Test Your fiziks concepts!

Topic: Statistical Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. Adding $1eV$ of energy to a large system did not change its temperature $\left(27^0 C\right)$ whereas it changed the number of microstates by a factor r. Then r is of the order [Note: $1eV \simeq 11600\,K$]

(a) 10^4 (b) 10^{23} (c) 10^{17} (d) 10^{-19}

Ans.: (c)

Solution.: According to the relationship

$$\beta = \left(\frac{\partial \ln \Omega}{\partial E}\right), \qquad \Omega \text{ being the number of microstates in the system.}$$

This allows us to write, $\Delta \ln \Omega = \beta \Delta E = \dfrac{\Delta E}{k_B T}$

$\therefore$ by changing energy by ΔE, Ω will change by a factor of $e^{\frac{\Delta E}{k_B T}} = e^{\frac{1\text{eV}}{25\text{meV}}} \approx 2.3 \times 10^{17}$

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Mechanics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. A particle of mass m is moving in a potential $V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$ where ω_0 and a are positive constants. The angular frequency of small oscillations for the simple harmonic motion of the particle about a stable minimum of the potential $V(x)$ is

(a) $\sqrt{2}\omega_0$ (b) $2\omega_0$ (c) $4\omega_0$ (d) $4\sqrt{2}\omega_0$

Ans.: (b)

Solution.: $V(x)=\frac{1}{2}m\omega_0^2x^2+\frac{a}{2mx^2}$; $\frac{dV}{dx}=m\omega_0^2x-\frac{a}{mx^3}=0\Rightarrow x^4=\frac{a}{m^2\omega_0^2}$

$$\left.\frac{d^2V}{dx^2}\right|_{x=x_0}=m\omega_0^2+\frac{3a}{mx^4}\Rightarrow m\omega_0^2+\frac{3m^2\omega_0^2}{m}=4m\omega_0^2$$

Thus, $\omega=\sqrt{\frac{\left.\frac{d^2V}{dx^2}\right|_{x=x_0}}{m}}=\sqrt{\frac{4m\omega_0^2}{m}}=2\omega_0$.

Note:

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Test Your fiziks concepts!

Topic: Thermodynamics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. Steam at 100°C is passed into 20g of water at 10°C. When water acquires a temperature of 80°C, the mass of water present will be:

[Take specific heat of water = 1 cal/g/°C and latent heat of steam = 540 cal g^{-1}]:

(a) 24g (b) 31.5g (c) 42.5g (d) 22.5g

Ans.: (d)

Solution.: Heat lost = Heat gained

$$mL_v + ms_w\Delta\theta_1 = m_w s_w \Delta\theta_2 \Rightarrow m\times 540 + m\times 1\times(100-80) = 20\times 1\times(80-10) \Rightarrow m = 2.5\,g$$

Total mass of water $=(20+2.5)\,g = 22.5g$

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Statistical Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. Consider a system of $2N$ non-interacting spin $1/2$ particles each fixed in position and carrying a magnetic moment μ. The system is immersed in a uniform magnetic field B. The number of spin up particles for which the entropy of the system will be maximum is

(a) 0 (b) N (c) $2N$ (d) $N/2$

Ans.: (b)

Solution.:

Total number of non-interacting spins $= 2N$

Since they are fixed, so they are distinguishable.

In field, let the number of spins aligned or spin-up $= n$

In field, let the number of spin-down $= 2N - n$

Number of microstates for choosing n spins-up out of $2N$ spins are $\Omega_1 = \frac{(2N)!}{(n!)(2N-n)!}$

Number of microstates for choosing $(2N-n)$ spins-down out of $2N$ spins are

$$\Omega_2 = \frac{(2N)!}{(2N-n)!(2N-(2N-n)!)} = \frac{(2N)!}{(2N-n)!(n!)}$$

Since Ω_1 and Ω_2 are independent of each other, thus $\Omega_{Total} = \Omega_1\Omega_2 = \left[\frac{(2N)!}{(n!)(2N-n)!}\right]^2$

Entropy, $S = k\ln\Omega_1$ $S = k\ln\frac{2N!}{(n!)(2N-n)!} = k\left[\left(\ln 2N! - \ln n! - \ln(2N-n)!\right)\right]$

$$S = k\left[2N\ln 2N - 2N - n\ln n + n - \left\{(2N-n)\ln(2N-n) - (2N-n)\right\}\right]$$

