thickness with no net charge density placed along  $z$  direction carries a current  $I$ . Another observer is moving along the current with a relativistic speed  $v$ . The magnitude of the linear charge density on the wire seen by the moving observer is:



(a) 0

(b)  $\frac{I}{\sqrt{c^2 - v^2}}$

(c)  $\frac{I\sqrt{c^2 - v^2}}{vc}$

(d)  $\frac{Iv}{c\sqrt{c^2 - v^2}}$

**Ans.:(d)**

**Solution.:**

**Rest Frame ( $S$ ):**

The wire has  $\lambda_0 = 0$ , but carries current  $I$ . This is composed of positive ions with charge density  $\lambda_+$  at rest and moving electrons with density  $\lambda_-$  moving at drift velocity  $u_0$  such that  $I = \lambda_- u_0$

**Transformation to Moving Frame ( $S'$ ):**

$$\lambda' = \gamma \left( \lambda - \frac{v}{c^2} I \right)$$

Since  $\lambda = 0$  in the lab frame:  $|\lambda'| = \gamma \frac{v}{c^2} I = \frac{v}{c^2} I \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{Iv}{c\sqrt{c^2 - v^2}}$

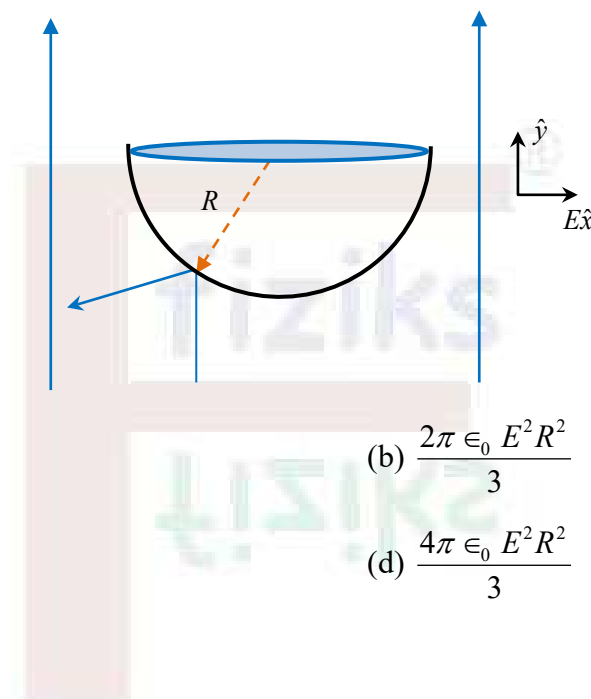
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**Q8.** A plane electromagnetic wave with an electric field

$$\vec{E}(t, \vec{r}) = E\hat{x} \cos(ky - \omega t)$$

is incident on a hemispherical mirror of radius  $R$  with a perfectly reflecting outer surface, placed as shown.

What is the electromagnetic force acting on the mirror averaged over one time-period of the wave?



(a)  $\frac{\pi \epsilon_0 E^2 R^2}{2}$

(b)  $\frac{2\pi \epsilon_0 E^2 R^2}{3}$

(c)  $\pi \epsilon_0 E^2 R^2$

(d)  $\frac{4\pi \epsilon_0 E^2 R^2}{3}$

**Ans.:(a)**

**Solution.:**

**Momentum Incident on Surface Element  $dA$  :**

Consider a surface element  $dA$  with normal vector  $\hat{n}$ . In a small time interval  $\Delta t$ , the wave covers a spatial volume along its direction of propagation:  $V = (dA \cos \theta)(c\Delta t)$

The total momentum  $\Delta p_{inc}$  contained within this volume striking the surface is:

$$\Delta p_{inc} = \frac{u}{c} V = V = u dA \cos \theta \Delta t$$

**Change in Momentum Vector (Reflection):** The normal component reverses direction.

Incident normal momentum:  $\Delta p_{n, initial} = \Delta p_{inc} \cos \theta$

Reflected normal momentum:  $\Delta p_{n, final} = -\Delta p_{inc} \cos \theta$

The total change in normal momentum experienced by the wave is:

$$\Delta p_n = \Delta p_{n, final} - \Delta p_{n, initial} = -2\Delta p_{inc} \cos \theta$$

By Newton's third law, the normal momentum transferred to the surface is:

$$\Delta p_{surface} = +2\Delta p_{inc} \cos \theta = 2u dA \cos^2 \theta \Delta t$$

### Average Force and Radiation Pressure

By Newton's second law, the normal force  $\langle F_n \rangle$  acting on area  $dA$  is:

$$\langle F_n \rangle = \frac{\langle \Delta p_{\text{surface}} \rangle}{\Delta t} = 2 \langle u \rangle dA \cos^2 \theta$$

$$\text{Radiation pressure } P(\theta) = \frac{\langle F_n \rangle}{dA} = \frac{\langle \Delta p_{\text{surface}} \rangle}{\Delta t} = 2 \langle u \rangle \cos^2 \theta$$

$$\text{Poynting Vector \& Time-Averaged Intensity: } \langle S \rangle = \frac{1}{2} c \epsilon_0 E^2, \text{ Amplitude} = E$$

$$\text{Energy density } \langle u \rangle = \frac{1}{2} \epsilon_0 E^2$$

Radiation Pressure on Perfectly Reflecting Surface:

$$P(\theta) = 2 \times \frac{1}{2} \epsilon_0 E^2 \cos^2 \theta = \epsilon_0 E^2 \cos^2 \theta$$

### Total Force Integration:

Integrating over the hemispherical surface area element

$$\langle F \rangle = \int_0^{\pi/2} P(\theta) dA \cos \theta = \int_0^{\pi/2} (\epsilon_0 E^2 \cos^2 \theta) 2\pi R^2 \sin \theta \cos \theta d\theta$$

$$\Rightarrow \langle F \rangle = 2\pi \epsilon_0 E^2 R^2 \int_0^{\pi/2} \cos^3 \theta \sin \theta d\theta = 2\pi \epsilon_0 E^2 R^2 \left[ -\frac{\cos^4 \theta}{4} \right]_0^{\pi/2} = \frac{\pi \epsilon_0 E^2 R^2}{2}$$

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TIFR Physics-2026

Solution-Modern Physics and Quantum Mechanics

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**SECTION A-(For both Int. Ph.D. and Ph.D. candidates)**

**Q18.** Hydrogen atom has a decay rate from  $n = 2$  state to  $n = 1$  state as about  $6 \times 10^8$  decays per second. If the energy level expression for the hydrogen atom is  $E_n = -\frac{13.6}{n^2} eV$  the uncertainty in the wavelength of this emission would be closest to

- (a) 30 fm                      (b) 0.1 fm                      (c) 122 nm                      (d) 19.4 nm

**Ans.:(a)**

**Solution.:**

Energy of the emitted photon

$$\Delta E = E_2 - E_1 = \left[ -\frac{13.6}{2^2} - \left( -\frac{13.6}{1^2} \right) \right] eV = (-3.4 + 13.6) eV = 10.2 eV$$

$$\Delta E = 10.2 \times 1.6 \times 10^{-19} J = 1.632 \times 10^{-18} J$$

$$\text{Photon wavelength } \lambda = \frac{hc}{\Delta E} \Rightarrow |\Delta \lambda| = \frac{hc}{(\Delta E)^2} \Delta(\Delta E)$$

$$\text{Energy uncertainty } \Delta(\Delta E) \cdot \tau \approx \frac{\hbar}{2} \Rightarrow \Delta(\Delta E) \approx \frac{\hbar \Gamma}{2} \text{ where Decay rate } \Gamma = \frac{1}{\tau}, \tau \text{ is life time.}$$

$$\Rightarrow |\Delta \lambda| = \frac{hc}{(\Delta E)^2} \frac{\hbar \Gamma}{2} \Rightarrow |\Delta \lambda| = \frac{hc \hbar \Gamma}{2(\Delta E)^2} = \frac{h^2 c \Gamma}{4\pi (\Delta E)^2}$$

$$\Rightarrow |\Delta \lambda| = \frac{(6.23 \times 10^{-34})^2 (3 \times 10^8) (6 \times 10^8)}{4\pi (1.632 \times 10^{-18})^2} \approx 2.36 \times 10^{-14} m \approx 23.6 \text{ fm}$$

So best option is (a).

