Test Your fiziks concepts!

Topic: Quantum Mechanics

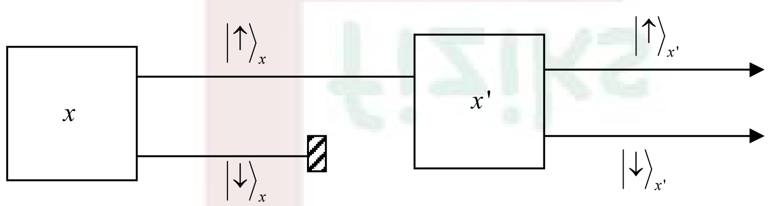
(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

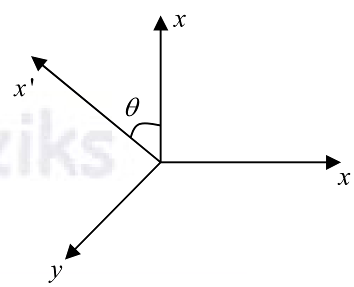
Q. A spin $\frac{1}{2}$ particle is in a spin up state along the x-axis (with unit vector $\hat{x}$) and is denoted as $\left|\frac{1}{2},\frac{1}{2}\right\rangle_x$. What is the probability of finding the particle to be in a spin up state along the direction $\hat{x}'$, which lies in the xy-plane and makes an angle θ with respect to the positive x-axis, if such a measurement is made?

(a) $\frac{1}{2}\cos^2\frac{\theta}{4}$ (b) $\cos^2\frac{\theta}{4}$ (c) $\frac{1}{2}\cos^2\frac{\theta}{2}$ (d) $\cos^2\frac{\theta}{2}$

Ans.: (d)

Solution.:





The probability of finding the particle in spin up state in the direction of $\hat{x}'$ is $P\left(\left|\uparrow\right\rangle_{x'}\right)=\cos^2\frac{\theta}{2}$

The probability of finding the particle in spin down state in the direction of $\hat{x}'$ is $P\left(\left|\downarrow\right\rangle_{x'}\right)=\sin^2\frac{\theta}{2}$

Note:

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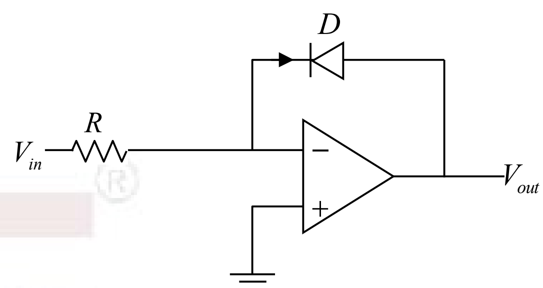
Test Your fiziks concepts!

Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. The I-V characteristics of the diode D in the circuit below is given by $I = I_s\left(e^{\frac{qV}{k_BT}} - 1\right)$ where I_s is the reverse saturation current, V is the voltage across the diode and T is the absolute temperature. If the input voltage is V_{in}, then the output voltage V_{out} is



(a) $I_S R \ln\left(\frac{qV_{in}}{k_BT} + 1\right)$ (b) $\frac{1}{q} k_BT \ln\left(\frac{q\left(V_{in} + I_S R\right)}{k_BT}\right)$

(c) $\frac{1}{q} k_BT \ln\left(\frac{V_{in}}{I_S R} + 1\right)$ (d) $-\frac{1}{q} k_BT \ln\left(\frac{V_{in}}{I_S R} + 1\right)$

Ans.: (c)

Solution.: $\because I_R = I_D \quad \Rightarrow I_S\left(e^{eV_D/k_BT} - 1\right) = \frac{0 - \left(-V_{in}\right)}{R}$

$$\Rightarrow e^{eV_D/k_BT} - 1 = +\frac{V_{in}}{I_S R} \Rightarrow e^{eV_D/k_BT} = \frac{V_{in}}{I_S R} + 1 \Rightarrow V_D = \frac{k_BT}{e}\ln\left(\frac{V_{in}}{I_S R} + 1\right)$$

$$V_{out} = V_D = \frac{k_BT}{e}\ln\left(\frac{V_{in}}{I_S R} + 1\right)$$

Note:

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Test Your fiziks concepts!

Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. In the Thomson model of hydrogen atom, the nuclear charge is distributed uniformly over a sphere of radius R. The average potential energy of an electron confined within this atom can be taken as $V=-\frac{e^2}{4\pi\in_0 R}$. Taking the uncertainty in position to be the radius of the atom, the minimum value of R for which an electron will be confined within the atom is estimated to be:

Given: The uncertainty product of momentum and position is $\hbar=1\times10^{-34}$ Js^{-1}, $e=1.6\times10^{-19}$ C, and $\frac{1}{4\pi\in_0}=9\times10^{9}$ Nm^2 C^{-2}.

