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An Institute of NET-JRF, IIT-JAM, GATE, JEST,
TIFR & CUET in Physics & Physical Sciences

THERMODYNAMIC AND STATISTICAL PHYSICS

(NET/JRF, GATE, JEST, TIFR)

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THERMODYNAMICS

CONTENTS

STATISTICAL MECHANICS

1. KINETIC THEORY OF GASES

1.1 Basic Assumption of Kinetic Theory:

1. Any infinitely small volume of a gas contains a large number of molecule.
2. A gas is made up identical molecule which behaves as rigid, perfectly elastic, hard sphere.
3. The molecules continuously move about in random directions. All directions of motion are equally probable.
4. The size of the molecules is much less than the average distance between them.
5. The molecule of a gas exert no force on each other except when they collide.
6. The collision between molecules and with walls are perfectly elastic.
7. The direction of molecular velocities are assumed to be distribute uniformly.
8. The molecules move with all speeds ranging from 0 to ∞.
9. The time of collision is much less than the time between collisions.

1.1.1 Pressure Exerted by a Gas

Suppose there are n molecules per cubic meter each of mass m, and its is assumed that n_i no. of molecule have velocity v_i.

Mathematically

$$\sum n_i = n \text{ and } v_i^2 = v_{ix}^2 + v_{iy}^2 + v_{iz}^2$$

where v_{ix} v_{iy} and v_{iz} are x, y, z component of velocity of gases.

From assume of kinetic theory of gases $v_{ix}^2 = v_{iy}^2 = v_{iz}^2 \quad = \frac{v_i^2}{3}$

suppose molecules are kept in the cubic container of parameter L.

A molecule moving in the x direction will have momentum mv_{ix} normal to face of the cube before collision

$$\Delta P_{ix} = mv_{ix} - (-mv_{ix}) = 2mv_{ix}$$

Force acting on the wall by molecule is $f_{ix} = \frac{n_i 2mv_{ix}}{\Delta t} = \frac{n_i 2mv_{ix}^2}{2L} == \frac{n_i mv_{ix}^2}{L}$

Pressure exert on the wall of container by molecule $P_{ix} = \frac{mn_i v_{ix}^2}{L^3}$

so that pressure in the x direction expected by all group

$$P_x = \sum P_{ix} = \frac{m}{L^3} \sum n_i v_{ix}^2$$

Average value of v^2 is given by

$$\langle v_x^2 \rangle = \frac{\sum_i n_i v_{ix}^2}{\sum n_i} = \frac{\sum_{i=1} n_i v_{ix}^2}{n}$$

For three dimensional system $\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle = \langle v^2 \rangle$ and

for isotropic system $\quad \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{\langle v^2 \rangle}{3}$

So P_x can be written as

$$P_x = \frac{m}{L^3} n \langle v_x^2 \rangle \,,\; P = P_x = \frac{1}{3}\frac{m}{L^3} n \langle v^2 \rangle \quad P = \frac{1}{3}\frac{mn\langle v^2 \rangle}{V}$$

$$PV = \frac{1}{3} mN \langle v^2 \rangle$$

where V is volume of the container and $\langle v^2 \rangle$ is average value of square of velocity.

1.2 Gas Law for Ideal Gases:

1.2.1 Boyle's Law

At constant temperature (T), the pressure (P) of a given mass a gas is inversely proportional to its volume (V)

$$P \propto \frac{1}{V}$$

1.2.2 Charle's Law

At constant pressure (P) the volume of a given mass of a gas is proportional to its temperature (T)

$$V \propto T$$

1.2.3 Avogadro's Law

At the same temperature and pressure, equal volume of all gases contain equal number of molecules (N).

$$N_1 = N_2$$

1.2.4 Graham's Law of Diffusion

When two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion (r) at each gas is inversely proportional to square root at density of gas (ρ)

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Dalton's Law of Partial Pressure: The sum of pressure exerted (P) by each gas occupying the same volume as that of the mixture (P_1, P_2, P_3,….)

$$\mathrm{P} = P_1 + P_2 + P_3 + \ldots.$$

1.2.5 Ideal Gas Equation:

Consider a sample of an Ideal gas at pressure P, volume V and temperature T the gas follows the equation

$$PV = nRT$$

Where n is number of molecules and R is proportionality constant known as gas constant

$$R = 8.314 \ \text{J/mol/K}$$

Boltzmann constant K is ratio between R to Avogadro number N_A $k_B = \frac{R}{N_A} = \frac{8.314}{6.03 \times 10^{23}}$

$$k_B = 1.3 \times 10^{-23} \, J/K$$

Example: Find the maximum attainable temperature of ideal gas in each process given by $p = p_0 - \alpha V^2$; where p_0, α and β are positive constants, and V is the volume of one mole of gas.