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**Q19.** A quantum particle of mass  $m$  in three-dimensions is governed by the Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2) + \lambda(x^6 + y^6)$$

with  $\lambda > 0$ . If  $\psi_3(\vec{r})$  is the third excited state of the system, which of the following is correct?

(a)  $\psi_3(\vec{r})$  is an eigenstate of  $L^2$

(b)  $|\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$

(c)  $\psi_3(\vec{r})$  is an eigenstate of  $L_z$

(d)  $\psi_3(\vec{r})$  is an eigenstate of  $p_z$

**Ans.:(b)**

**Solution.:** Check parity:  $(x, y, z) \rightarrow (-x, -y, -z): \hat{H}(-\vec{r}) = \hat{H}(\vec{r})$ .

So the Hamiltonian has **inversion symmetry**.

Hence its eigenstates can be chosen to have definite parity:  $\psi(-\vec{r}) = \pm\psi(\vec{r})$

**Determine parity of the third excited state**

The unperturbed 3D harmonic oscillator has:

- Ground state: even
- First excited states: one quantum of excitation, hence odd.

The additional  $(x^6 + y^6)$  potential is also even under inversion, so it does not mix even and odd parity states.

The third excited state belongs to the odd-parity sector so  $\psi_3(-\vec{r}) = -\psi_3(\vec{r})$ .

$$|\psi_3(-\vec{r})|^2 = |-\psi_3(\vec{r})|^2 \Rightarrow |\psi_3(\vec{r})|^2 = |\psi_3(-\vec{r})|^2$$

**Check other options:**

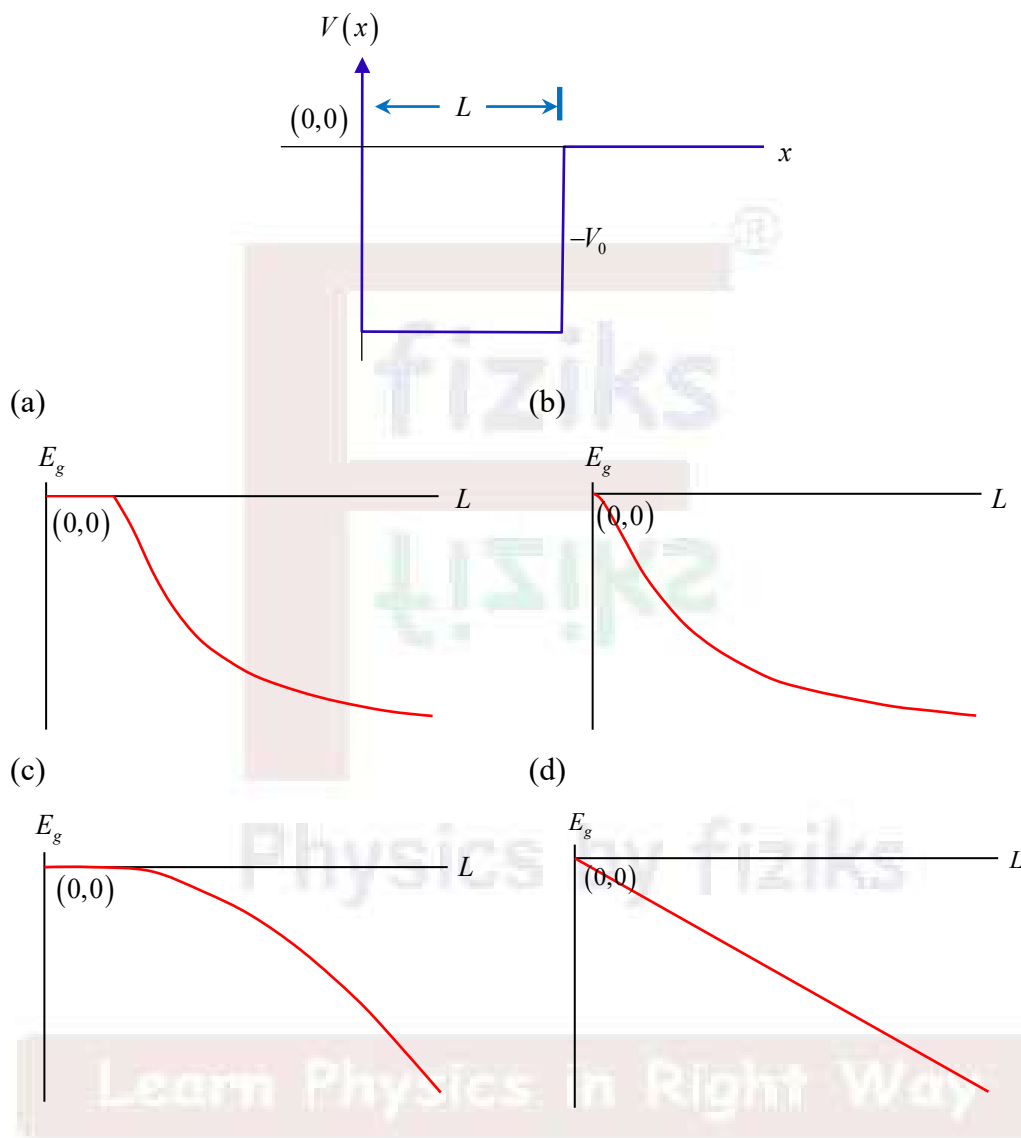
The potential contains  $\lambda(x^6 + y^6)$ , which is not spherically symmetric. Therefore, generally,

$[H, L^2] \neq 0$  and  $L^2$  is not conserved.

Also,  $[H, L_z] \neq 0$  because the  $x^6 + y^6$  term does not have full rotational symmetry about  $z$ .

And  $p_z$  is not an eigenoperator because the harmonic potential contains  $z^2$ .

**Q22.** Consider a quantum particle in a one-dimensional potential of the shape shown, with the well of width  $L$ , and  $V(x < 0) \rightarrow \infty$ . The potential in the well is  $-V_0$  and the potential for  $x \in (L, \infty)$  is 0. Which plot describes the change in the ground state energy ( $E_g$ ) and  $L$  is changed keeping  $V_0$  constant.



**Ans.:(a)**

**Solution.:**

For very small well width  $L$ , there is no bound state and  $E_g = 0$ . Above a critical width, a bound state appears and  $E_g$  decreases, eventually approaching  $-V_0$ . The correct plot has a horizontal part at  $E_g = 0$  followed by a decreasing curve.