(a) $2.38\times10^{-11}m$ (b) $2.38\times10^{-10}m$ (c) $2.38\times10^{-9}m$ (d) $2.38\times10^{-8}m$

Ans.: (a)

Solution.: Energy of electron $E=\frac{p_e^2}{2m_e}-\frac{e^2}{4\pi\varepsilon_0 R}$

As electron is bounded within the atom, so $E\le 0 \Rightarrow \frac{p_e^2}{2m_e}-\frac{e^2}{4\pi\varepsilon_0 R}\le 0 \Rightarrow p_e\le\sqrt{\frac{e^2}{4\pi\varepsilon_0}\frac{2m_e}{R}}$

From uncertainty principle $\Delta r.\Delta p=10^{-34}$ where $\Delta r=R$, and $\Delta p=p_e^{\max}\le\sqrt{\frac{e^2}{4\pi\varepsilon_0}\frac{2m_e}{R}}$

Thus $R\sqrt{\frac{e^2}{4\pi\varepsilon_0}\frac{2m_e}{R}}=10^{-34}\Rightarrow R=\frac{10^{-68}}{\frac{e^2}{4\pi\varepsilon_0}\times 2m_e}=\frac{10^{-68}}{\left(9\times10^{9}\right)\left(1.6\times10^{-19}\right)^2\left(2\times9.1\times10^{-31}\right)}$

$$\Rightarrow R=\frac{10^{-68}}{\left(9\times10^{9}\right)\left(2.56\times10^{-38}\right)\left(2\times9.1\times10^{-31}\right)}=\frac{10^{-68}}{419.33\times10^{-60}}=\frac{10^{-8}}{419.33}=\frac{1000}{419.33}\times10^{-11}m$$

$\Rightarrow R=2.38\times10^{-11}m$.

Note:

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Test Your fiziks concepts!

Topic: Quantum Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. Consider two non-identical spin $\frac{1}{2}$ particles labelled 1 and 2 in the spin product state $\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$. The Hamiltonian of the system is

$$H=\frac{4\lambda}{\hbar^2}\vec{S}_1\cdot\vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the spin operators of particles 1 and 2, respectively, and λ is a constant with appropriate dimensions. What is the expectation value of H in the above state?

(a) $-\lambda$ (b) -2λ (c) λ (d) 2λ

Ans.: (a)

Solution.: For $s_1=\frac{1}{2},\ m_1=\pm\frac{1}{2}$; For $s_2=\frac{1}{2}, m_2=\pm\frac{1}{2}$

There are four possible states

$$|s_1,m_1\rangle|s_2,m_2\rangle=\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle,\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle,\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle,\left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$=|\uparrow\uparrow\rangle,|\uparrow\downarrow\rangle,|\downarrow\uparrow\rangle,|\downarrow\downarrow\rangle$$

Since, $s_1=\frac{1}{2},s_2=\frac{1}{2}$, therefore $s=\vec{s}_1+\vec{s}_2=0,1$

The possible states for singlet $(s=0)$ and triplet $(s=1)$ are

For $s=0,m_s=0 \quad \therefore|0,0\rangle$

For $s=1,m_s=0,\pm1 \ \therefore|1,1\rangle,|1,0\rangle$ and $|1,-1\rangle$

where $|1,1\rangle=|\uparrow\uparrow\rangle$; $\quad |1,0\rangle=\frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle\right]$

$|1,-1\rangle=|\downarrow\downarrow\rangle$ and $|0,0\rangle=\frac{1}{\sqrt{2}}\left[|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle\right]$

According to the question, we have to calculate expectation value of H in the state $|\uparrow\downarrow\rangle=\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle$. This state can be written is linear combination of $|1,0\rangle$ and $|0,0\rangle$ as

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$\left|\uparrow\downarrow\right\rangle = c_1\left|1,0\right\rangle + c_2\left|0,0\right\rangle$ where $c_1 = \left\langle 1,0 \middle| \uparrow\downarrow\right\rangle = \frac{1}{\sqrt{2}}\left(\left\langle\uparrow\downarrow\right| + \left\langle\downarrow\uparrow\right|\right)\left|\uparrow\downarrow\right\rangle = \frac{1}{\sqrt{2}}$

and $c_2 = \left\langle 0,0 \middle| \uparrow\downarrow\right\rangle = \frac{1}{\sqrt{2}}\left(\left\langle\uparrow\downarrow\right| - \left\langle\downarrow\uparrow\right|\right)\left|\uparrow\downarrow\right\rangle = \frac{1}{\sqrt{2}}$

