Solution

GATE Full Length Test

Ans. 1: (c)

Solution: The sentence tries to blame the police for not responding to the tragic event.

Ans. 2: (c)

Solution: $1+\cos 2x = 2 \Rightarrow 2\cos^2 x = 2 \Rightarrow \cos^2 x = 1$

$\Rightarrow \cos x = 1$ or $\cos x = -1$

In the given interval $\cos x$ is never equal to 1.

$\cos x = -1$ at $x = 3\pi$

Ans. 3: (a)

Solution: Let a be the side length of the hexagon.

From the question, $6\times\frac{\sqrt{3}}{4}a^2 = 24\sqrt{3}$

$\Rightarrow a^2 = 16 \Rightarrow a = 4$

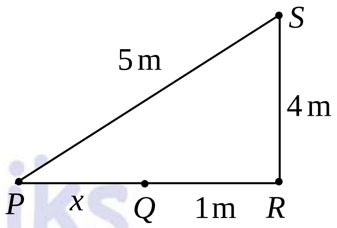
Hence sum of all sides $= 6\times 4 = 24$

Ans. 4: (b)

Solution: 'Emerging situation' means evolving situation.

Ans. 5: (c)

Solution: From the given information we can draw the diagram as shown in the figure.



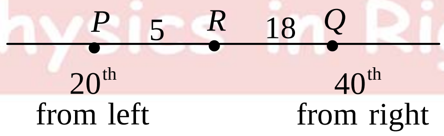
Here we have assumed that the distance between P and Q is x

From the figure, $(x+1)^2 + 4^2 = 5^2$

$\Rightarrow (x+1)^2 = 9 \Rightarrow x+1 = 3 \Rightarrow x = 2$

Ans. 6: (b)

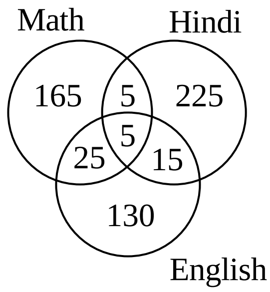
Solution:



Total number of soldiers $= 20+5+18+40+1 = 84$

Ans. 7: (d)

Solution: Using the given information we construct the Venn-diagram



Total number of students passing in at least one subject

$$= 165 + 5 + 5 + 25 + 225 + 15 + 130 = 570$$

Hence number of students who are unable to pass even in a single subject

$$= 1000 - 570 = 430$$

Ans. 8: (c)

Solution: $x^8 + \frac{1}{x^8} = 2 \Rightarrow x^8 + 2 \cdot x^4 \frac{1}{x^4} + \frac{1}{x^8} = 4$

$$\Rightarrow \left(x^4 + \frac{1}{x^4}\right)^2 = 4 \Rightarrow x^4 + \frac{1}{x^4} = 2 \Rightarrow x^4 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \frac{1}{x^4} = 4 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 4$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2$$

Ans. 9: (b)

Solution: This is a problem on combination. Triangles are formed by joining three points without considering their order.

Total number of triangles $= nc_3 = \frac{n^3 - 3n^2 + 2n}{6}$

Ans. 10: (b)

Solution: The formula for average speed is

Average speed $= \frac{2v_1 v_2}{v_1 + v_2}$

Where v_1 and v_2 are speed during the forward and return Journey.

From the question,

$$\frac{2 \cdot 60 \cdot x}{60 + x} = 48$$

$$\Rightarrow 120x = 48 \times 60 + 48x \Rightarrow 72x = 48 \times 60 \Rightarrow x = 40$$

Ans. 11: (b)

Solution: For Lagrangian $L = \frac{1}{2} m \left(\frac{dq}{dt}\right)^2 - \frac{1}{2} m\omega^2 q^2 + \alpha q \dot{q}$

Equation of motion is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \Rightarrow m\ddot{q} + m\omega^2 q = 0$$

Ans. 12: (d)

Solution: XOR is inequality comparator and XNOR is equality comparator. In AND gate output will be high when all the input is 1.