**SECTION B- (For only Integrated Ph.D. candidates)**

**Q13.** Consider a quantum particle of mass  $m$  in one-dimension with a Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \lambda \hat{x}^8 \text{ with } \lambda > 0. \text{ The ground state energy of the particle scales with } \lambda \text{ as:}$$

- (a)  $\propto \lambda^{1/4}$  (b)  $\propto \lambda^{1/5}$  (c)  $\propto \lambda^{1/3}$  (d)  $\propto \lambda^{1/6}$

**Ans.:(b)**

**Solution.:** Using dimensional scaling / virial theorem:

Let  $x \approx L$ , then momentum scales as  $p \approx \frac{\hbar}{L}$ . The energy scales as:  $E(L) \approx \frac{\hbar^2}{2mL^2} + \lambda L^8$

To find the minimum energy scale:  $\frac{dE}{dL} = 0 \Rightarrow \frac{\hbar^2}{mL^3} + 8\lambda L^7 = 0 \Rightarrow L^{10} \approx \frac{1}{\lambda} \Rightarrow L \approx \lambda^{-1/10}$

Substitute  $L$  back into the kinetic energy or potential term:

$$E \propto \lambda (\lambda^{-1/10})^8 \propto \lambda \cdot \lambda^{-4/5} \propto \lambda^{1/5}$$

**Q14.** A one-dimensional quantum mechanical wave function is given by:  $\psi(x) = Ne^{-|x-b|/a}$ , where  $x \in (-\infty, \infty)$  and  $N$  is a normalization factor. What is the expectation value of the kinetic energy in this state?

- (a)  $\frac{\hbar^2}{2ma^2} + \frac{\hbar^2}{2mb^2}$  (b)  $\frac{2\hbar^2}{ma^2}$  (c)  $\frac{\hbar^2}{2ma^2}$  (d)  $\frac{2\hbar^2}{mb^2}$

**Ans.:(c)**

**Solution.:** **Note:**  $\because \psi(x) = Ne^{-|x-b|/a}$ ,  $b$  merely shifts the center of the wavefunction. It should not affect the kinetic energy.

**Kinetic-energy expectation**  $\langle \hat{T} \rangle = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx$  where  $\hat{T} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$

For  $x > b$ ;  $\psi(x) = Ne^{-(x-b)/a} \Rightarrow \frac{d\psi}{dx} = -\frac{1}{a}\psi$ ; For  $x < b$ ;  $\psi(x) = Ne^{(x-b)/a} \Rightarrow \frac{d\psi}{dx} = \frac{1}{a}\psi$

In both regions,  $\left| \frac{d\psi}{dx} \right|^2 = \frac{1}{a^2} |\psi|^2$  so  $\int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx = \frac{1}{a^2} \int_{-\infty}^{\infty} |\psi|^2 dx = \frac{1}{a^2}$

Since the wavefunction is normalized,  $\int_{-\infty}^{\infty} |\psi|^2 dx = 1$ .

Thus  $\langle \hat{T} \rangle = -\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx = -\frac{\hbar^2}{2ma^2}$

**SECTION C- (For only Ph.D. candidates)**

**Q1.** Consider a quantum particle moving in one spatial dimension with the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

Let  $|\psi(t)\rangle$  be the state of the system where  $|\psi(0)\rangle$  is a normalized eigenstate of this Hamiltonian. What is the value of  $\langle \psi(t) | \hat{x}\hat{p} | \psi(t) \rangle$ ? (Hint: Consider  $\frac{d\langle \psi(t) | \hat{x}^2 | \psi(t) \rangle}{dt}$ )

- (a)  $\frac{i\hbar}{2}$  (b)  $-i\hbar$  (c)  $\hbar$  (d) 0

**Ans.:(a)**

**Solution.:** Use Ehrenfest theorem  $\frac{d\langle x^2 \rangle}{dt} = \frac{1}{i\hbar} \langle [x^2, H] \rangle = 0 \quad \because \langle x^2 \rangle = \text{constant}$

Calculate the commutator (Only  $\frac{\hat{p}^2}{2m}$  contributes):

$$[x^2, H] = \frac{1}{2m} [x^2, p^2] = \frac{1}{2m} 2i\hbar (xp + px) \quad \because [x, p] = i\hbar$$

$$\frac{d\langle x^2 \rangle}{dt} = \frac{1}{i\hbar} \langle [x^2, H] \rangle = 0 \Rightarrow \frac{1}{i\hbar} \frac{1}{2m} 2i\hbar \langle xp + px \rangle = 0 \Rightarrow \langle xp + px \rangle = 0 \Rightarrow \langle xp \rangle + \langle px \rangle = 0$$

$$\because [x, p] = i\hbar \Rightarrow xp - px = i\hbar \Rightarrow \langle xp \rangle - \langle px \rangle = i\hbar \Rightarrow \langle xp \rangle - \langle px \rangle = i\hbar$$

$$\Rightarrow \langle xp \rangle = \frac{i\hbar}{2}$$

**Q2.** In the standard  $|S^2, S_z\rangle$  eigenbasis of a spin 1 particle, the matrix corresponding to the operator  $\hat{S}_x^2 - \hat{S}_y^2$  is given by:

- (a)  $\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (b)  $\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  (c)  $\hbar^2 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$  (d)  $\hbar^2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

**Ans.:(d)**

**Solution.:**  $S_x = \frac{S_+ + S_-}{2}$ ,  $S_y = \frac{S_+ - S_-}{2i} \Rightarrow S_x^2 - S_y^2 = \frac{1}{2} (S_+^2 + S_-^2)$ . For spin  $s = 1$ ,  $m = +1, 0, -1$

The ladder operators satisfy  $S_+ |m\rangle = \hbar \sqrt{s(s+1) - m(m+1)} |m+1\rangle$

$S_+ |-1\rangle = \sqrt{2}\hbar |0\rangle$ ,  $S_+ |0\rangle = \sqrt{2}\hbar |+1\rangle \Rightarrow S_+^2 |-1\rangle = 2\hbar^2 |+1\rangle$ . Similarly,  $S_-^2 |+1\rangle = 2\hbar^2 |-1\rangle$

$$(S_x^2 - S_y^2)|-1\rangle = \hbar^2|+1\rangle \quad \text{and} \quad (S_x^2 - S_y^2)|+1\rangle = \hbar^2|-1\rangle \quad \because S_x^2 - S_y^2 = \frac{1}{2}(S_+^2 + S_-^2)$$

The  $m = 0$  state goes to zero.

Thus, the matrix is  $\hbar^2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ .

**Q3.** The Hamiltonian of a quantum system in the basis states,  $|1\rangle, |2\rangle$ , and  $|3\rangle$  is given by the matrix:

$$\omega_0 \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

If the system is in the state  $|\psi(0)\rangle = (|1\rangle - |3\rangle)/\sqrt{2}$  at  $t = 0$ , the probability of the system to be in state  $|2\rangle$  at time  $t$  is given by:

- (a)  $\sin^2(\omega_0 t)$  (b)  $\sin^2(2\omega_0 t)$   
(c) 0 (d)  $\sin^2\left(\frac{\omega_0 t}{2}\right)$

**Ans.:(c)**

**Solution.:** The initial state as  $|\psi(0)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad \because |\psi(0)\rangle = (|1\rangle - |3\rangle)/\sqrt{2}$

$$H|\psi(0)\rangle = \frac{1}{\sqrt{2}} \omega_0 \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

So, the initial state is an eigenstate of  $H$  with energy  $E = 0$ .

An energy eigenstate evolves as  $|\psi(t)\rangle = e^{-iEt/\hbar} |\psi(0)\rangle = |\psi(0)\rangle \quad \because E = 0$

Thus, there is never any component of  $|2\rangle$ . Therefore,  $P_2(t) = 0$

**Note:** If the initial state is an eigenstate of  $H$ , it does not change its probability distribution with time.



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Solution-Thermodynamics and Statistical Mechanics

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**SECTION A-(For both Int. Ph.D. and Ph.D. candidates)**

**Q11.** The equation of state of a gas is given by:  $\left(p + \frac{a}{v^2}\right)(v-b) = RT$ , where  $v$  is the volume per mole and  $R, a, b$  are constants. The internal energy per mole of the gas is given by:

$$U(T, v) = \frac{3}{2} RT - \frac{a}{v}$$

Which of the following gives the adiabatic equation of state at fixed particle number for the gas?

