Test Your fiziks concepts!

Topic: Quantum Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. Which of the following two physical quantities cannot be measured simultaneously with arbitrary accuracy for the motion of a quantum particle in three dimensions?

(a) square of the radial position and z-component of angular momentum (r^2 and L_z)

(b) x- components of linear and angular momenta (p_x and L_x)

(c) y-component of position and z-component of angular momentum (y and L_x)

(d) squares of the magnitudes of the linear and angular momenta (p^2 and L^2)

Ans.: (c)

Solution.: The two physical quantities cannot be measured simultaneously with arbitrary accuracy in quantum mechanics whose commutator is not zero.

(a) $\left[r^2, L_z\right] = \left[x^2 + y^2 + z^2, L_z\right] = \left[x^2, L_z\right] + \left[y^2, L_z\right] + \left[z^2, L_z\right]$

$$= x\left[x, L_z\right] + \left[x, L_z\right]x + y\left[y, L_z\right] + \left[y, L_z\right]y = x\left(-i\hbar y\right) + \left(-i\hbar yx\right) + y\left(i\hbar x\right) + \left(i\hbar x\right)y = 0$$

where, we have used $\left[x, L_y\right] = -i\hbar y : \left[y, L_z\right] = i\hbar x \; ; \left[z, L_z\right] = 0$

(b) $\left[p_x, L_x\right] = \left[p_x, yp_z, zp_y\right] = \left[p_x, yp_z\right] - \left[p_x - zp_y\right]$

$$= y\left[p_x, p_z\right] + \left[p_x, y\right]p_z + z\left[p_x, p_z\right] - \left[p_x, z\right]p_y$$

$\left[p_x, L_x\right] = 0$

where, we have used $\left[p_x, p_z\right] = \left[p_x, y\right] = \left[p_x, z\right] = 0$

(c) $\left[y, L_z\right] = \left[y, xp_y - yp_x\right] = \left[y, xp_y\right] - \left[y, yp_x\right] = x\left[y, p_y\right] + \left[y, x\right]p_y - y\left[y, p_x\right] - \left[y, y\right]p_x$

$$= x\left[y, p_y\right] + 0 + 0 + 0 = i\hbar x$$

where we have used $\left[y, p_y\right] = i\hbar x, \left[y, x\right] = \left[y, p_x\right] = \left[y, y\right] = 0$

(d) $\left[p^2, L^2\right] = \left[p^2, r^2p^2 - \left(\vec{r}\cdot\vec{p}\right)^2 + i\hbar\left(\vec{r}\cdot\vec{p}\right)\right]$

$$= \left[p^2, r^2p^2\right] - \left[p^2, \left(\vec{r}\cdot\vec{p}\right)^2\right] + i\hbar\left[p^2, \left(\vec{r}\cdot\vec{p}\right)\right] = 0$$

where, we have used $\left[p, r\right] = 0$

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

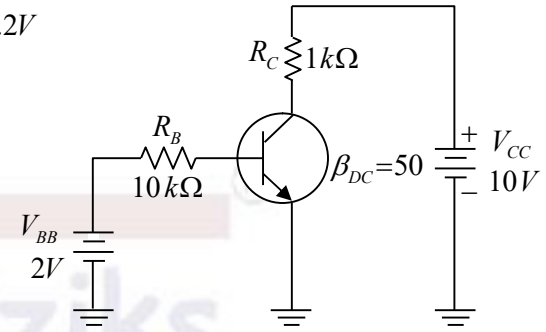
Q. Calculate the collector current and determine whether or not the transistor in figure shown below is in saturation. Assume $V_{CE}(sat) = 0.2V$

(a) $6.5\,mA$, not in saturation

(b) $11.5\,mA$, in saturation

(c) $11.5\,mA$, not in saturation

(d) $6.5\,mA$, in saturation



Ans.: (a)

Solution.: Emitter base junction is forward bias.

$$I_B = \frac{2V - 0.7V}{10k} = 0.13\,mA \Rightarrow I_C = \beta I_B = 6.5\,mA$$

$V_B = V_{BE} = 0.7V$ and $V_C = V_{CC} - I_C R_C = 10V - (6.5 \times 1)V = 3.5V$

$\Rightarrow V_{CB} = V_C - V_B = 3.5V - 0.7V = 2.8V = +ve$ (Collector base junction reverse bias)

Thus, transistor is in active region.

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Test Your fiziks concepts!

Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. If the ground state energy of a particle in an infinite potential well of width L_1 is equal to the energy of the second excited state in another infinite potential well of width L_2, then the ratio $\frac{L_1}{L_2}$ is equal to

(a) 1 (b) 1/3 (c) $1/\sqrt{3}$ (d) 1/9

Ans.: (b)

Solution.: The energy of a particle in Infinite Square well potential of length L is $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

For $L = L_1$ and $n = 1$; $E_1 = \frac{\pi^2\hbar^2}{2mL_1^2}$ and For $L = L_2$ and $n = 3$; $E_3 = \frac{9\pi^2\hbar^2}{2mL_2^2}$

Since, $E_1 = E_3 \Rightarrow \frac{\pi^2\hbar^2}{2mL_1^2} = \frac{9\pi^2\hbar^2}{2mL_2^2} \Rightarrow \frac{L_1^2}{L_2^2} = \frac{1}{9} \quad \Rightarrow \frac{L_1}{L_2} = \frac{1}{3}$

Thus correct option is (b).

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Quantum Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. In a quantum harmonic oscillator problem, $\hat{a}$ and $\hat{N}$ are the annihilation operator and the number operator, respectively. The operator $e^{\hat{N}}\hat{a}e^{-\hat{N}}$ is

(a) $\hat{a}$ (b) $e^{-1}\hat{a}$ (c) $e^{-\left(\hat{I}+\hat{a}\right)}$ (d) $e^{\hat{a}}$

(where $\hat{I}$ is the identity operator)

Ans.: (b)

Solution.:

Since $e^{\hat{A}}\hat{B}e^{-\hat{A}} = \hat{B}+\left[\hat{A},\hat{B}\right]+\frac{1}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]+\frac{1}{3!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]+\ldots$

Here $\hat{A}=\hat{N}$ and $\hat{B}=\hat{a}$ also $\hat{N}=a^{+}a$ and $\left[\hat{N},\hat{a}\right]=-\hat{a}$

$$\therefore\ e^{\hat{N}}\hat{a}e^{-\hat{N}} = \hat{a}+\left[\hat{N},\hat{a}\right]+\frac{1}{2!}\left[\hat{N},\left[\hat{N},\hat{a}\right]\right]+\frac{1}{3!}\left[\hat{N},\left[\hat{N},\hat{a}\right]\right]+\ldots$$

$$= \hat{a}-\hat{a}+\frac{1}{2!}\left[\hat{N},-\hat{a}\right]+\frac{1}{3!}\left[\hat{N},-\hat{a}\right]+\ldots = \hat{a}-\hat{a}+\frac{1}{2!}\hat{a}+\frac{1}{3!}\hat{a}+\ldots$$

$$\Rightarrow e^{\hat{N}}\hat{a}\,e^{-\hat{N}} = \hat{a}\left[1-1+\frac{1}{2!}-\frac{1}{3!}+\ldots\right] = \hat{a}e^{-1} \qquad \Rightarrow e^{\hat{N}}\hat{a}\,e^{-\hat{N}} = e^{-1}\hat{a}$$

Thus correct option is (b).

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

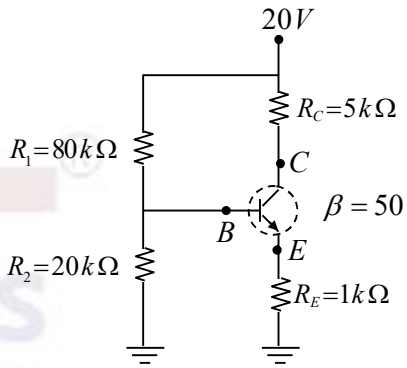
H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

Test Your fiziks concepts!

Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. An *npn* -transistor is connected in a voltage divider configuration as shown in the figure below. If the resistor R_2 is disconnected, the voltages V_B at the base and V_C at the collector change as follows.



(a) both V_B and V_C increase

(b) both V_B and V_C decrease

(c) V_B decreases, but V_C increases

(d) V_B increases, but V_C decreases

Ans.: (d)

Solution.: $V_B = \dfrac{V_{CC}R_2}{R_1+R_2} = \dfrac{V_{CC}}{R_1/R_2+1}$ as $R_2 \uparrow, V_B \uparrow$

$\because V_E = V_B - V_{BE}$ and $I_E = \dfrac{V_E}{R_E} \approx I_C$.

As $V_B \uparrow, V_E \uparrow$ thus $I_E \approx I_C \uparrow$

$\because V_C = V_{CC} - I_C R_C \Rightarrow I_C \uparrow, V_C \downarrow$

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. A spacecraft has speed $v_s = fc$ with respect to the earth, where c is the speed of light in vacuum. An observer in the spacecraft measures the time of one complete rotation of the earth to be 48 hours. The value of f is

(a) 0.67 (b) 0.77 (c) 0.87 (d) 0.97

Ans.: (c)

Solution.: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\dfrac{v_s^2}{c^2}}} \Rightarrow 24 = \dfrac{48}{\sqrt{1-f^2}} \Rightarrow f^2 = \dfrac{3}{4} \Rightarrow f = \dfrac{\sqrt{3}}{2} = 0.87$

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com