Solution: $$P = P_0 - \alpha V^2 \qquad \text{(i)}$$

Number of mole of gas = 1

We know $PV = nRT \quad \Rightarrow P = \frac{RT}{V}$ put in (i)

$$\frac{RT}{V} = P_0 - \alpha V^2 \quad \Rightarrow T = \frac{P_0 V}{R} - \frac{\alpha V^3}{R} \qquad \text{(ii)}$$

For T maximum, $\frac{dT}{dV} = 0 \quad \Rightarrow \frac{P_0}{R} - \frac{3\alpha V^2}{R} = 0$

$V = \sqrt{\frac{P_0}{3\alpha}}$ put in (ii) one will get $T_{\max} = \frac{2}{3} P_0 \sqrt{\frac{P_0}{3\alpha}}$

Example: Two thermally insulated vessel 1 and 2 are filled with air. They are connected by a short tube with a value. The volume of vessels and the pressure and temperate of air in them are (V_1, P_1, T_1) and (V_2, P_2, T_2) respectively. Calculate the air temperate and pressure established after opening of value if air follow Ideal gas equation.

Solution: For vessel (1) $P_1V_1 = n_1RT_1$ $\qquad n_1 = \dfrac{P_1V_1}{RT_1}$

For vessel (2) $P_2V_2 = n_2RT_2$ $\qquad n_2 = \dfrac{P_2V_2}{RT_2}$

After opening the value let pressure volume and temperature is P, V, T

$$PV = nRT$$

$$V = V_1 + V_2$$

$$n = n_1 + n_2 = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

Hence system is isolated then

Energy of (1) + energy of (2) = energy of composite

$$\frac{3}{2}n_1KT_1 + \frac{3}{2}n_2KT_2 = \frac{3}{2}(n_1 + n_2)KT$$

$$n_1T_1 + n_2T_2 = (n_1 + n_2)/T.$$

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{\dfrac{P_1V_1}{RT_1}T_1 + \dfrac{(P_2V_2)}{RT_2}T_2}{\dfrac{P_1V_1}{RT_1} + \dfrac{P_2V_2}{RT_2}} \Rightarrow T = T_1T_2\frac{(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$$

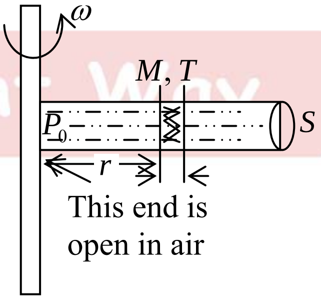
$$PV = nRT \qquad \Rightarrow P = \frac{nRT}{V} \Rightarrow \quad P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2}$$

Example: A horizontal cylinder closed from one end is rotated with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The outside air pressure is equal to p_0, the temperature to T, and the molar mass of air to M. Find the air pressure as a function of the distance r from the rotation axis. The molar mass is assumed to be independent of r.

Solution: Force equation of dr element.

$$dF = (dm)r\omega^2 \qquad \text{if } S \text{ is cross section area then}$$

$$dP = \frac{dF}{S} = \left(\frac{dm}{S}\right)r\omega^2 \qquad dm = \left(\frac{S}{r\omega^2}\right)dP$$



Also we know

$$P(Sdr) = \left(\frac{dm}{M}\right)RT$$

$$PS(dr) = \frac{RT}{M}\left(\frac{S}{r\omega^2}\right)dP$$

$$M\omega^2\int_0^r rdr = RT\int_{P_0}^{P}\frac{dP}{P}$$

$$\frac{M\omega^2r^2}{2} = RT\ln\frac{P}{P_0} \Rightarrow P = P_0e^{\frac{M\omega^2r^2}{2RT}}$$

Example: Prove that $PA = \frac{1}{2} mN\langle v^2 \rangle$ and $\langle E \rangle = \frac{2}{2} k_B T = k_B T$ in two dimension.

