

Solution

IIT - JAM – 2025 (Physics)

Full Length Test – 01

Ans. 1: (c)

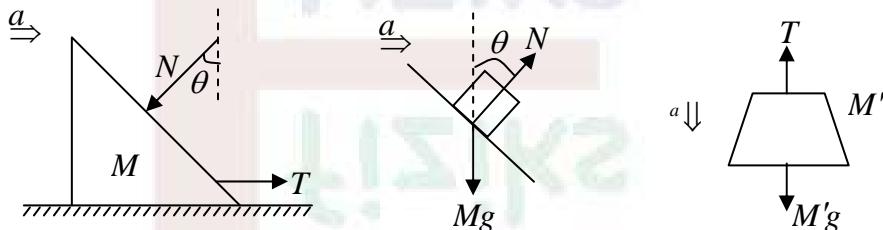
Solution: Potential inside $(\phi) = ar^2 + b$ $\therefore E_r = -\frac{\partial \phi}{\partial r} = -2ar$

Electric field inside uniformly charged solid volume varies with ' r '. So charge density is constant.

$$\text{Thus } \therefore E_r = \frac{\rho r}{3\epsilon_0} = -2ar \Rightarrow \rho = -6\epsilon_0 a$$

Ans. 2: (a)

Solution: $T - N \sin \theta = N \cos \theta = mg ; M'g - T = M'a$



Solving for M' , we get:

$$M' = \frac{M+m}{\cot \theta - 1}$$

Ans. 3: (c)

Solution: $t^2 u(t-3) = [(t-3)^2 + 6(t-3) + 9] u(t-3)$

$$= (t-3)^2 \cdot u(t-3) + 6(t-3) \cdot u(t-3) + 9u(t-3)$$

$$L[t^2 \cdot u(t-3)] = L[(t-3)^2 \cdot u(t-3)] + 6L[(t-3) \cdot u(t-3)] + 9L[u(t-3)]$$

$$= e^{-3s} \left[\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s} \right]$$

$$\text{A liter } L[t^2 u(t-3)] = e^{-3s} L(t+3)^2 = e^{-3s} L[t^2 + 6t + 9] = e^{-3s} \left[\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s} \right]$$

Ans. 4: (c)

Solution:

A	B	D	X
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = A\bar{B} + \bar{A}B, \quad X = \bar{A}\bar{B}$$

Ans. 5: (b)

Solution: $dQ = dW$, since dQ is positive, so dW is positive and the gas will do positive work.

Ans. 6: (b)

Solution: Stopping potential is the negative potential which stops the emission of $(K.E)_{\max}$ electrons when applied.

$$\therefore \text{Stopping potential} = 4V.$$

Ans. 7: (a)

Ans. 8: (c)

Ans. 9: (c)

Solution: The frequency of the combined motion is

$$\omega_a = \frac{\omega_1 + \omega_2}{2} = \frac{20\pi + 10\pi}{2} = 15\pi$$

$$f_a = \frac{\omega_a}{2\pi} = \frac{15\pi}{2\pi} = 7.5$$

Ans. 10: (b)

Ans. 11: (d)

Solution: The force on the two sides cancels.

$$\text{At the bottom, } B = \frac{\mu_0 I_1}{2\pi a} \Rightarrow F = \left(\frac{\mu_0 I_1}{2\pi a} \right) I_2 a = \frac{\mu_0 I_1 I_2}{2\pi} \text{ (up)}$$

$$\text{At the top, } B = \frac{\mu_0 I_1}{2\pi(a+a)} \Rightarrow F = \frac{\mu_0 I_1 I_2 a}{4\pi a} \Rightarrow F = \frac{\mu_0 I_1 I_2}{4\pi} \text{ (down)}$$

$$\text{Thus Net Force} = \left(\frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \right) = \frac{\mu_0 I_1 I_2}{4\pi} \text{ (up)}$$

Ans. 12: (a)

Solution: The initial angular momentum of the asteroid about the centre of the planet is $L = mv_0 d$.

