Test Your fiziks concepts!

Topic: Quantum Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$,

is $$\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right]$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

(a) $3\in, 2\in, -\in$ (b) $6\in, 5\in, 2\in$

(c) $3\in, 2\in, \in$ (d) $-3\in, -2\in, \in$

Ans.: (a)

Solution.: $\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right] = \frac{\in}{\hbar^2}\left[\hat{L}_1\hat{L}_2 + \hat{L}_2^2 - \left(L_{1z} + L_{2z}\right)^2\right]$

where $\hat{L} = \hat{L}_1 + \hat{L}_2 \Rightarrow \hat{L}^2 = \hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_1\hat{L}_2 + \hat{L}_2\hat{L}_1 = \hat{L}_1^2 + \hat{L}_2^2 + 2\hat{L}_1\hat{L}_2 \quad \because \left[\hat{L}_1, \hat{L}_2\right] = 0$

$\Rightarrow \hat{L}_1\hat{L}_2 = \frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right)$ and $\hat{L}_z = \hat{L}_{1z} + \hat{L}_{2z}$

$$\hat{H} = \frac{\in}{\hbar^2}\left[\frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right) + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left[\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} - \frac{\hat{L}_2^2}{2} + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left(\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} + \frac{\hat{L}_2^2}{2} - \hat{L}_z^2\right)$$

Now $$\left\langle\hat{H}\right\rangle = \frac{\in}{\hbar^2}\left[\frac{\left\langle\hat{L}^2\right\rangle - \left\langle\hat{L}_1^2\right\rangle + \left\langle\hat{L}_2^2\right\rangle}{2} - \left\langle\hat{L}_z^2\right\rangle\right] = \frac{\in}{\hbar^2}\left[\ell(\ell+1)\hbar^2 - \ell_1(\ell_1+1)\hbar^2 + \ell_2(\ell_2+1)\hbar^2 - m_\ell^2\hbar^2\right]$$

for $\ell_1 = \ell_2 = 1$ and $\ell = 2$; $\left\langle\hat{H}\right\rangle = \frac{\in}{\hbar^2}\left[\frac{6\hbar^2 - 2\hbar^2 + 2\hbar^2}{2} - m_\ell^2\hbar^2\right] = \in\left[3 - m_\ell^2\right]$

For $\ell = 2$, $m_\ell = -2, -1, 0, +1, +2$ and $m_\ell^2 = 0, 1, 4$

$\therefore$ The possible values of $\langle H\rangle$ are

$\langle H\rangle = \in[3-4] = -\in$ for $m_\ell^2 = 4$; $\langle H\rangle = \in[3-1] = 2\in$ for $m_\ell^2 = 1$

$\langle H\rangle = \in[3-0] = 3\in$ for $m_\ell^2 = 0$. Thus, correct option is (a).

Note:

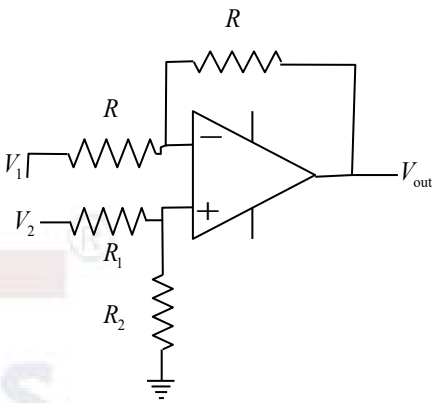
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Test Your fiziks concepts!

Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. In the given circuit, with an ideal op-amp for what value of $\frac{R_1}{R_2}$ the output of the amplifier $V_{out} = V_2 - V_1$?



(a) 1

(b) 1/2

(c) 2

(d) 3/2

Ans.: (a)

Solution.: From Superposition theorem

$$V_{out} = V_{01} + V_{02} = -\frac{R}{R}V_1 + \left(1+\frac{R}{R}\right)\frac{R_2}{R_1+R_2}V_2 = -V_1 + \frac{2R_2}{R_1+R_2}V_2$$

$$\because V_{out} = V_2 - V_1 \text{ so } \frac{2R_2}{R_1+R_2} = 1 \Rightarrow R_2 = R_1 \Rightarrow \frac{R_1}{R_2} = 1$$

Note:

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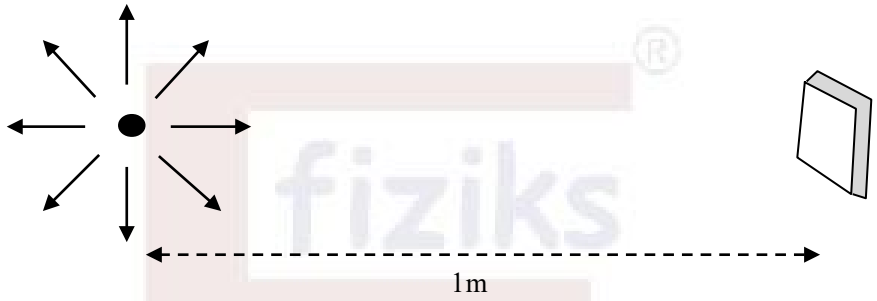
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Test Your fiziks concepts!

Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. A point source emitting photon of 2 eV energy and 1 W of power is kept at a distance of $1m$ from a small piece of a photoelectric material of area 10^{-4} m^2. If the efficiency of generation of photoelectrons is 10%, then the number of photoelectrons generated are $f\times10^{12}$ per second. The value of f is: (Given: $1\text{eV}=1.6\times10^{-19}$ J)



(a) 1.50 (b) 2.00 (c) 2.50 (d) 3.00

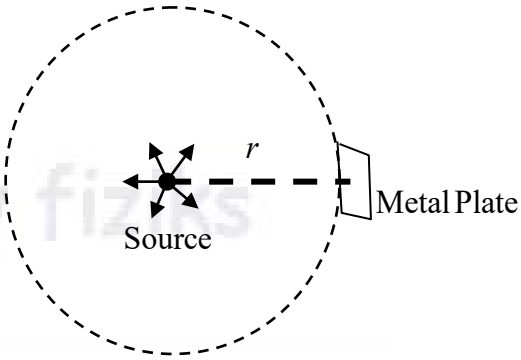
Ans.: (c)

Solution.:

Energy emitted in one second is $E=1J$.

The number of Photon emitted in one second is

$$n=\frac{E}{hv}=\frac{1J}{2\times1.6\times10^{-19}J}=3.125\times10^{18}\text{ photons}$$



Now, area of sphere of radius $r=4\pi r^2$

Number of photons per unit area $=\frac{n}{4\pi r^2}$

Number of photons incident on metal plate of area 10^{-4}m^2 is $n'=\frac{n}{4\pi r^2}\times10^{-4}\text{m}^2$

$$\Rightarrow n'=\frac{3.12\times10^{18}}{4\times3.14\times1^2}\times10^{-4}=2.48\times10^{13}\text{ photons}$$

Given the efficiency of generation of Photoelectron = 10%

Thus, number of photoelectrons = 10% of $n'=\frac{10}{100}\times2.48\times10^{13}=2.48\times10^{12}\Rightarrow f=2.48$

Note:

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Test Your fiziks concepts!

Topic: Quantum Mechanics

(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

Q. A quantum particle of mass m is moving in a one-dimensional potential

$$V(x)=V_0\theta(x)-\lambda\delta(x),$$

where V_0 and λ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E>>V_0>\lambda$ and $\sqrt{2mE}>>\frac{V_0\hbar}{\lambda}$ is

(a) $\frac{V_0^2}{4E^2}$ (b) $\frac{V_0^2}{8E^2}$ (c) $\frac{m\lambda^2}{2E\hbar^2}$ (d) $\frac{m\lambda^2}{4E\hbar^2}$

Ans.: (c)

Solution.:

$$V(x)=V_0\theta(x)-\lambda\delta(x) \Rightarrow V(x)=\begin{cases}0, & x<0\\ V_0-\lambda\delta(x), & x\geq 0\end{cases}$$

$$\psi(x)=\begin{cases}A\exp(ik_1x)+B\exp(-ik_1x), & x\leq 0\\ C\exp ik_2x, & x\geq 0\end{cases} \quad \text{where } k_1=\sqrt{\frac{2mE}{\hbar^2}}, k_2=\sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

The wave function is continuous $A+B=C$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+(V_0-\lambda\delta(x))\psi(x)=E\psi(x)$$