Solution: A molecule moving in the x direction will have momentum mv_{ix} normal to face of the cube before collision

$$\Delta P_{ix} = mv_{ix} - (-mv_{ix}) = 2mv_{ix}$$

Force acting on the wall by molecule is $f_{ix} = \frac{n_i 2mv_{ix}}{\Delta t} = \frac{n_i 2mv_{ix}^2}{2L} == \frac{n_i mv_{ix}^2}{L}$

Pressure exert on the wall of container by molecule $P_{ix} = \frac{mn_i v_{ix}^2}{L^3}$

So that pressure in the *x* direction expected by all group

$$P_x = \sum P_{ix} = \frac{m}{L^3} \sum n_i v_{ix}^2$$

Average value of v^2 is given by

$$\langle v_x^2 \rangle = \frac{\sum_i n_i v_{ix}^2}{\sum n_i} = \frac{\sum_{i=1} n_i v_{ix}^2}{n}$$

For two dimensional system $\langle v_x^2 \rangle + \langle v_y^2 \rangle = \langle v^2 \rangle$ and $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \frac{\langle v^2 \rangle}{2}$

So P_x can be written as

$$P_x = \frac{m}{L^2} n\langle v_x^2 \rangle \, , \; P = P_x = \frac{1}{2}\frac{m}{L^2} n\langle v^2 \rangle \quad P = \frac{1}{2}\frac{mn\langle v^2 \rangle}{A}$$

$$PA = \frac{1}{2} mN\langle v^2 \rangle$$

1.3 Kinetic Interpretation of Temperature

According to assumption of Kinetic theory of gases, there is only translation motion of the molecule and there is not any potential acting between them, so

Average energy $\langle E \rangle$ of gases are equivalent to Average translation energy of a molecule $\langle E \rangle = \frac{1}{2} m\langle v^2 \rangle$

Pressure at P as $P = \frac{1}{3} mn\langle v^2 \rangle = \frac{2}{3} n\left(\frac{1}{2} m\langle v^2 \rangle\right) = \frac{2}{3} n\langle E \rangle$

$PV = \frac{2}{3} Vn\langle E \rangle \quad PV = \frac{2}{3} N\langle E \rangle$ where $n = \frac{N}{V}$ number density

$\langle E \rangle = \frac{3}{2}\frac{RT}{N_A}$ and $\langle E \rangle = \frac{3}{2}\left(\frac{R}{N_A}\right) T$

$\langle E \rangle = \frac{3}{2} K_B T$ where k_B is Boltzman constant

So average kinetic energy is given by

$\langle E \rangle = \frac{3}{2} k_B T$ where T is absolute temperature.

Example: It is possible to treat electromagnetic radiation in container whose wall is mirrors, as a gas of particle (photons) with a constant speed c and whose energy is related to their momentum p which is directed parallel to their velocity by $E = pc$. Show that if container is full of radiation the equation of state is $PV = \frac{1}{3}E$

Solution: Pressure $P = \frac{1}{3}nm\langle v^2\rangle = \frac{1}{3}n\langle mv \cdot v\rangle = \frac{1}{3}n\langle \vec{p} \cdot \vec{v}\rangle$

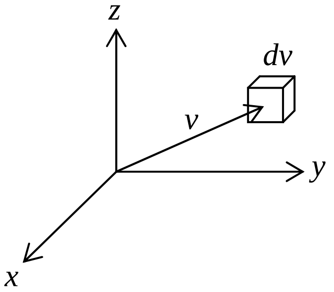
For Photon $v = c$ and velocity is parallel to momentum, so

$$P = \frac{1}{3}n\langle Pc\rangle \qquad \Rightarrow \qquad P = \frac{1}{3}\frac{N}{V}\langle pc\rangle$$

$$PV = \frac{1}{3}\langle Npc\rangle \qquad \Rightarrow \qquad PV = \frac{1}{3}E$$

1.4 Maxwell-Boltzmann Distribution Law:

Distribution of Molecular velocity in perfect gas.



Maxwell-Boltzmann distribution law is applicable for Ideal gas where molecules have no vibrational or rotational energies.

In the equilibrium state of the molecules have complete random motion and probability that a molecule has a given velocity component is independent of other two components.

In given figure dv is volume element in velocity space for a molecule at velocity $\vec{v} \equiv (v_x, v_y, v_z)$.

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

We need to calculate number of molecules simultaneously having component in the range v_x to $v_x + dv_x, v_y$ to $v_y + dv_y$ and v_z to $v_z + dv_z$

It is assumptions in Maxwell-Boltzmann distribution law is that probability that molecule selected at random has velocities in a given range is a function purely at the magnitude of velocity and the width of the interval.