At the turning point the velocity v of the asteroid will be perpendicular to the radial vector. Therefore the angular momentum $L' = mvR$ if the asteroid is to just graze the planet. Conservation of angular momentum requires that $L' = L$. Therefore

$$mvR = mv_0 d \text{ or } v = \frac{v_0 d}{R}$$

Energy conservation requires

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - \frac{GMm}{R} \text{ or , } v^2 = v_0^2 + \frac{2GM}{R}$$

Eliminating v between (2) and (4) the minimum value of v_0 is obtained.

$$v_0 = \sqrt{\frac{2GMR}{d^2 - R^2}}.$$

Ans. 13: (a)

Solution: DC Analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{470k\Omega} = 24.04 \mu A$$

$$I_E = (\beta + 1)I_B = 101 \times (24.04 \mu A) = 2.43 mA$$

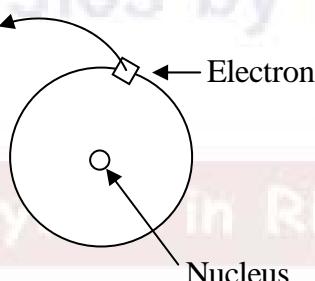
$$r_e = \frac{26mV}{I_E} = \frac{26mV}{2.43mA} = 10.7 \Omega$$

AC Analysis:

$$r_o = \infty; A_v = -\frac{R_C}{r_e} = -\frac{2k\Omega}{10.7\Omega} \approx -187$$

Ans. 14: (c)

Solution:



The electrostatic force = centripetal force

$$\text{i.e., } \frac{mv^2}{r} = \frac{Ze^2}{kr^2}$$

$$\Rightarrow mv^2 r = \frac{Ze^2}{k} \quad (\text{i})$$

$$\text{By Bohr theory } mv r = n\hbar \quad (\text{ii})$$

Dividing equation (i) by, equation (ii), we get

$$v = \frac{Ze^2}{kn\hbar} \Rightarrow v = \frac{1}{n} \left(\frac{Ze^2}{k\hbar} \right) \Rightarrow v \propto \frac{1}{n} \Rightarrow \frac{v_1}{v_2} = \frac{n_2}{n_1} \Rightarrow v_2 = \frac{n_1}{n_2} v_1 \Rightarrow v_n = \frac{v_1}{n}$$

Ans. 15: (d)

Solution: $f(x) = \begin{cases} 0 & \text{if } -\pi < x < -\pi/2 \\ 1 & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 < x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$

$$\text{Let } f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/2} (0) dx + \int_{-\pi/2}^{\pi/2} (1) dx + \int_{\pi/2}^{\pi} (0) dx \right] = \frac{1}{2\pi} \pi = \frac{1}{2}$$

$$\therefore a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$\Rightarrow a_n = \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} (0) \cos nx dx + \int_{-\pi/2}^{\pi/2} (1) \cos nx dx + \int_{\pi/2}^{\pi} (0) \cos nx dx \right]$$

$$\Rightarrow a_n = \frac{1}{\pi} \left\{ \frac{\sin nx}{n} \right\}_{-\pi/2}^{\pi/2} = \frac{1}{n\pi} \left\{ \sin n \frac{\pi}{2} + \sin n \frac{-\pi}{2} \right\} = \frac{2}{n\pi} \sin n \frac{\pi}{2}$$

$$\therefore b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$\Rightarrow b_n = \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} (0) \sin nx dx + \int_{-\pi/2}^{\pi/2} (1) \sin nx dx + \int_{\pi/2}^{\pi} (0) \sin nx dx \right] \Rightarrow b_n = -\frac{1}{\pi} \left\{ \frac{\cos nx}{n} \right\}_{-\pi/2}^{\pi/2} = 0$$

Thus Fourier series is

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \left[\frac{1}{1} \sin \frac{\pi}{2} \cos x + \frac{1}{2} \sin 2 \frac{\pi}{2} \cos 2x + \frac{1}{3} \sin 3 \frac{\pi}{2} \cos 3x + \dots \right]$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \left[\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x - \dots \right]$$

Ans. 16: (d)

Solution: Characteristic time: $\tau = \frac{\varepsilon}{\sigma} = \frac{8.85 \times 10^{-12}}{10^6} = 8.85 \times 10^{-18}$

Ans. 17: (d)

Solution: Group velocity is $v_g = d\omega/dk$. So, take the derivative of the quantity

$\omega = \sqrt{c^2 k^2 + m^2}$ to get

$$v_g = d\omega/dk = \frac{c^2 k}{\sqrt{c^2 k^2 + m^2}}$$

Use the above equation to test the choices:

- (a) As $k \rightarrow 0 \Rightarrow v_g = 0$, not infinity. The first condition doesn't work, no need to test the second (don't have to remember L'Hopital's rule).
- (b) Wrong for the same reason as (a).
- (c) As $k \rightarrow \infty, v_g \approx \frac{kc^2}{\sqrt{c^2 k^2}} = c$, since $m \ll (ck)^2$. So, v_g doesn't tend towards ∞ . This choice is wrong.
- (d) This is it. The conditions work (and it's the only choice left).