Integrating both sides: $\frac{-\hbar^2}{2m}\int_{-\delta}^{\delta}\frac{\partial^2\psi}{\partial x^2}dx+\int_{-\delta}^{\delta}(V_0-\lambda\delta(x))\psi(x)dx=\int_{-\delta}^{\delta}E\psi(x)dx$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\Big|_{-\delta}^{\delta}+\int_{-\delta}^{\delta}V_0\psi(x)dx-\int_{-\delta}^{\delta}\lambda\psi(x)\delta(x)dx=\int_{-\delta}^{\delta}E\psi(x)dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\Big|_{-\delta}^{\delta}+0-\lambda\psi(0)=0 \Rightarrow -\frac{\hbar^2}{2m}\left[Cik_2-(Aik_1-Bik_1)\right]-\lambda(A+B)=0$$

$$-\frac{\hbar^2}{2m}\left[(A+B)ik_2-(Aik_1-Bik_1)\right]-\lambda(A+B)=0 \qquad \because C=A+B$$

$$-\frac{\hbar^2}{2m}A\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)=-\frac{\hbar^2}{2m}B\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)\Rightarrow\frac{B}{A}=\frac{\left(i(k_2-k_1)+\frac{2m\lambda}{\hbar^2}\right)}{\left(-\frac{2m\lambda}{\hbar^2}-i(k_1+k_2)\right)}$$

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$$R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\left(k_1-k_2\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\left(k_1+k_2\right)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}-\sqrt{\frac{2m\left(E-V_0\right)}{\hbar^2}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}+\sqrt{\frac{2m\left(E-V_0\right)}{\hbar^2}}\right)^2}$$

$$\Rightarrow R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1-\sqrt{1-\frac{V_0}{E}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{8mE}{\hbar^2}}=\frac{1}{1+\frac{2E\hbar^2}{m\lambda^2}}=\frac{m\lambda^2}{2E\hbar^2}$$

Since $E>>V_0>\lambda$ and $\sqrt{2mE}>>\frac{V_0\hbar}{\lambda}$.

Note:

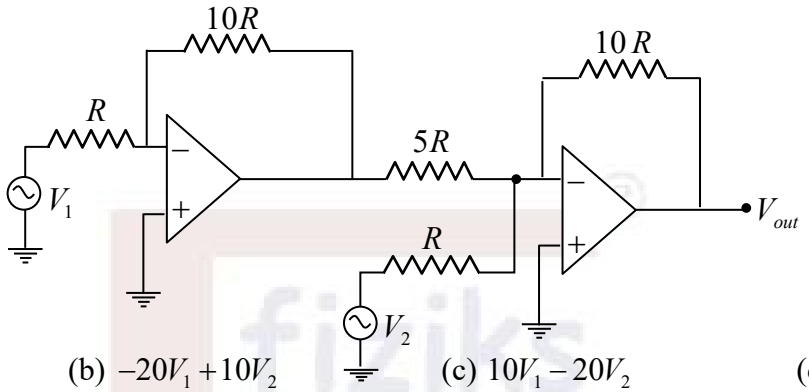
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Test Your fiziks concepts!

Topic: Electronics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. In the circuit shown in the figure, both OPAMPs are ideal. The output for the circuit V_{out} is



(a) $20V_1+10V_2$ (b) $-20V_1+10V_2$ (c) $10V_1-20V_2$ (d) $20V_1-10V_2$

Ans.: (d)

Solution.: Output of first op-amp is $V_{01}=-\frac{10R}{R}V_1=-10V_1$.

Thus $V_{out}=-\frac{10R}{5R}(-10V_1)+\left(-\frac{10R}{R}\right)V_2 \Rightarrow V_{out}=20V_1-10V_2$

Note:

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Test Your fiziks concepts!

Topic: Modern Physics

(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

Q. Consider the α-decay ${}^{90}Th^{232} \rightarrow {}^{88}Ra^{228}$. In an experiment with one gram of ${}^{90}Th^{232}$, the average count rate (integrated over the entire volume) measured by the α-detector is 3000 counts s^{-1}. If the half life of ${}^{90}Th^{232}$ is given as 4.4×10^{17} s, then the efficiency of the α-detector is: Given: Avogadro's number = 6.023×10^{23} mol^{-1}

(a) 0.53 (b) 0.63 (c) 0.73 (d) 0.83

Ans.: (c)

Solution.:

Solution.: Actual decay rate $\left|\frac{dN}{dt}\right| = N\lambda = \left(\frac{1\,gm}{232}\times6.02\times10^{23}\right)\times\frac{0.693}{4.4\times10^{17}} = 4086\,s^{-1}$

Measured decay rate $= 3000\,s^{-1}$. So, efficiency of α-detector $= \frac{3000}{4086} = 0.73$

Note:

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