- (a)  $(v-b)^2 T^{\frac{3}{2}} = \text{constant}$  (b)  $(v-b)^2 T^{-\frac{3}{2}} = \text{constant}$   
(c)  $(v-b)^2 T^3 = \text{constant}$  (d)  $(v-b)^2 T^{-3} = \text{constant}$

**Ans.:(c)**

**Solution.:** First Law of Thermodynamics for a Reversible Adiabatic Process:

$$dQ = dU + pdv = 0 \Rightarrow dU = -pdv$$

$$\therefore U(T, v) = \frac{3}{2} RT - \frac{a}{v} \Rightarrow dU = \frac{3}{2} R dT + \frac{a}{v^2} dv \text{ and } p = \frac{RT}{(v-b)} - \frac{a}{v^2}$$

$$\text{So, } dU = -pdv \Rightarrow \frac{3}{2} R dT + \frac{a}{v^2} dv = -\left(\frac{RT}{v-b} - \frac{a}{v^2}\right) dv \Rightarrow \frac{3}{2} R dT = -\frac{RT}{v-b} dv$$

$$\Rightarrow \frac{3}{2} \frac{dT}{T} + \frac{dv}{v-b} = 0 \Rightarrow \frac{3}{2} \ln T + \ln(v-b) = c \Rightarrow \ln T^{3/2} (v-b) = c$$

$$\Rightarrow T^{3/2} (v-b) = e^c = c' \Rightarrow T^3 (v-b)^2 = \text{constant}$$

**Q23.** A non-interacting, classical gas is made up of  $N$  Nitrogen molecules. The specific heat at constant volume is  $C_v$ .  $C_v / (Nk_B T)$  is given by:

(Ignore rotations about the axis of symmetry.)

- (a) 7/2 for some very low  $T$ , 5/2 for temperatures around the room temperature, 3/2 for some high temperatures  
(b) 3/2 for some very low  $T$ , 2 for temperatures around the room temperature, 5/2 for some high temperatures  
(c) 3/2 for some very low  $T$ , 2 for temperatures around the room temperature, 3 for some high temperatures  
(d) 3/2 for some very low  $T$ , 5/2 for temperatures around the room temperature, 7/2 for some high temperatures

**Ans.:(d)**

**Solution.:** Diatomic Nitrogen ( $N_2$ ) Degrees of Freedom (ignoring axis rotation):

**Very High  $T$ :** Translational (3) + Rotational (2) + Vibrational (2) = 7 d.o.f  $\Rightarrow C_v = \frac{7}{2} Nk_B$

**Room temp  $T$ :** Translational (3) + Rotational (2) = 5 d.o.f  $\Rightarrow C_v = \frac{5}{2} Nk_B$

(Vibration frozen out)

**Very low  $T$ :** Translational (3) = 3 d.o.f  $\Rightarrow C_v = \frac{3}{2} Nk_B$

(Rotation and Vibration frozen out)

### SECTION B- (For only Integrated Ph.D. candidates)

**Q5.** Scientists have discovered a new type of particle, which can occupy a quantum state with 0, 1 or 2 particles. If a quantum state has energy  $\omega$ , the average occupation of this state in a system at temperature  $T$  and chemical potential  $\mu$  is given by

$$\begin{aligned} \text{(a)} \quad & \frac{2}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}} & \text{(b)} \quad & \frac{2 + e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}} \\ \text{(c)} \quad & \frac{1 + 2e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}} & \text{(d)} \quad & \frac{1}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}} \end{aligned}$$

**Ans.:(b)**

**Solution.:** The possible occupation numbers are  $n = 0, 1, 2$ . The energy of one particle is  $\omega$ , and the chemical potential is  $\mu$ .

For occupation  $n$ , the grand-canonical statistical weight is  $e^{-\frac{n(\omega-\mu)}{T}}$ .

Let  $x = e^{-\frac{(\omega-\mu)}{T}}$ , then the three possible weights are  $1, x, x^2$ .

Therefore, the partition function is  $Z = 1 + x + x^2$

The average occupation is  $\langle n \rangle = \frac{0(1) + 1(x) + 2(x^2)}{1 + x + x^2} = \frac{x + 2x^2}{1 + x + x^2} = \frac{x^{-1} + 2}{x^{-2} + x^{-1} + 1}$

$$\langle n \rangle = \frac{2 + e^{\frac{(\omega-\mu)}{T}}}{1 + e^{\frac{(\omega-\mu)}{T}} + e^{\frac{2(\omega-\mu)}{T}}} \quad \because x^{-1} = e^{\frac{(\omega-\mu)}{T}}$$

**Q6.** A cycle for an ideal gas consists of the following three reversible processes:

- (i) Isothermal expansion at  $T_H$  from  $V_1$  to  $V_2$
- (ii) Isochoric (constant volume) cooling from  $T_H$  to  $T_C$
- (iii) Adiabatic compression from  $T_C$  and  $V_2$  back to the initial state  $(T_H, V_1)$

If  $C_V$  is the constant volume heat capacity of the gas, the efficiency  $\eta$  of this cycle is:

- (a)  $\eta = 1 - \frac{C_V (T_H - T_C)}{C_P T_H}$
- (b)  $\eta = 1 - \frac{T_C}{T_H}$
- (c)  $\eta = 1 - \frac{C_V (T_H - T_C)}{RT_H \log(V_2/V_1)}$
- (d)  $\eta = 1 - \frac{C_V T_C}{RT_H \log(V_2/V_1)}$

**Ans.:(c)**

**Solution.:** We know that efficiency  $\eta = \frac{W_{net}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$ .

**Heat absorbed:**

In the first process is isothermal: For an ideal gas,  $Q_{in} = W = RT_H \log\left(\frac{V_2}{V_1}\right)$

**Heat rejected:** The second process is isochoric, so  $W = 0$

Therefore,  $Q_{out} = C_V (T_H - T_C)$ .

Hence,  $\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{C_V (T_H - T_C)}{RT_H \log(V_2/V_1)}$

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## SECTION C- (For only Ph.D. candidates)

**Q11.** Consider a single particle in a potential well with an energy spectrum  $E_n = n\epsilon$ , for  $n = 0, 1, 2, 3, \dots$ , with  $\epsilon > 0$ . The  $n^{\text{th}}$  energy level has a degeneracy  $g(n) = 2n + 1$ . This system is at temperature  $T$ . What is the value of  $\langle \hat{H} \rangle$ ? ( $k$  is the Boltzmann constant.)