Ans. 18: (c)

Solution: A diagonalised = $S^{-1}A S$ (i)

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \text{ and finding } S^{-1} \text{ and putting in (i), we get } A \text{ diagonalized} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Ans. 19: (c)

Solution: $V_o = -\frac{R_l}{R_i}V_i + \left(1 + \frac{R_l}{R_i}\right) \frac{X_c}{R + X_c} V_i \Rightarrow \frac{V_o}{V_i} = \frac{1 - RCs}{1 + RCs}$ where $s = j\omega$.

Thus $\phi = -2 \tan^{-1}(\omega RC)$.

Minimum value of $\phi = -\pi$ (at $\omega \rightarrow \infty$)

Maximum value of $\phi = 0$ (at $\omega = 0$)

Ans. 20: (a)

Solution: 16. Let $f_1 = u - x - y - z$, $f_2 = 2v - xyz$, and $f_3 = w - vx$.

$$\frac{\partial(f_1 f_2 f_3)}{\partial(x, y, z)} = \begin{vmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{vmatrix} = \begin{vmatrix} -1 & -1 & -1 \\ -yz & -xz & -xy \\ -v & 0 & 0 \end{vmatrix} = -v(xy - xz)$$

$$\frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} \\ \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} \\ \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial w} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -x & 1 \end{vmatrix} = 2$$

$$\text{Hence } \frac{\partial(u, v, w)}{\partial(x, y, z)} = (-1)^3 \frac{\partial(f_1, f_2, f_3)}{\partial(u, v, w)} \Rightarrow \frac{\partial(u, v, w)}{\partial(x, y, z)} = \frac{v(xy - xz)}{2}$$

at the point $x = 1, y = 2, z = 1$, we have $v = \frac{1.2.1}{2} = 1$

$$\text{Hence, } \frac{\partial(u, v, w)}{\partial(x, y)} \Big|_{(1,2,1)} = \frac{1(1.2 - 1.1)}{2} = \frac{1}{2}$$

Ans. 21: (d)

Solution: $1 = \int_{-\infty}^{\infty} |\psi|^2 dx = A^2 = A^2 \int_{-a}^a dx \left(1 + 2 \cos \frac{\pi x}{a} + \frac{1}{2} \cos^2 \frac{\pi x}{a} \right)$

$$= A^2 \int_{-a}^a dx \left[\frac{3}{2} + 2 \cos \frac{\pi x}{a} + \frac{1}{2} \cos \frac{2\pi x}{a} \right] \text{ so } A = \frac{1}{\sqrt{3a}}$$

Ans. 22: (b)

Solution: Since $n_1 = 1$, $n_2 = 1.52$

Brewster angle $\theta_B = \tan^{-1} \left(\frac{n_2}{n_1} \right) = \tan^{-1} \left(\frac{1.52}{1} \right) = 56.7^\circ$

Now $\theta_R = 180 - 90 - 56.7 = 33.4^\circ$

Ans. 23: (a)

Solution: $\beta_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$

At a constant temperature, for a given mass of gas, $PV = \text{constant}$, according to Boyle's law.

$$\therefore PV = \text{constant} \therefore PdV + VdP = 0$$

or $\frac{dV}{dP} = -\frac{V}{P}$ or $-\frac{1}{V} \left(\frac{dV}{dP} \right) = +\frac{1}{P}$ or $\beta_T = \frac{1}{P}$ or $\beta_T P = 1$

Ans. 24: (a)

Solution: $T = 2\pi \sqrt{\frac{l}{g}}$

The acceleration of lift is

$$a = \frac{dv}{dt} = \frac{d}{dt} \left(\frac{dy}{dt} \right) = \frac{d}{dt} \left[(1.5m/s^2)(2t) \right] = 3m/s^2$$