So fraction of molecule having velocity component in the range v_x to $v_x + dv_x, v_y$ to $v_y + dv_y$ and v_z to $v_z + dv_z$ is $f(v_x)dv_x$, $f(v_y)dv_y$ and $f(v_z)dv_z$ respectively.

$$\frac{dN}{N} = f(v_x)f(v_y)f(v_z)dv_x\, dv_y\, dv_z$$

where dN is number of molecule having between velocity v to $v + dv$ and N is total number of molecules.

$$dN = N\, f(v_x)f(v_y)f(v_z)dv_x\, dv_y\, dv_z$$

Number of molecule having velocity v_x to $v_x + dv_x$, v_y to $v_y + dv_y$ and v_z to $v_z + dv_z$ is same as number of molecule having velocity v to $v + dv$.

So $\quad N\, f(v_x)f(v_y)f(v_z)dv_x\, dv_y\, dv_z = N\, F(v^2)dv_x dv_y dv_z$

F is some function of v^2 (magnitude of velocity) and for fixed value of $\vec{v}$, $F(v^2)$ is constant.

So $\quad dF(v^2) = 0$ is equivalent to $d[f(v_x)f(v_y)f(v_z)] = 0$

$$f'(v_x)dv_x f(v_y)f(v_z) + f'(v_y)dv_y f(v_x)f(v_z) + f'(v_z)dv_z f(v_x)f(v_y) = 0$$

Dividing both side with $f(v_x)\, f(v_y)\, f(v_z)$

$$\frac{f'(v_x)}{f(v_x)}dv_x + \frac{f'(v_y)}{f(v_y)}dv_y + \frac{f'(v_z)}{f(v_z)}dv_z = 0 \qquad \text{(i)}$$

$$v^2 = \text{constant}\ v_x^2 + v_y^2 + v_z^2 = v^2$$

$$v_x dv_x + v_y dv_y + v_z dv_z = 0 \qquad \text{(ii)}$$

by method of Lagrange's method of undetermined multiplies multiply by 2β in equation (ii) and add in equation (i)

$$\left(\frac{f'(v_x)}{f(v_x)}+2\beta v_x\right)dv_x+\left(\frac{f'(v_y)}{f(v_y)}+2\beta v_y\right)dv_y+\left(\frac{f'(v_z)}{f(v_z)}+2\beta v_z\right)dv_z=0$$

hence v_x, v_y and v_z are independent

$$\frac{f'(v_x)}{f(v_x)}+2\beta v_x=0\Rightarrow\frac{f'(v_y)}{f(v_y)}+2\beta v_y=0\Rightarrow\frac{f'(v_z)}{f(v_z)}+2\beta v_z=0$$

$$f(v_x)=A_xe^{-\beta v_x^2}\qquad f(v_y)=A_ye^{-\beta v_y^2}\qquad f(v_z)=A_ze^{-\beta v_z^2}$$

$f(v_x), f(v_y), f(v_z)$ are probability density, so

$$\int_{-\infty}^{\infty}f(v_x)dv_x=1,\qquad\int_{-\infty}^{\infty}f(v_y)dv_y=1,\qquad\int_{-\infty}^{\infty}f(v_z)dv_z=1,$$

Use the integration

$$\int_0^{\infty}e^{-\beta v^2}v^ndv=\frac{1}{2\beta^{\left(\frac{n+1}{2}\right)}}\left|\overline{\frac{n+1}{2}}\right.$$

$$A_x\int_{-\infty}^{\infty}e^{-\beta v_x^2}dv_x=1\;=A_x\;\;2\cdot\int_0^{\infty}e^{-\beta v_x^2}dv_x=1$$

$$A_x=\left(\frac{\beta}{\pi}\right)^{1/2}\text{ Similarly, }A_y=\left(\frac{\beta}{\pi}\right)^{1/2}\;A_z=\left(\frac{\beta}{\pi}\right)^{1/2}$$

$$f(v_x)=\left(\frac{\beta}{\pi}\right)^{1/2}e^{-\beta v_x^2},\quad f(v_y)=\left(\frac{\beta}{\pi}\right)^{1/2}e^{-\beta v_y^2},\quad f(v_z)=\left(\frac{\beta}{\pi}\right)^{1/2}e^{-\beta v_z^2}$$

$$\frac{dN}{N}=\left(\frac{\beta}{\pi}\right)^{3/2}e^{-\beta\left(v_x^2+v_y^2+v_z^2\right)}dv_xdv_ydv_z$$

where $-\infty<v_x<\infty$, $\qquad -\infty<v_y<\infty$, $\qquad -\infty<v_z<\infty$