$$(a) \epsilon \in \frac{3e^{\frac{\epsilon}{kT}} + 1}{\left(e^{\frac{2\epsilon}{kT}} - 1\right)}$$

$$(b) \epsilon \in \frac{1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)}$$

$$(c) \epsilon \in \frac{3e^{\frac{\epsilon}{kT}} + 1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)}$$

$$(d) \epsilon \in \frac{1}{\left(e^{\frac{\epsilon}{kT}} - 1\right)^2}$$

**Ans.:(a)**

**Solution.:** Partition function  $Z = \sum_{n=0}^{\infty} (2n+1)x^n$  where  $x = e^{-\epsilon/k_B T}$ .

$$\because \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{and} \quad \because \sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$$

$$\Rightarrow Z = \frac{2x}{(1-x)^2} + \frac{1}{1-x} = \frac{2x+1-x}{(1-x)^2} \Rightarrow Z = \frac{1+x}{(1-x)^2}$$

$$\text{Average energy } \langle \hat{H} \rangle = -\frac{\partial \ln Z}{\partial \beta} \quad \text{where } \beta = \frac{1}{k_B T}.$$

$$\because Z = \frac{1+x}{(1-x)^2} \Rightarrow \ln Z = \ln(1+x) - 2\ln(1-x)$$

$$\langle \hat{H} \rangle = -\frac{\partial \ln Z}{\partial \beta} = -\left[ \frac{1}{1+x} + \frac{2}{1-x} \right] \frac{dx}{d\beta}$$

$$\because x = e^{-\epsilon/k_B T} = e^{-\beta\epsilon} \Rightarrow \frac{dx}{d\beta} = -\epsilon e^{-\beta\epsilon} = -\epsilon x$$

$$\Rightarrow \langle \hat{H} \rangle = \epsilon x \left[ \frac{1}{1+x} + \frac{2}{1-x} \right] = \epsilon x \left( \frac{3+x}{1-x^2} \right) = \epsilon \frac{3x+x^2}{1-x^2} = \epsilon \frac{3x^{-1}+1}{x^{-2}-1}$$

$$\Rightarrow \langle \hat{H} \rangle = \epsilon \frac{3e^{\frac{\epsilon}{kT}} + 1}{\left(e^{\frac{2\epsilon}{kT}} - 1\right)} \quad \because x^{-1} = e^{\epsilon/k_B T}$$

**Q12.** A system of  $N$  classical non-interacting particles of mass  $m$  is confined to a cubic box of volume  $V$ . Inside the box is a region of volume  $V_0 (< V)$ , where a constant potential  $U$  is present. If  $P$  is the pressure of the system,  $T$  is its temperature, then:

$$(a) \frac{P}{Nk_B T} = \frac{1}{V - V_0}$$

$$(b) \frac{P}{Nk_B T} = \frac{1}{V - V_0 e^{-U/k_B T}}$$

$$(c) \frac{P}{Nk_B T} = \frac{1}{V_0 e^{-U/k_B T}}$$

$$(d) \frac{P}{Nk_B T} = \frac{1}{V - V_0 (1 - e^{-U/k_B T})}$$

**Ans.:(d)**

**Solution.:**

**Effective single-particle partition function**

There are two regions:

- Volume  $V - V_0$ , where potential = 0
- Volume  $V_0$ , where potential =  $U$

Thus, apart from momentum-dependent factors,

$$Z_1 \propto (V - V_0) + V_0 e^{-U/k_B T} \Rightarrow Z_1 \propto V - V_0 (1 - e^{-U/k_B T})$$

For  $N$  non-interacting particles,  $Z_N \propto (Z_1)^N$

$$\text{Pressure is } P = k_B T \left( \frac{\partial \ln Z_N}{\partial V} \right)_T$$

$$\Rightarrow P = \frac{Nk_B T}{V - V_0 (1 - e^{-U/k_B T})} \Rightarrow \frac{P}{Nk_B T} = \frac{1}{V - V_0 (1 - e^{-U/k_B T})}$$

**Q13.** A Bosonic excitation has a density of states given by:

$$\rho(E) = \rho_0 \left[ 1 - \left( \frac{E}{E_0} - 1 \right)^2 \right]$$

for  $0 \leq E \leq 2E_0$ , and 0 otherwise. If the two-dimensional system is in a temperature range  $k_B T \ll E_0$ , the variation of the specific heat with temperature is given by:

$$(a) C_V \propto T^0$$

$$(b) C_V \propto T^3$$

$$(c) C_V \propto T^2$$

$$(d) C_V \propto T$$

**Ans.:(a)**

**Solution.:**

Bose occupation:  $n(E) = \frac{1}{e^{E/k_B T} - 1}$

$\Rightarrow k_B T \ll E_0$  and all relevant energies satisfy  $E \leq 2E_0$  so  $\frac{E}{k_B T} \ll 1 \Rightarrow e^{E/k_B T} - 1 \approx \frac{E}{k_B T}$

Thus  $n(E) = \frac{1}{e^{E/k_B T} - 1} \approx \frac{k_B T}{E}$

The excitation energy is  $U = \int E \rho(E) n(E) dE \approx \int E \rho(E) \frac{k_B T}{E} dE \approx k_B T \int \rho(E) dE$

The integral over the density of states is temperature independent because  $E_0$  is fixed, so

$$U \propto T.$$

Specific heat  $C_V = \frac{dU}{dT} \propto T^0$

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Solution-Electronics and Experimental Physics

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**SECTION A-(For both Int. Ph.D. and Ph.D. candidates)**

**Q3.** A student performs an experiment to measure the acceleration due to gravity,  $g$ , using a simple pendulum. The student measures  $L = 1.00 \pm 0.01 \text{ m}$  and the time for 50 oscillations  $t_{50} = 100.0 \pm 1.0 \text{ s}$ . Based on this experiment, what is the calculated value of  $g$  and its uncertainty, assuming that the errors on the length and time are statically independent?

- (a)  $9.87 \pm 0.44 \text{ m/s}^2$  (b)  $9.87 \pm 0.11 \text{ m/s}^2$   
(c)  $9.87 \pm 0.22 \text{ m/s}^2$  (d)  $9.87 \pm 0.05 \text{ m/s}^2$

**Ans.:(c)**

**Solution.:**

**Value of  $g$ :** The period of a simple pendulum is given by:  $T = \frac{t_{50}}{50} = \frac{100.0}{50} = 2.0 \text{ sec}$

$$\therefore T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 (1.00)}{(2.0)^2} = \pi^2 \approx 9.869 \approx 9.87 \text{ m/s}^2$$

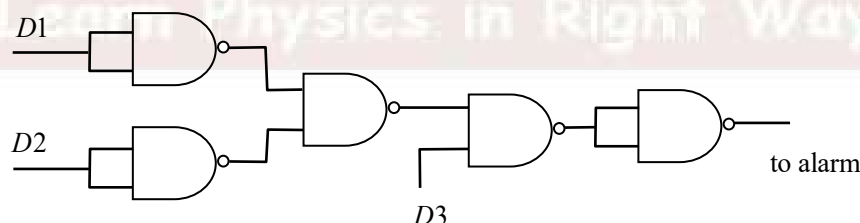
**Calculation of Uncertainty ( $\Delta g$ ):**  $\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(2\frac{\Delta T}{T}\right)^2}$

where  $\frac{\Delta L}{L} = \frac{0.01}{1.00} = 0.01$ ,  $\frac{\Delta T}{T} = \frac{\Delta t_{50}}{t_{50}} = \frac{1.0}{100.0} = 0.01$

$$\Rightarrow \frac{\Delta g}{g} = \sqrt{(0.01)^2 + (2 \times 0.01)^2} = \sqrt{0.0001 + 0.0004} = \sqrt{0.0005} \approx 0.02236$$

$$\Rightarrow \Delta g = g \times 0.02236 = 9.87 \times 0.02236 \approx 0.22 \text{ m/s}^2$$

**Q8.** Three nearby laboratory rooms **1, 2, 3** have one smoke detector each labelled **D1, D2, D3** respectively, for fire safety. In case smoke is detected, the detector output goes to a logic state *True*. However, to prevent false alarm, these outputs are connected to the logic circuit below whose output activates a single alarm.

