

Test Your fiziks concepts!**Topic: Statistical Mechanics****(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)**

Q. Two energy levels, 0 (non-degenerate) and ϵ (double degenerate), are available to N non-interacting distinguishable particles. If U is the total energy of the system, for large values of N

the entropy of the system is $k_B \left[N \ln N - \left(N - \frac{U}{\epsilon} \right) \ln \left(N - \frac{U}{\epsilon} \right) + X \right]$. In this expression, X is

(a) $-\frac{U}{\epsilon} \ln \frac{U}{2\epsilon}$

(b) $-\frac{U}{\epsilon} \ln \frac{2U}{\epsilon}$

(c) $-\frac{2U}{\epsilon} \ln \frac{2U}{\epsilon}$

(d) $-\frac{U}{\epsilon} \ln \frac{U}{\epsilon}$

Ans.: (a)**Solution.:** Two energy levels:(i) $E_0 = 0$, degeneracy $g_0 = 1$; (ii) $E_1 = \epsilon$, degeneracy $g_1 = 2$ Let N_0 = number of particles in $E_0 = 0$ state, N_1 = number of particles in $E_1 = \epsilon$ and $N_0 + N_1 = N$ So, $U = 0 \times N_0 + \epsilon \times N_1 = \epsilon N_1 \Rightarrow N_1 = \frac{U}{\epsilon}$ and $\Rightarrow N_0 = N - \frac{U}{\epsilon}$ So, number of microstates $\Omega = \left(\frac{N!}{N_0! N_1!} \right) \times (2^{N_1})$

$$S = k_B \ln \Omega = k_B \left[\ln \left(\frac{N!}{N_0! N_1!} \right) \times (2^{N_1}) \right] = k_B \left[\ln \left(\frac{N!}{N_0! N_1!} \right) + N_1 \ln 2 \right]$$

$$S = k_B [N \ln N - N - N_0 \ln N_0 + N_0 - N_1 \ln N_1 + N_1 + N_1 \ln 2]$$

$$\Rightarrow S = k_B [N \ln N - N_0 \ln N_0 - N_1 \ln N_1 + N_1 \ln 2]$$

$$\Rightarrow S = k_B \left[N \ln N - \left(N - \frac{U}{\epsilon} \right) \ln \left(N - \frac{U}{\epsilon} \right) - \frac{U}{\epsilon} \ln \frac{U}{\epsilon} + \frac{U}{\epsilon} \ln 2 \right]$$

Compare with $S = k_B \left[N \ln N - \left(N - \frac{U}{\epsilon} \right) \ln \left(N - \frac{U}{\epsilon} \right) + X \right]$

$$\Rightarrow X = -\frac{U}{\epsilon} \ln \frac{U}{\epsilon} + \frac{U}{\epsilon} \ln 2 = -\frac{U}{\epsilon} \ln \frac{U}{2\epsilon}$$

Test Your fiziks concepts!**Topic: Mechanics****(For IIT-JAM, JEST, TIFR and CUET Aspirants)**

Q. A particle is moving in a plane with a constant radial velocity of 12 m/s and constant angular velocity of 2 rad/s . When the particle is at a distance $r = 8\text{ m}$ from the origin, the magnitude of the instantaneous velocity of the particle in m/s is

- (a) $8\sqrt{15}$ (b) 20 (c) $2\sqrt{37}$ (d) 10

Ans.: (b)

Solution.: $v_r = 12\text{ m/s}$ $v_\theta = \omega r \Rightarrow 2 \times 8 = 16\text{ m/sec}$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{144 + 256} = \sqrt{400} = 20\text{ m/sec}$$

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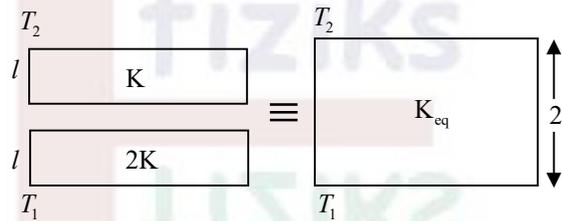
Test Your fiziks concepts!**Topic: Thermodynamics****(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)**

Q. Consider a compound slab consisting of two different materials having equal thicknesses and thermal conductivities K and $2K$, respectively. The equivalent thermal conductivity of the slab is:

- (a) $-K$ (b) $\sqrt{2}K$ (c) $3K$ (d) $\frac{4}{3}K$

Ans.: (d)