The effective acceleration experienced by the pendulum is

$$T^1 = 2\pi \sqrt{\frac{1}{g+a}}$$

$$\therefore \frac{T^1}{T} = \sqrt{\frac{g}{g+a}} = \sqrt{\frac{10}{13}}$$

$$T^1 = \sqrt{\frac{10}{13}} T$$

Ans. 25: (b)

Solution: $\frac{dU}{dx} = 0 \Rightarrow k(6x^2 - 10x + 4) = 0 \Rightarrow x = 1, x = \frac{2}{3}$

$$\frac{d^2U}{dx^2} = k(12x - 10)$$

For $x = 1$, $\frac{d^2U}{dx^2} = 2k > 0$ and for $x = \frac{2}{3}$, $\frac{d^2U}{dx^2} = -4k < 0$ So

$$\omega = \sqrt{\frac{2k}{m}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{2k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{2k}} \Rightarrow T = \pi\sqrt{\frac{2m}{k}}$$

Ans. 26: (d)

Solution: From first law of thermodynamics,

$$TdS = dE + PdV, dE = TdS - PdV, \text{ it is given } dV = 0$$

$$dE = TdS \Rightarrow dS = \frac{1}{T}dE$$

$$E = \frac{aT^2}{2} + \frac{bT^4}{4} \Rightarrow dE = aTdT + bT^3dT$$

$$dS = \frac{1}{T}(aTdT + bT^3dT) = adT + bT^2dT \Rightarrow S = aT + \frac{bT^3}{3}$$

Ans. 27: (b)

Solution 27: $y_p = \frac{1}{(D^2 - 4D + 3)} 3e^x \cos 2x = 3e^x \frac{1}{(D-1)^2 - 4(D-1) + 3} \cos 2x$

$$= 3e^x \frac{1}{D^2 - 2D} \cos 2x = 3e^x \frac{1}{(-4) - 2D} \cos 2x$$

$$= -\frac{3}{2} e^x \frac{D-2}{D^2 - 4} \cos 2x = -\frac{3}{2} e^x \frac{1}{(-4)-4} (D-2) \cos 2x$$

$$= \frac{3}{16} e^x (-2 \sin 2x - 2 \cos 2x) \Rightarrow y_p = -\frac{3}{8} e^x (\sin 2x + \cos 2x)$$

The general solution of the corresponding homogeneous differential equation is

$$y_h = c_1 e^x + c_2 e^{3x}$$

Thus the general solution of the given nonhomogeneous equation is

$$y = y_h + y_p \Rightarrow y = c_1 e^x + c_2 e^{3x} - \frac{3}{8} e^x (\cos 2x + \sin 2x).$$

Ans. 28: (a)

Solution:

(a) $k_h = -k_e$ (correct)

If an electron is missing from an orbital of wave vector k_e the total wavevector of the system is $-ve$ and is attributed to the hole. $k_h + k_e = 0 \Rightarrow k_h = -k_e$

(b) $v_h = -v_e$ (Incorrect)

The velocity of the hole is equal to the velocity of the missing electron. From the figure we see

that $v_h = \frac{\partial E_h}{\partial k_h} = \frac{\partial E_e}{\partial k_e} = v_e \Rightarrow v_h = v_e$

(c) $m_h = m_e$ (incorrect)

The effective mass is inversely proportional to the curvature $\frac{\partial^2 E}{\partial k^2}$ and for hole band this has the opposite sign to that for an electron in the valence band. Near the top of the valence band m_e is negative, so that m_h is positive

(d) $E_h = E_e$ (Incorrect)

If the band is symmetric. Then

$$E_e(k_e) = E_e(-k_e) = -E_h(-k_h) = -E_h(k_h) \Rightarrow E_e = -E_h$$

Ans. 29: (c)

Solution: Equating the volume, we have

$$8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3 \text{ or } R = 2r$$

$$V_T \propto (\text{radius})^2$$

Radius has become two times. Therefore terminal velocity will become 4 times.