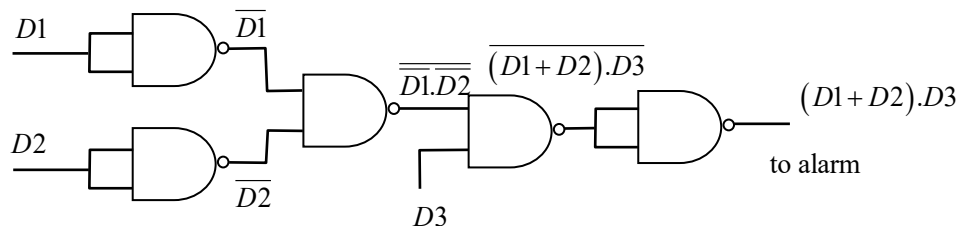


In which of the following cases will the alarm definitely ring:

- (a) Smoke is detected in rooms 2 and 3  
(b) Smoke is detected in rooms 1 and 2  
(c) Smoke is detected in rooms 1 or 2  
(d) Smoke is detected in any two of three rooms

Ans.:(a)

Solution.:



The alarm output is **True** (1) when:  $(D1 + D2).D3 = 1 \Rightarrow D3 = 1 \text{ AND } (D1 = 1 \text{ OR } D2 = 1)$ .

This means smoke must be detected in **Room 3 AND at least one of Room 1 or Room 2**. Thus, if smoke is detected in **rooms 2 and 3** (or 1 and 3), the alarm definitely ring.

**Q12.** Electrons in a metal are accelerated under a constant electric field  $\vec{E}$  and experience a drag  $-\eta\vec{v}$  due to the surrounding medium. If the density of electrons is  $n$  and the current is constant, what is the conductivity  $\sigma$  of the metal?

- (a)  $\sigma = \frac{ne^2}{m\eta}$  (b)  $\sigma = \frac{neE}{m\eta}$   
(c)  $\sigma = \frac{ne^2E}{\eta}$  (d)  $\sigma = \frac{ne^2}{\eta}$

Ans.:(d)

**Solution.:** Under a constant electric field  $\vec{E}$ , electrons experience two forces:

$$\text{Electric force: } \vec{F}_e = -e\vec{E} \text{ and Drag force: } \vec{F}_{drag} = -\eta\vec{v}$$

$$\text{At steady state, acceleration is zero: } m\frac{d\vec{v}}{dt} = -e\vec{E} - \eta\vec{v} = 0 \Rightarrow \vec{v} = -\frac{e\vec{E}}{\eta}$$

$$\text{Current density } J = ne|\vec{v}| = \frac{ne^2E}{\eta}$$

$$\text{Conductivity } \sigma = \frac{J}{E} = \frac{ne^2}{\eta}$$

**Q13.** (Note: Either (a) or (b) can be correct, depending on the applications; both (a) or (b) will be given full marks.)

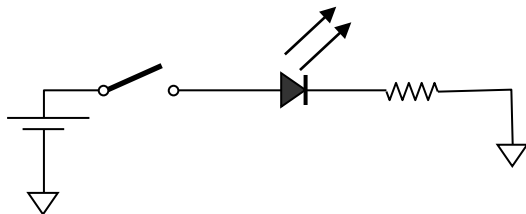
A thermistor measures the temperature by measuring the change of electric resistivity of a material. Which of the following types of materials are most suited for marking thermistors to work in the temperature range  $-100$  degree C to  $300$  degree C?

- (a) Semiconductors (b) Metals  
(c) Superconductors (d) Gas of atoms

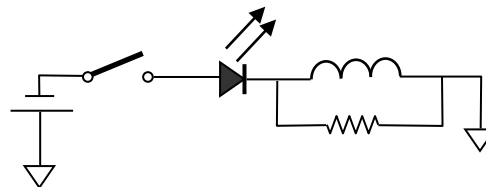
Ans.: (a) and (b)

**Q16.** Which of these circuits made with resistor, capacitor and inductor will be the best to get light out of the LED using a  $9V$  battery, after the switch is closed?

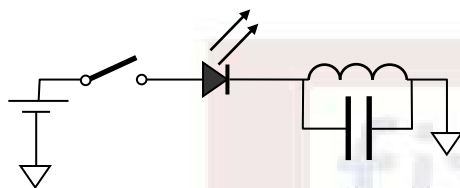
(a)



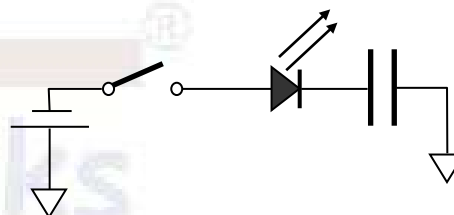
(b)



(c)



(d)



**Ans.: (a)**

**Solution.:**

To light the LED steadily from a  $9V$  battery:

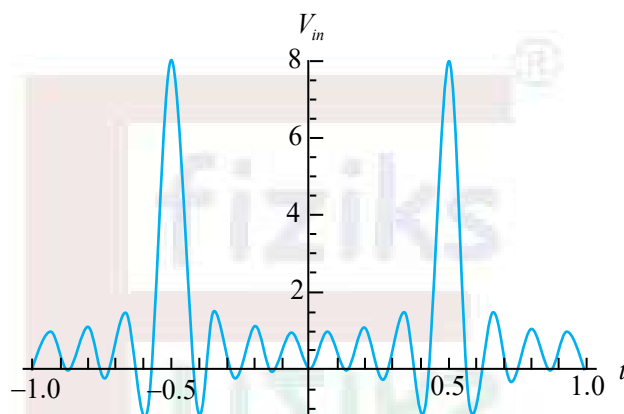
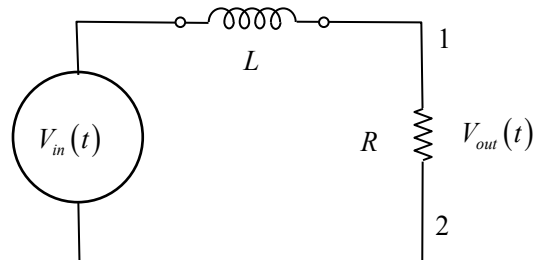
- A capacitor blocks DC, so circuits (c) and (d) will not work.
- An inductor in parallel with a resistor (b) will short the resistor at steady state, allowing excessive current that may burn the LED.
- A series resistor (a) limits the current and provides steady light.

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**SECTION B- (For only Integrated Ph.D. candidates)**

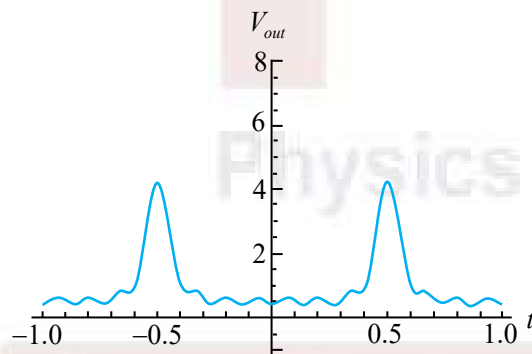
**Q15.** The figure below shows a circuit. The input waveform  $V_{in}(t)$  is shown.  $R = 5\Omega$  and  $L = 0.25 \text{ V.s/A}$ .

An ideal oscilloscope is connected between points 1 and 2, and shows a signal  $V_{out}(t)$ . What is the closest representation of  $V_{out}(t)$ ?