Ans.13: (a)

Solution: $D^2 - 3D + 2 = 0 \Rightarrow (D-1)(D-2) = 0 \Rightarrow D = 1, 2 \Rightarrow x = c_1 e^{2t} + c_2 e^t$

Using boundary condition $x = 0$ at $t = 0 \Rightarrow c_1 = -c_2$

Again using boundary condition at $x = 1$ at $t = 1 \Rightarrow c_2 = -\frac{1}{e^2 - e}, c_1 = \frac{1}{e^2 - e}$

$$\Rightarrow x = \frac{e^{2t}}{e^2 - e} - \frac{1}{e^2 - e} e^t = \frac{1}{e^2 - e}\left(e^{2t} - e^t\right)$$

If $t = 2$ then $x = \frac{e^4}{e^2 - e} - \frac{e^2}{e^2 - e} = \frac{e^3 - e}{e - 1} = \frac{e(e-1)(e+1)}{e-1} = \left(e^2 + e\right)$

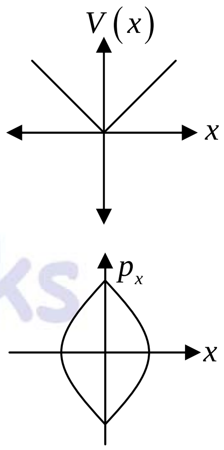
Ans. 14: (b)

Solution: The magnetic moment of $La^{3+}\,({}^1S_0)$ is $p = g\sqrt{i(j+1)} = 0$

Therefore, La^{3+} is a diamagnetic in nature.

Ans. 15: (d)

Solution: $E = \frac{p^2}{2m} + k|x|$



For $x > 0$, $E = \frac{p^2}{2m} + kx$

$\Rightarrow p^2 = 2m(E - kx)$

For $x < 0$, $E = \frac{p^2}{2m} - kx$

$\Rightarrow p^2 = 2m(E + kx)$

Ans. 16: (b)

Solution: The mean number of impacts between the molecules of the gas and wall per unit time per unit area is number current density which is assuming that the molecules collides with the face which is parallel to xy plane

$$dJ_z = \frac{N}{V}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left\{-\frac{m\left(v_x^2 + v_y^2 + v_z^2\right)}{2k_B T}\right\} v_z\, dv_x dv_y dv_z, -\infty < v_x < \infty, -\infty < v_y < \infty,$$

$0 < v_z < \infty$

$$J_z = \frac{N}{V}\left(\frac{m}{2\pi k_B T}\right)^{1/2}\int_0^{\infty} \exp-\left(\frac{mv_z^2}{2k_B T}\right) v_z\, dv_z$$

$$\frac{N}{V}\left(\frac{m}{2\pi k_B T}\right)^{1/2}\times\frac{1}{2}\times\frac{1}{\left(\frac{m}{2k_B T}\right)^{\frac{1+1}{2}}}\left|\frac{1+1}{2}\right. = \frac{N}{V}\sqrt{\frac{k_B T}{2\pi m}}$$

$$U = \frac{3}{2}k_B T \Rightarrow k_B T = \frac{2U}{3}$$

$$\frac{N}{V}\sqrt{\frac{2U}{6\pi m}} \Rightarrow \frac{N}{V}\sqrt{\frac{U}{3\pi m}}$$

Ans. 17: (c)

Solution: $\vec{A}' = \vec{A} + \vec{\nabla}\lambda = \vec{A} + \hat{r} \Rightarrow \partial\lambda/\partial r = 3 \Rightarrow \lambda = 3r + C$

$V' = V - \partial\lambda/\partial t = V - 3\partial r/\partial t$

Ans. 18: (d)

Solution: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$

And $|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-\frac{i\in t}{\hbar}} + |\psi_-\rangle e^{\frac{i\in t}{\hbar}}\right]$ At $t = \frac{h}{6\in}$,