Ans. 30: (d)

Solution: $\frac{N_{x_1}}{N_{x_2}} = \frac{1}{e} = \frac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}}$

$$t = \frac{1}{9\lambda}$$

Ans. 31: (a), (b) and (c)

Ans. 32: (b) and (d)**Solution:** For orthogonal condition, scalar product

$$(\psi_1, \psi_2) = 0, \text{ so } 1 + 2 + 3\alpha = 0 \Rightarrow \alpha = -1$$

$$\psi_2 = \phi_0 - \phi_1 + \alpha \phi_2, \text{ put } \alpha = -1, \langle H \rangle = \frac{\langle \psi_2 | H | \psi_2 \rangle}{\langle \psi_2 | \psi_2 \rangle} = \frac{\frac{\hbar\omega}{2} + \frac{3\hbar\omega}{2} + \frac{5\hbar\omega}{2}}{3} = \frac{3}{2}\hbar\omega$$

Ans. 33: (b) and (d)**Solution:** Kinetic theory of gases are based on the following assumptions:

- (i) All gases are made up of tiny elastic particles known as molecules.
- (ii) The volume of the molecule is negligible.
- (iii) The collision between the molecules is elastic.
- (iv) They exert no force on each other.

Ans. 34: (a), (b), (c) and (d)

Solution: For the disc about its diameter $Mk^2 = \frac{MR^2}{4} \Rightarrow k = \frac{R}{2}$

For the disc about the tangent in its plane $Mk^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2 \Rightarrow k = \frac{\sqrt{5}}{2}R$

For the thin rod about an axis through one end $MK^2 = \frac{ML^2}{3} \Rightarrow K = \frac{L}{\sqrt{3}}$

For a rectangular lamina about an axis through the centre and perpendicular to the rod

$$Mk^2 = \frac{M}{12}(l^2 + b^2) \Rightarrow k = \frac{1}{2}\sqrt{\frac{l^2 + b^2}{3}}.$$

Ans. 35: (a), (b) and (d)**Ans. 36: (a), (d)**

Solution: Due to solenoid field is non zero in region $0 < r < R$ and non zero in region $r > 2R$ due to conductor.

Ans. 37: (a), (c)**Ans. 38: (a),(c) and (d)**

Solution: $V_1 = \sqrt{2gh}$

$$V_2 = \sqrt{2g \times 4h} = 2V_1$$

$$t_1 = \sqrt{\frac{2h}{g}}, t_2 = \sqrt{\frac{2 \times 4h}{g}}$$

$$t_2 = 2t$$

$$R_1 = 2\sqrt{4 \times 4h}$$

$$R_2 = 2\sqrt{4h \times h}$$

Ans. 39: (b) and (c)

Solution:

(a) $y = a \cos(kx - \omega t)$

$$\text{Wave speed } (v_p) = \frac{dx}{dt} = \frac{\omega}{k}$$

Increasing ω also increases k as a result wave speed remain independent of frequency. Thus, this option is wrong.

(b) $v_p = \frac{\omega}{k}$

Thus v_p is also independent of amplitude ‘ a ’. This is a correct option

(c) Power $P = \frac{1}{2} \mu \omega^2 A^2 v \Rightarrow P \propto A^2$

If ‘ A ’ changes by 2 than P increases by 4. This is a correct option

(d) wave frequency ω is independent of amplitude ‘ a ’. This is a wrong option

Ans. 40: (a), (b)

Solution: $I = \frac{V_0}{\sqrt{\left(L\omega - \frac{1}{C\omega}\right) + R^2}}$

$$f_R = \frac{1}{\sqrt{LC}} = 10^6 \text{ rad s}^{-1} \Rightarrow \text{Resonance frequency}$$

If $\omega \gg 10^6 \text{ rad / s}$, then circuit behaves as inductive circuit.

Ans. 41: 20

Solution: $f = \frac{qB}{2\pi m} \Rightarrow \frac{f'}{f_\alpha} = \frac{q'}{q_\alpha} \times \frac{m_\alpha}{m'} = \frac{q}{q} \times \frac{4m_p}{3m_p} = \frac{4}{3} \Rightarrow f' = 20 \text{ MHz}$

Ans. 42: 2

Solution: $t'_2 - t'_1 = 0 \Rightarrow \frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{t_1 - \frac{vx_1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left((t_2 - t_1) - \frac{v}{c^2} (x_2 - x_1) \right) = 0$

$$\text{Dividing by } (t_2 - t_1) \text{ we get } 1 - \frac{v}{c^2} \frac{(x_2 - x_1)}{(t_2 - t_1)} = 0 \Rightarrow \frac{(x_2 - x_1)}{(t_2 - t_1)} = \frac{c^2}{v} = 2c$$