Solution.: Thermal resistance of material is given as $R = \frac{l}{KA}$



Since the slabs are in series: $\frac{2l}{K_{eq}(A)} = \frac{l}{2KA} + \frac{l}{KA}$ [Series connection $R_{eq} = R_1 + R_2$]

$$\Rightarrow K_{eq} = \frac{4}{3}K$$

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Test Your fiziks concepts!**Topic: Statistical Mechanics****(For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)**

Q. The number of microstates of a gas of N particles in a volume V and of internal energy U , is given by $\Omega(U, V, N) = (V - Nb)^N \left(\frac{aU}{N}\right)^{3N/2}$ (where a and b are positive constants). Its pressure P , volume V and temperature T , are related by

$$(a) \left(P + \frac{aN}{V}\right)(V - Nb) = Nk_B T \quad (b) \left(P - \frac{aN}{V^2}\right)(V - Nb) = Nk_B T$$

$$(c) PV = Nk_B T \quad (d) P(V - Nb) = Nk_B T$$

Ans.: (d)

Solution: $\Omega(U, V, N) = (V - Nb)^N \left(\frac{aU}{N}\right)^{3N/2}$

$$\Rightarrow S = k \ln \Omega = Nk \left[\ln(V - Nb) + \frac{3}{2} \ln\left(\frac{aU}{N}\right) \right]$$

$$dS = \frac{1}{T} dU + \frac{P}{T} dV \quad \Rightarrow \left(\frac{\partial S}{\partial V}\right)_U = \frac{P}{T} \Rightarrow \frac{P}{T} = \frac{Nk}{(V - Nb)} \Rightarrow P(V - Nb) = NkT$$

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Topic: Mechanics

(For IIT-JAM, JEST, TIFR and CUET Aspirants)

Q. A ball is dropped from a height h to the ground. If the coefficient of restitution is e , the time required for the ball to stop bouncing is proportional to:

- (a) $\frac{2+e}{1-e}$ (b) $\frac{1+e}{1-e}$ (c) $\frac{1-e}{1+e}$ (d) $\frac{2-e}{1+e}$

Ans: (b)

Solution: $t_1 = \sqrt{\frac{2H}{g}}$, $v_1 = \sqrt{2gH}$,

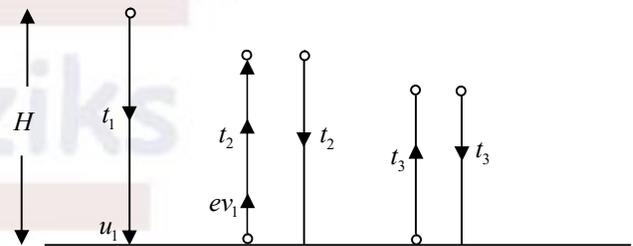
$$0 = ev_1 - gt_2 \Rightarrow t_2 = e \frac{v_1}{g} = e \sqrt{\frac{2H}{g}}$$

$$t_3 = e^2 \sqrt{\frac{2H}{g}}$$

⋮

$$T = t_1 + 2t_2 + 2t_3 + 2t_4 + \dots = \sqrt{\frac{2H}{g}} + 2e\sqrt{\frac{2H}{g}}(1 + e + e^2 + \dots)$$

$$\Rightarrow T = \sqrt{\frac{2H}{g}} \left[1 + 2e \frac{1}{1-e} \right] = \sqrt{\frac{2H}{g}} \left(\frac{1+e}{1-e} \right)$$



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Test Your fiziks concepts!**Topic: Thermodynamics****(For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)**

Q. We consider the radiation emitted by the human, body. Which of the following statements is true:

- (a) The radiation emitted is in the infrared regions
- (b) The radiation is emitted only during the day
- (c) The radiation is emitted during the summers and absorbed during the winters.
- (d) The radiation emitted lies in the ultraviolet region and hence is not visible

Ans.: (a)

Solution.: As we know that the wavelength of the emitted radiations is inversely proportional to the temperature of the body.

Wavelength, $\lambda = \frac{b}{T}$ [Where b = constant, T = Temperature of body]

We know that, temperature of human body is $T = 37^\circ\text{C} = 310\text{ K}$

\therefore Wavelength $\lambda = \frac{b}{T} \Rightarrow \lambda = \frac{2898}{10} \mu\text{m}$ [$\because b = 2898 \mu\text{mK}$]

$\lambda = 9.35 \mu\text{m}$ [corresponds to infrared region]

So, the infrared radiations are emitted by human body.

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