1.4.1 The Distribution in Term of Magnitude

$v^2=v_x^2+v_y^2+v_z^2$ which is equation of sphere and $dv_xdv_ydv_z$ can be replace by $4\pi v^2dv$

$$f(v)dv=\frac{dN}{N}=\left(\frac{\beta}{\pi}\right)^{3/2}4\pi e^{-\beta v^2}v^2dv\qquad 0<v<\infty$$

1.4.2 To Determine Value of *β* in Term of Temperature *T*.

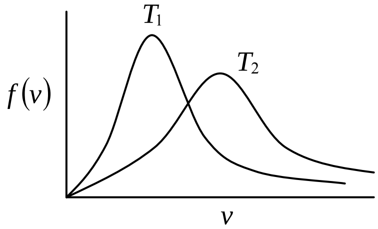
Mean square velocity $\left(\langle v^2\rangle\right)$ can be calculated by

$$\int_0^{\infty}v^2f(v^2)dv$$

$$4\pi\left(\frac{\beta}{\pi}\right)^{3/2}\int_0^{\infty}v^4e^{-\beta v^2}\,dv$$

$$\Rightarrow\quad 4\pi\left(\frac{\beta}{\pi}\right)^{3/2}\frac{1}{2\beta^{5/2}}\left|\overline{5/2}\right.$$

$$4\pi\left(\frac{\beta}{\pi}\right)^{3/2}\frac{1}{2\beta^{5/2}}\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\quad\Rightarrow\quad\langle v^2\rangle=\frac{3}{2}\cdot\frac{1}{\beta}$$



$T_1<T_1$

Now average energy of temperature T equivalent to

$$\frac{3}{2}k_BT=\frac{1}{2}m\left\langle v^2\right\rangle$$

$$\frac{3}{2}k_BT=\frac{1}{2}m\quad\frac{3}{2}\frac{1}{\beta}\quad=\beta=\frac{m}{2k_BT}$$

So $$f\left(v_x,v_y,v_z\right)=\left(\frac{m}{2\pi k_BT}\right)^{3/2}e^{-\frac{m\left(v_x^2+v_y^2+v_z^2\right)}{2k_BT}}dv_xdv_ydv_z$$

$$f\left(v\right)dv=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}v^2e^{-\frac{mv^2}{2k_BT}}dv$$

1.4.2 Average Velocity

$$\left\langle v\right\rangle=\int_0^\infty vf\left(v\right)dv\qquad=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int e^{-\frac{mv^2}{2k_BT}}v^3dv\;=\sqrt{\frac{8k_BT}{\pi m}}$$

1.4.3 Root Mean Square Velocity

$$\left[\left\langle v^2\right\rangle\right]^{1/2}=\left[\int_0^\infty v^2f\left(v\right)dv\right]^{1/2}=\left[4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\right]^{1/2}\left[\int_0^\infty e^{-\frac{mv^2}{2k_BT}}v^2dv\right]^{1/2}=\sqrt{\frac{3k_BT}{m}}$$

1.4.4 Most Probable Velocity v_p : $\frac{df}{dv}=0\;\Rightarrow v_p=\sqrt{\frac{2k_BT}{m}}$

Example: For Maxwellian gas find the $\left\langle v\right\rangle\times\left\langle\frac{1}{v}\right\rangle$

Solution: $\left\langle v\right\rangle=\sqrt{\frac{8k_BT}{\pi m}}\quad\Rightarrow\left\langle\frac{1}{v}\right\rangle=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_0^\infty\frac{1}{v}f\left(v\right)\;dv=\left(\frac{2m}{\pi k_BT}\right)^{1/2}$

$$\Rightarrow\left\langle v\right\rangle\times\left\langle\frac{1}{v}\right\rangle=\frac{4}{\pi}$$