Ans. 43: (b)

Solution: $(8 + 0 + 2 + 1) + (0.5 + 0.25 + 0.125) = (11.875)_{10}$

Ans. 44: 1.86

Solution: Since, $E = 315 \text{ MeV}$ and $m_0 = 105 \frac{\text{MeV}}{c^2}$.

$$E = mc^2 \Rightarrow E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 315 = \frac{105}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow v = 0.94 c .$$

$$\text{Now, } t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t_0 = 2.2 \mu\text{s} \Rightarrow t = \frac{2.2 \times 10^{-5}}{\sqrt{1 - \frac{8}{9}}} = 6.6 \mu\text{s}$$

Now, the distance traversed by muon is $vt = 0.94 c \times 6.6 \times 10^{-6} = 1.86 \text{ km}$.

Ans. 45: 10

Solution: $\varepsilon = -\frac{d\phi}{dt} = -(20t - 50) = -10 \text{ volt}$.

Ans. 46: 1: 8

Solution: $N = \frac{1 \times 8}{1} = 1$

Ans. 47: 19.8

Solution: $\frac{2 \times 3\pi^2 \hbar^2}{2ma^2} = 2 \times 9.9 \times 10^{-17} \text{ J} = 19.8 \times 10^{-17} \text{ J}$

Ans. 48: 5

Solution: When $V_2 = 0$, $v_{01} = -V_1$ when $V_1 = 0$, $v_{02} = \left(1 + \frac{R}{R}\right) \left(\frac{R_2}{R_1 + R_2}\right) V_2$

Since $V_0 = -V_1 + \frac{V_2}{3} \Rightarrow 2 \cdot \frac{R_2}{R_1 + R_2} = \frac{1}{3} \Rightarrow \frac{R_1}{R_2} = 5$

Ans. 49: 2

Solution: $I_C = 4I_0$, $I_{INCO} = I_0 + I_0 = 2I_0$

$$\text{Ratio} = \frac{I_C}{I_{INCO}} = 2$$

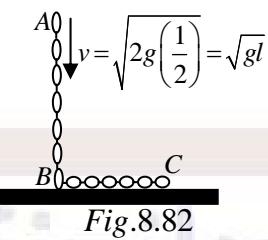
Ans. 50: 6.0

Solution: $\left| \frac{dN}{dt} \right| = \lambda N = \frac{0.693}{3.942 \times 10^{16}} \times \frac{70 \times 10^3}{40} \times \frac{3.29 \times 10^5}{100} \times 6.022 \times 10^{23}$
 $= 6.0 \times 10^3 \text{ disintegration / S}$
 $= 6.0 \times 10^3 \text{ Bq} = 6 \text{ KBq}$

Ans. 51: 0.5

Solution: Magnetic dipole moment $M' = IA = \frac{Q}{T} \pi r^2 \Rightarrow \frac{Q}{2\pi T} \times 2\pi \times \pi r^2 = \frac{Q\omega r^2}{2} \quad \therefore \omega = \frac{2\pi}{T}$

Angular momentum $J = Mr^2\omega \Rightarrow \frac{M'}{J} = \frac{Q}{2M} = 0.5 \left(\frac{Q}{M} \right)$

Ans. 52: 1.5
Solution:

Fig.8.82

Force exerted by the chain on the table. It consists of two parts:

(1) Weight of the portion BC of the chain lying on the table.

$$W = \frac{mg}{2} \text{ (downwards)}$$

(2) Thrust force $F_t = \lambda v^2$

$$\text{Here, } \lambda = \text{mass per unit length of chain} = \frac{m}{l}$$

$$v^2 = (\sqrt{gl})^2 = gl$$

$$\therefore F_t = \left(\frac{m}{l} \right) (gl) = mg \quad (\text{Downwards})$$

\therefore Net force exerted by the chain on the table is

$$F = W + F_t = \frac{mg}{2} + mg = \frac{3}{2}mg \quad (\text{Downwards})$$

So, from Newton's third law the force exerted by the table on the chain will be $\frac{3}{2}mg$

(Vertical upwards).

Here, the thrust force (F_t) applied by the chain on the table will be vertically downwards, as

$F_t = v_r \left(\frac{dm}{dt} \right)$ and in this expression v_r is downwards plus $\frac{dm}{dt}$ is positive. So, will be downwards.