$[\because \ln N! = N\ln N - N!]$

$$S = k\left[2N\ln 2N - 2N - n\ln n + n - (2N-n)\ln(2N-n) + 2N - n\right]$$

$S = k\left[2N\ln 2N - n\ln n - (2N-n)\ln(2N-n)\right]$

Now for maximum entropy at equilibrium for spin $\frac{1}{2}$ up particle, $\frac{dS}{dn} = 0$

$$\frac{dS}{dn} = k\left[0 - \ln n - 1 - (0-1)\ln(2N-n) - (2N-n)\frac{1}{2N-n}\times(0-1)\right] = 0$$

$$\Rightarrow \frac{dS}{dn} = k\left[-1 - \ln n + \ln(2N-n) + 1\right] = k\ln\left(\frac{2N-n}{n}\right) = 0$$

$$\because k \neq 0 \quad \therefore \ln\left(\frac{2N-n}{n}\right) = 0 \Rightarrow \frac{2N-n}{n} = 1 \Rightarrow 2N = 2n \Rightarrow n = N$$

This means that entropy is maximum when the number of spin-up and spin-down particles are equal, i.e., the most disordered configuration.

This corresponds to the zero net magnetization state-which is what we expect at high temperatures when thermal agitation randomizes the spins.

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com

Test Your fiziks concepts!

Topic: Mechanics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. A particle of mass m is moving in $x-y$ plane. At any given time t, its position vector is given by $\vec{r}(t)=A\cos\omega t\,\hat{i}+B\sin\omega t\,\hat{j}$ where A,B and ω are constants with $A\neq B$. Which of the following statement is not true?

(a) Orbit of the particle is an ellipse

(b) Speed of the particle is constant

(c) At any given time t the particle experiences a force towards origin

(d) The angular momentum of the particle is $m\omega AB\hat{k}$

Ans.: (b)

Solution: (a) $\vec{r}(t)=A\cos\omega t\,\hat{i}+B\sin\omega t\,\hat{j}\Rightarrow x=A\cos\omega t,\ y=B\sin\omega t$

$$\Rightarrow \frac{x}{A}=\cos\omega t,\ \frac{y}{B}=\sin\omega t\Rightarrow \frac{x^2}{A^2}+\frac{y^2}{B^2}=1\ \text{(Ellipse)}$$

(b) $\dfrac{d\vec{r}}{dt}=-A\omega\sin\omega t\,\hat{i}+B\omega\cos\omega t\,\hat{j}$

Speed $=\left|\dfrac{d\vec{r}}{dt}\right|=\sqrt{A^2\omega^2\sin^2\omega t+B^2\omega^2\cos^2\omega t}$. Speed is function of time, so not constant.

(c) $\dfrac{d\vec{r}^2}{dt^2}=-A\omega^2\cos\omega t\,\hat{i}-B\omega^2\sin\omega t\,\hat{j}=-\omega^2\vec{r}$. Force act towards origin.

(d) $\vec{L}=(\vec{r}\times\vec{p})=m\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ A\cos\omega t & B\sin\omega t & 0\\ -A\omega\sin\omega t & B\omega\cos\omega t & 0\end{pmatrix}\Rightarrow\vec{L}=m\omega AB\hat{k}$

Note:

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Test Your fiziks concepts!

Topic: Thermodynamics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. Which of the following rods, (given radius r and length l) each made of the same material and whose ends are maintained at the same temperature will conduct most heat?

(a) $r = r_0, l = l_0$

(b) $r = 2r_0, l = l_0$

(c) $r = r_0, l = 2l_0$

(d) $r = 2r_0, l = 2l_0$

Ans.: (b)

Solution.: Heat conducted $= \dfrac{KA(T_1 - T_2)}{l} = \dfrac{K\pi r^2 (T_1 - T_2)}{l}$

The rod with the maximum ratio of A/l will conduct most. Here the rod with $r = 2r_0$ and $l = l_0$ will conduct most heat.

Note:

For detailed solutions, visit the *Free Download* section at www.physicsbyfiziks.com