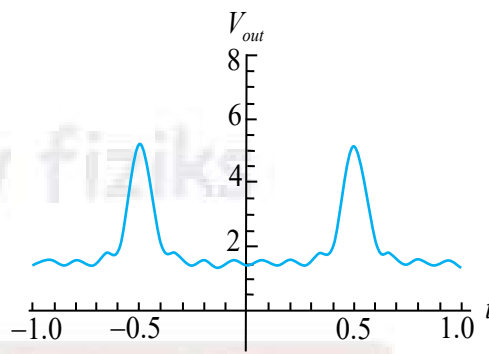


(a)

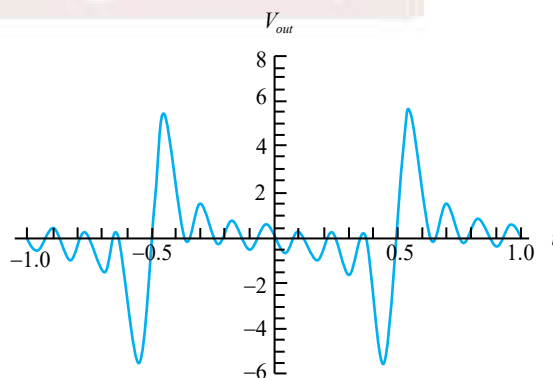
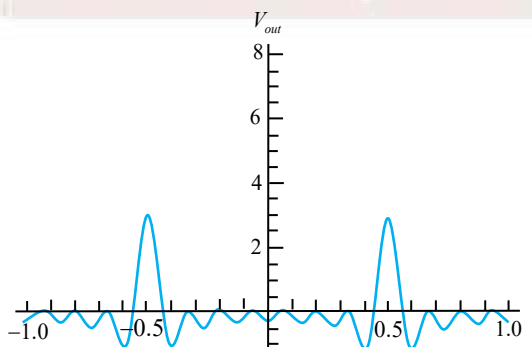
(b)



(c)



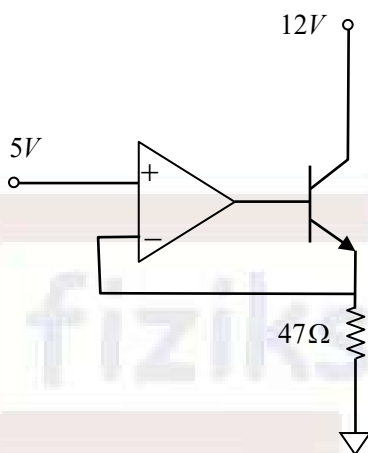
(d)



**Ans.:(a)**

## SECTION C- (For only Ph.D. candidates)

**Q14.** Op-Amps normally cannot output very high currents, which is typically limited to tens of mA. To overcome this, a transistor is used to boost the current output onto a load resistor. The circuit below uses a Si transistor with common-emitter mode current gain  $\beta = 120$ . What is approximately the current being drawn from the output of the Op-Amp here?



- (a)  $1.23\text{ mA}$  (b)  $0.76\text{ mA}$   
(c)  $0.88\text{ mA}$  (d)  $2.11\text{ mA}$

**Ans.:(c)**

**Solution.:** Op-Amp circuit is operating in negative feedback to regulate output voltage across the  $47\Omega$  load resistor:

Voltage across load resistor  $R_L = 47\Omega$ ,  $V_L = 5V$

Emitter current through the load:  $I_E = \frac{V_L}{R_L} = \frac{5V}{47\Omega} = 106.38\text{ mA}$

The base current  $I_B$  drawn directly from the Op-Amp output is:

$$I_B = \frac{I_E}{\beta + 1} = \frac{106.38\text{ mA}}{121} \approx 0.88\text{ mA}$$



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Solution-Solid State Physics

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**SECTION A-(For both Int. Ph.D. and Ph.D. candidates)**

**Q14.** An electron beam is accelerated over a potential  $V$  and strikes a crystal with lattice constant  $5\text{\AA}$ . What is the minimum  $V$  needed to resolve the crystal?

- (a) 5 V (b) 30 V (c) 300 V (d) 0.5 V

**Ans.:(a)**

**Solution.:** To resolve features of size  $a$ , the de Broglie wavelength  $\lambda$  of the electrons must be at most on the order of the lattice constant ( $\lambda \leq a$ ).

The de Broglie wavelength for an electron accelerated through a potential difference  $V$  (in Volts) is given by:

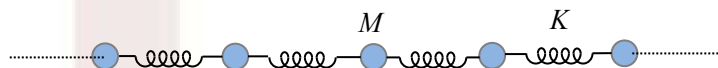
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \approx \frac{12.27}{\sqrt{V}} \text{\AA} \Rightarrow 5\text{\AA} = \frac{12.27}{\sqrt{V}} \text{\AA} \Rightarrow \sqrt{V} = 2.454 \Rightarrow V \approx 6.02 \text{ V}$$

**SECTION B- (For only Integrated Ph.D. candidates)**

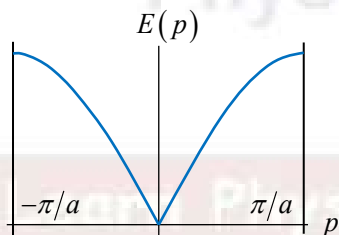
**No Question**

**SECTION C- (For only Ph.D. candidates)**

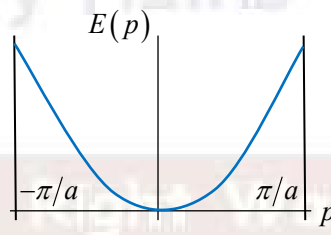
**Q15.** Consider a one-dimensional lattice where each site has mass  $M$  and the lattice constant is  $a$ . Nearest neighbours are connected by springs of constant  $K$ . Which of the following best represents the dispersion of longitudinal oscillations in the lattice?



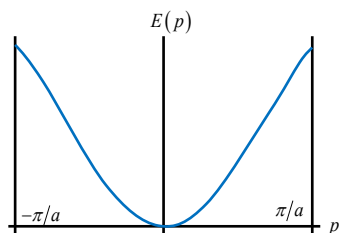
(a)



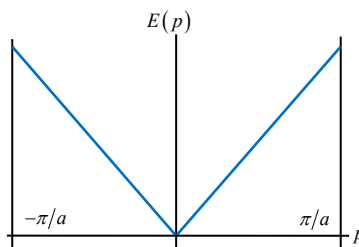
(b)



(c)



(d)



**Ans.:(a)**



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Solution-Nuclear and Particle Physics

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SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

No Question

SECTION B- (For only Integrated Ph.D. candidates)

**Q12.** Reactions in a closed container convert one molecule of  $A$  to one molecule of  $B$  at a rate  $\Gamma_1$  and one molecule  $B$  to one molecule of  $C$  at a rate  $\Gamma_2$ . At time  $t = 0$ , there are  $N_0$  number of  $A$  molecules and no  $B$  or  $C$  molecules in the container. At what time is the number of  $B$  molecules in the container the maximum?