Ans. 53: 9.2

Solution: When $V = 5V \Rightarrow$ open circuit voltage $V_i = \frac{1000}{1500} \times 5 = 3.33 < V_Z = 10V$

$$\Rightarrow V_L = V_i = 3.33V \Rightarrow P_{L,\min} = \frac{V_i^2}{R_L}$$

When $V = 20V \Rightarrow$ open circuit voltage $V_i = \frac{1000}{1500} \times 20 = 13.33 > V_Z = 10V$

$$\Rightarrow V_L = V_z = 10V \Rightarrow P_{L,\max} = \frac{V_z^2}{R_L}$$

$$\Rightarrow \frac{P_{L,\max}}{P_{L,\min}} = \frac{V_z^2}{V_i^2} = \left(\frac{10}{3.33}\right)^2 = 9.2$$

Ans. 54: 20

Solution: Total number of degeneracy

$$g = (\text{No. of energy state } n) \times (\text{No. of degeneracy due to spin } (2s+1))$$

$$n = 3, s = \frac{1}{2}, \quad g = 3 \times (2 \cdot \frac{1}{2} + 1) = 6$$

No. of particle $N = 3$ so no. of ways ${}^g c_N = {}^6 c_3 = 20$

Ans. 55: 182

Solution: $I = \frac{V}{R} e^{-t/RC} = \frac{3}{10 \times 10^3} e^{-5/10 \times 10^3 \times 1000 \times 10^{-6}} = \frac{3}{10^4} e^{-5/10} = \frac{3}{\sqrt{e} \times 10^4} = \frac{3}{1.65 \times 10^4} = 182 \mu A$

Ans. 56: 23

Solution: For no dispersion, $v_g = v$ i.e. $\frac{dv}{d\lambda} = 0$

$$\therefore v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}$$

differentiating this w.r.t λ and sifting

$$\frac{dv}{d\lambda} = 0 \text{ and } \lambda = \lambda_0 \text{ gives}$$

$$\lambda_0 = 2\pi \sqrt{\frac{\sigma}{\rho g}} = 2\pi \sqrt{\frac{7.2 \times 10^{-2}}{1000 \times 9.8}}$$

$$\lambda_0 = 1.70 \times 10^{-2} m$$

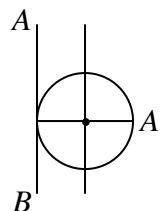
$$\therefore v_g = \left(\frac{g\lambda_0}{2\pi} + \frac{2\pi\sigma}{\rho\lambda_0} \right)^{1/2}$$

$$= \left(\frac{9.8 \times 1.7 \times 10^{-2}}{2\pi} + \frac{2\pi \times 7.2 \times 10^{-2}}{1000 \times 1.7 \times 10^{-2}} \right)$$

$$= 0.23 m/sec = 23 cm/sec$$

Ans. 57: 2000

Solution:



The moment of inertia of the disc about axis AB is

$$I_{AB} = I_{A'B'} + MR^2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2$$

The angular speed ω of the disc $= \frac{v}{R}$

$$\text{Hence } K = \frac{1}{2} \times \frac{5}{4}MR^2 \times \frac{v^2}{R^2} = \frac{1}{2} \times \frac{5}{4} \times 8 \times 400 = 2000 \text{ J}$$

Ans. 58: 2

Solution: $P = \frac{RT}{V-b} \exp\left(-\frac{a}{RTV}\right)$ critical point are point of inflection so, at critical points

$$\left(\frac{\partial P}{\partial V}\right)_T = 0 \text{ and } \left(\frac{d^2P}{dV^2}\right)_T = 0 \text{ (point of inflection)}$$

$$\text{So, } V_c = 2b$$

Ans. 59: 2

Solution: Hence mass is loosing adiabatically the angular momentum is constant

$$m_B \gg m_A \quad \frac{J^2}{m_A R_A^3} = \frac{Gm_A m_B}{R_A^2} \quad \text{so orbital radius } R_A \text{ is proportional to } \frac{1}{m_A^2}$$

Ans. 60: 10

Solution: $\Delta p = \frac{1}{2} \rho v^2$

$$\therefore v = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2 \times 0.5 \times 10^5}{10^3}} = 10 \text{ m/s}$$