Example: If v_x and v_y are x and y component of velocity then find the average value of $\left(av_x+bv_y\right)^2$

$$\left\langle\left(av_x+bv_y\right)^2\right\rangle=a^2\left\langle v_x^2\right\rangle+b^2\left\langle v_y^2\right\rangle+2ab\left\langle v_x\cdot v_y\right\rangle$$

$$=a^2\left\langle v_x^2\right\rangle+b^2\left\langle v_y^2\right\rangle+2ab\left\langle v_x\right\rangle\left\langle v_y\right\rangle$$

$$\left\langle v_x\right\rangle=\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}v_xe^{-\frac{m\left(v_x^2+v_y^2+v_z^2\right)}{2k_BT}}dv_xdv_ydv_z=0$$

$$\left\langle v_x^2\right\rangle=\left(\frac{m}{2\pi k_BT}\right)^{3/2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}v_x^2e^{-\frac{m\left(v_x^2+v_y^2+v_z^2\right)}{2k_BT}}dv_xdv_ydv_z\;=\frac{k_BT}{m}$$

Similarly, $\left\langle v_y\right\rangle=0\qquad\left\langle v_y^2\right\rangle=\frac{k_BT}{m}$

$$\left\langle\left(av_x+bv_y\right)^2\right\rangle=a^2\left\langle v_x^2\right\rangle+b^2\left\langle v_y^2\right\rangle+2ab\left\langle v_x\right\rangle\left\langle v_y\right\rangle$$

$$=a^2\frac{k_BT}{m}+b^2\frac{k_BT}{m}+0\;=\frac{k_BT}{m}\left(a^2+b^2\right)$$

Example: Write down expression of energy distribution function for Maxwellian gas between E and $E+dE$. Hence find $\langle E\rangle$ down $\langle E^2\rangle$.

Solution: $E=\frac{1}{2}mv^2$, $dv=\frac{dE}{(2mE)^{1/2}}$

$$f(v)dv=4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2}e^{-\frac{mv^2}{2k_BT}}v^2dv \quad \text{put value of } v \text{ and } dv$$

$$f(E)dE=\frac{2}{\sqrt{\pi}}\frac{1}{(k_BT)^{3/2}}e^{-\frac{E}{k_BT}}E^{1/2}dE \qquad 0<E<\infty$$

$$\langle E\rangle=\int_0^\infty Ef(E)dE \quad \langle E\rangle=\frac{3}{2}k_BT$$

$$\langle E^2\rangle=\int_0^\infty E^2f(E)\ dE,$$

$$\langle E^2\rangle=\int_0^\infty E^2\frac{2}{\sqrt{\pi}}\frac{1}{(k_BT)^{3/2}}e^{-\frac{E}{k_BT}}E^{1/2}dE=\frac{2}{\sqrt{\pi}}.(k_BT)^2.\frac{5}{2}.\frac{3}{2}\sqrt{\pi}=\frac{15}{2}.(k_BT)^2$$

Example: Write down expression of energy distribution function for Maxwellian gas between E and $E+dE$ in two dimensional system. Hence find $\langle E\rangle$.

$$E=\frac{1}{2}mv^2,\ dv=\frac{dE}{(2mE)^{1/2}}$$

$$f(v)dv=2\pi\left(\frac{m}{2\pi k_BT}\right)^{2/2}e^{-\frac{mv^2}{2k_BT}}vdv \quad \text{put value of } v \text{ and } dv$$

$$f(E)dE=\frac{1}{(k_BT)}e^{-\frac{E}{K_BT}}dE \qquad 0<E<\infty$$

$$\langle E\rangle=\int_0^\infty Ef(E)dE \Rightarrow \langle E\rangle=k_BT$$

Example: Using the Maxwell distribution function, calculate the mean velocity projection $\langle v_x\rangle$ the mean value of the modulus of the modulus of this projection $\langle|v_x|\rangle$ if the mass of each molecule is equal to m and the gas temperature is T.

Solution: We know Mean Velocity

$$=\langle v_x\rangle=\int_{-\infty}^{\infty}\frac{v_xdN}{N}=\frac{\int_{-\infty}^{\infty}v_xN\left(\frac{m}{2\pi k_BT}\right)^{1/2}e^{-\frac{m}{2k_BT}v_x^2}dv_x}{N}=0$$

Mean speed $$\langle|v_x|\rangle=\frac{\int_{-\infty}^{\infty}|v_x|N\left(\frac{m}{2\pi k_BT}\right)^{1/2}e^{-\frac{m}{2k_BT}v_x^2}dv_x}{N}$$

$$\langle|v_x|\rangle=\frac{2\int_0^{\infty}v_xN\left(\frac{m}{2\pi k_BT}\right)^{1/2}e^{-\frac{m}{2k_BT}v_x^2}dv_x}{N} \quad \Rightarrow\langle|v_x|\rangle=\sqrt{\frac{2k_BT}{\pi m}}$$