$\therefore \left|\uparrow\downarrow\right\rangle = \frac{1}{\sqrt{2}}\left|1,0\right\rangle + \frac{1}{\sqrt{2}}\left|0,0\right\rangle$

The expectation value of H in the state $\left|\uparrow\downarrow\right\rangle$ is

$$\left\langle\uparrow\downarrow\right|H\left|\uparrow\downarrow\right\rangle = \left\langle H\right\rangle = \sum_n E_n P(n) = E_{1,0}P(1,0) + E_{0,0}P(0,0)$$

Given $H = \frac{4\lambda}{\hbar^2}\vec{s}_1\cdot\vec{s}_2 = \frac{4\lambda}{\hbar^2}\left[\frac{s^2 - s_1^2 - s_2^2}{2}\right] = \frac{2\lambda}{\hbar^2}\left[s^2 - s_1^2 - s_2^2\right]$

Now, $H\left|1,0\right\rangle = \frac{2\lambda}{\hbar^2}\left(s^2 - s_1^2 - s_2^2\right)\left|1,0\right\rangle = \frac{2\lambda}{\hbar^2}\left(2 - \frac{3}{4} - \frac{3}{4}\right)\hbar^2\left|1,0\right\rangle$

$H\left|1,0\right\rangle = \lambda\left|1,0\right\rangle = E_{1,0}\left|1,0\right\rangle$ and $H\left|0,0\right\rangle = \frac{2\lambda}{\hbar^2}\left(s^2 - s_1^2 - s_2^2\right)\left|0,0\right\rangle = \frac{2\lambda}{\hbar^2}\left(0 - \frac{3}{4} - \frac{3}{4}\right)\hbar^2\left|0,0\right\rangle$

$H\left|0,0\right\rangle = -3\lambda\left|0,0\right\rangle = E_{0,0}\left|0,0\right\rangle$

Also, $P(1,0) = \frac{|c_1|^2}{\sum |c_i|^2} = \frac{1}{2}$ and $P(0,0) = \frac{|c_2|^2}{\sum |c_i|^2} = \frac{1}{2}$

Therefore $\left\langle H\right\rangle = \lambda\times\frac{1}{2} - 3\lambda\times\frac{1}{2} = -\lambda$

Thus, correct option is (a)

Note:

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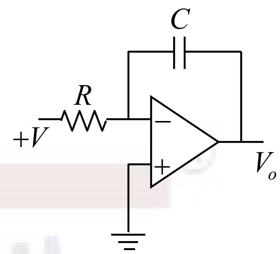
Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. If a constant voltage $+V$ is applied to the input of the following OPAMP circuit for a time t, then the output voltage V_0 will approach



(a) $+V$ exponentially

(b) $-V$ exponentially

(c) $+V$ linearly

(d) $-V$ linearly

Ans.: (d)

Solution.:

$$\because \quad I_R = I_C \Rightarrow \frac{V-0}{R} = C\frac{d\left(0-V_0\right)}{dt} \Rightarrow \frac{dV_0}{dt} = -\frac{V}{RC} \Rightarrow V_0 = -\frac{V}{RC}t + c$$

Note:

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Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. Three frames F_0, F_1 and F_2 are in relative motion. The frame F_0 is at rest, F_1 is moving with velocity $v_1\hat{i}$ with respect to F_0 and F_2 is moving with velocity $v_2\hat{i}$ with respect to F_1. A particle is moving with velocity $v_3\hat{i}$ with respect to F_2. If $v_1 = v_2 = v_3 = c/2$, where c is the speed of light, the speed of the particle with respect to F_0 is

(a) $0.73c$ (b) $0.83c$ (c) $0.93c$ (d) $0.99c$

Ans.: (c)

Solution.:

$$\vec{v}_{01} = v_1\hat{i} = \frac{c}{2}\hat{i}\ ;\ \ \vec{v}_{21} = v_2\hat{i} = \frac{c}{2}\hat{i}\ ;\ \vec{v}_{P2} = v_3\hat{i} = \frac{c}{2}\hat{i}$$

$$v_{P1} = \frac{v_{P2} - v_{12}}{1 - \frac{v_{P2}v_{12}}{c^2}} = \frac{\frac{c}{2} - \left(-\frac{c}{2}\right)}{1 - \frac{1}{c^2}\left(\frac{c}{2}\right)\left(-\frac{c}{2}\right)} = \frac{4}{5}c\ ;\ \ v_{P0} = \frac{\frac{4}{5}c - \left(-\frac{c}{2}\right)}{1 - \frac{1}{c^2}\left(\frac{4}{5}c\right)\left(-\frac{c}{2}\right)} = \frac{13}{14}c = 0.93c$$

Note:

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