- (a)  $\frac{\log(\Gamma_2)}{\Gamma_2} - \frac{\log(\Gamma_1)}{\Gamma_1}$  (b)  $\frac{\log(\Gamma_1/\Gamma_2)}{\Gamma_2 - \Gamma_1}$   
(c)  $\frac{\log(\Gamma_2)}{\Gamma_2} + \frac{\log(\Gamma_1)}{\Gamma_1}$  (d)  $\frac{\log(\Gamma_2/\Gamma_1)}{\Gamma_2 - \Gamma_1}$

Ans.:(d)

**Solution.:** Sequential reactions  $A \xrightarrow{\Gamma_1} B \xrightarrow{\Gamma_2} C$ ;  $N_A(0) = N_0$ ,  $N_B(0) = 0$

**Number of A molecules:**

$$\frac{dN_A}{dt} = -\Gamma_1 N_A \Rightarrow N_A(t) = N_0 e^{-\Gamma_1 t}$$

**Equation for B:**

$$\frac{dN_B}{dt} = \Gamma_1 N_A - \Gamma_2 N_B \Rightarrow \frac{dN_B}{dt} + \Gamma_2 N_B = \Gamma_1 N_0 e^{-\Gamma_1 t}$$

The solution is:  $N_B(t) = \frac{N_0 \Gamma_1}{\Gamma_2 - \Gamma_1} (e^{-\Gamma_1 t} - e^{-\Gamma_2 t})$

**Maximum condition:**

$$\begin{aligned} \frac{dN_B}{dt} = 0 &\Rightarrow \Gamma_1 e^{-\Gamma_1 t} = \Gamma_2 e^{-\Gamma_2 t} \\ \Rightarrow e^{(\Gamma_2 - \Gamma_1)t} &= \frac{\Gamma_2}{\Gamma_1} \Rightarrow t = \frac{\log(\Gamma_2/\Gamma_1)}{\Gamma_2 - \Gamma_1} \end{aligned}$$

**SECTION C- (For only Ph.D. candidates)**

**Q4.** An  $Fe(A = 56, Z = 26)$  nucleus starts from infinity with an initial speed  $v$  and moves head on towards a  $Co(A = 59, Z = 27)$  nucleus which is initially stationary. The nuclei are uniformly charged spheres of radii  $R = 1.2 A^{1/3} fm$ , and a nuclear reaction can take place when they touch each other. What is the smallest  $v$  for which this reaction can occur? Neglect the mass difference between neutrons and protons and nuclear forces, and assume that the two nuclei remain spherical throughout the motion.



(a)  $2.70 \times 10^7 m/s$

(b)  $1.91 \times 10^7 m/s$

(c)  $1.37 \times 10^7 m/s$

(d)  $0.95 \times 10^7 m/s$

**Ans.:(a)**

**Solution.:**  $Fe(A = 56, Z = 26)$ ,  $Co(A = 59, Z = 27)$

The crucial point is that both nuclei move after the  $Fe$  nucleus starts approaching  $Co$ , so we must use the **reduced mass**.

At contact,  $R_{Fe} + R_{Co} = 1.2(56^{1/3} + 59^{1/3}) fm \approx 9.27 fm$   $\because R = 1.2 A^{1/3} fm$

Coulomb energy:  $U = \frac{e^2 Z_{Fe} Z_{Co}}{4\pi\epsilon_0 r} = \frac{e^2 Z_{Fe} Z_{Co}}{4\pi\epsilon_0 (R_{Fe} + R_{Co})} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2 (26 \times 27)}{9.27 \times 10^{-15}}$

$\Rightarrow U = \frac{6.310 \times 10^{-25}}{9.27 \times 10^{-15}} \approx 1.75 \times 10^{-11} J$

Now  $K_{relative} = \frac{1}{2} \mu v^2$

where  $\mu = \frac{m_{Fe} m_{Co}}{m_{Fe} + m_{Co}} = \frac{56 \times 59}{56 + 59} u \approx 28.73 u = 28.73 \times 1.67 \times 10^{-27} kg = 4.77 \times 10^{-26} kg$

At the threshold,  $K_{relative} = \frac{1}{2} \mu v^2 = U \Rightarrow v^2 = \frac{2U}{\mu} = \frac{2 \times 1.75 \times 10^{-11}}{4.77 \times 10^{-26}} \approx 7.33 \times 10^{-14} m^2 / s^2$

$\Rightarrow v \approx 2.70 \times 10^7 m/s$



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Solution-Waves and Optics

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## SECTION A-(For both Int. Ph.D. and Ph.D. candidates)

**Q6.** A thin film of oil (refractive index,  $n_{oil} = 1.40$ ) floats on a puddle of water (refractive index,  $n_{water} = 1.33$ ) on a road. The film is illuminated at normal incidence from air by white light. In the light reflected vertically, green colour (with wavelength in air  $\lambda_0 = 560\text{ nm}$ ) is enhanced. The smallest possible thickness of the oil film is approximately:

- (a)  $100\text{ nm}$  (b)  $2000\text{ nm}$   
(c)  $200\text{ nm}$  (d)  $360\text{ nm}$

**Ans.:(a)**

**Solution.:**

For thin-film interference in reflected light, the condition for constructive enhancement is

$$2n_{oil}t = \left(m + \frac{1}{2}\right)\lambda_0$$

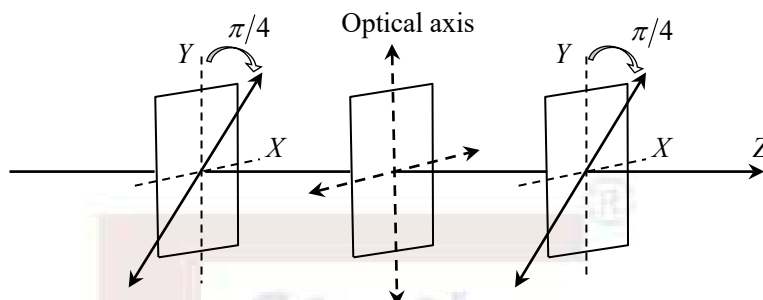
because there is only one phase-reversal (at the air–oil surface; oil–water reflection has no phase change).

Smallest thickness is for  $m = 0$ :

$$t = \frac{\lambda_0}{4n_{oil}} = \frac{560\text{ nm}}{4(1.40)} = 100\text{ nm}$$

**SECTION B- (For only Integrated Ph.D. candidates)**

**Q9.** In the experimental arrangement below, a plane polarised light of intensity  $I$  and wavelength  $500\text{nm}$  is propagating along the  $z$ -axis. It is polarised along the  $y$ -axis and is incident on the first polarizer which has the transmission axis at an angle of  $\pi/4$  w.r.t. the  $y$ -axis, as shown.



Subsequently, the beam passes through a flat birefringent crystal of thickness  $950\mu\text{m}$  and optical axis along the  $y$ -axis. The crystal is kept normal to the  $z$ -axis. If the refractive indices of the crystal are  $n_o = 1.514$  and  $n_e = 1.519$ , the beam intensity measured after the second polarizer whose transmission axis is parallel to that of the first one would be:

- (a) 0                      (b)  $I/4$                       (c)  $I/2$                       (d)  $I$

**Ans.:(a)**

**Solution.:** After the first polarizer, intensity is  $I_1 = I \cos^2 45^\circ = \frac{I}{2}$  and the light is linearly polarized at  $45^\circ$  to the  $y$ -axis.

The birefringent crystal introduces a phase difference  $\Delta\phi = \frac{2\pi}{\lambda}(n_e - n_o)d$ .

$$\Delta\phi = \frac{2\pi}{500 \times 10^{-9}}(1.519 - 1.514) \times 950 \times 10^{-6} = \frac{2\pi}{500 \times 10^{-9}}(0.005) \times 950 \times 10^{-6} = 19\pi \equiv \pi$$

So the crystal acts like a half-wave plate. The initially  $45^\circ$  polarized light is rotated by  $90^\circ$ , becoming perpendicular to the first polarizer's axis.

The second polarizer is parallel to the first, so the angle between the light polarization and the second polarizer axis is  $90^\circ$ . Thus

$$I_2 = I_1 \cos^2 90^\circ = 0$$

**SECTION C- (For only Ph.D. candidates)**

**No Question**