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-\frac{i\in h\times 2\pi}{6\in h}} + |\psi\rangle e^{\frac{i\in h\times 2\pi}{6\in h}}\right] = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{\frac{-i\pi}{3}} + |\psi_-\rangle e^{\frac{i\pi}{3}}\right]$$

Ans. 19: (a)

Solution: $Z = 7: \left(1s_{1/2}\right)^2\left(1p_{3/2}\right)^4\left(1p_{1/2}\right)^1 \qquad \therefore\ j_1 = \frac{1}{2}, l_1 = 1$

$N = 7: \left(1s_{1/2}\right)^2\left(1p_{3/2}\right)^4\left(1p_{1/2}\right)^1 \qquad \therefore\ j_2 = \frac{1}{2}, l_2 = 1$

Now, $j_1 + j_2 + l_1 + l_2 = \frac{1}{2} + \frac{1}{2} + 1 + 1 = 3$ (odd)

$\therefore$ spin $= |j_1 + j_2| = 1$

and parity $= (-1)^{\sum l} = (-1)^{1+1} = \text{even}$

$\therefore I^{\pi} = I^{+}$

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Ans. 20: (b)

Ans. 21: (c)

Solution: (a) Electric polarizability: $\alpha_e = \dfrac{e^2}{m\left(\omega_0^2 - \omega^2\right)}$. It is temperature independent. This option is incorrect

(b) Ionic polarizability: $\alpha_i = \dfrac{e^2}{\omega_0^2}\left(\dfrac{1}{m} + \dfrac{1}{M}\right)$. It is also temperature independent. This option is also incorrect

(c) Dipolar polarizability: $\alpha_d = \dfrac{p^2}{3kT}$. It depends is temperature. Thus, this option is correct

(d) α_e also depends on frequency. This option is incorrect.

Ans. 22: (b)

Solution:

$$\psi(x) = \begin{cases} k, & -\frac{a}{2} - \frac{d}{2} < x < -\frac{a}{2} + \frac{d}{2} \\ 0, & -\frac{a}{2} + \frac{d}{2} < x < \frac{a}{2} - \frac{d}{2} \\ k, & \frac{a}{2} - \frac{d}{2} < x < \frac{a}{2} + \frac{d}{2} \\ 0, & \frac{a}{2} + \frac{d}{2} > 0 \end{cases}$$

$$\langle \psi | \psi \rangle = 1$$

$$k^2 \int_{-\frac{a}{2}-\frac{d}{2}}^{-\frac{a}{2}+\frac{d}{2}} dx + k^2 \int_{\frac{a}{2}-\frac{d}{2}}^{\frac{a}{2}+\frac{d}{2}} dx = 1$$

$$k^2\left[\left(-\frac{a}{2} + \frac{d}{2}\right) - \left(-\frac{a}{2} - \frac{d}{2}\right)\right] + k^2\left[\left(\frac{a}{2} + \frac{d}{2}\right) - \left(\frac{a}{2} - \frac{d}{2}\right)\right] = 1$$

$$k^2\left[\frac{d}{2} + \frac{d}{2} + \frac{d}{2} + \frac{d}{2}\right] = 1 \Rightarrow k = \frac{1}{\sqrt{2d}}$$

Ans. 23: (c)

Solution: $\dfrac{R_{Mg}}{R_{Cu}} = \left(\dfrac{A_{Mg}}{A_{Cu}}\right)^{1/3}$

$$\Rightarrow R_{Mg} = \left(\frac{A_{Mg}}{A_{Cu}}\right)^{1/3} R_{Cu} = \left(\frac{27}{64}\right)^{1/3} \times 4.8\times10^{-15}\,m = 3.6\times10^{-15}\,m$$

Ans. 24: (a)

Solution: $K.E$ = rest mass energy

$$E - m_e c^2 = m_e c^2 \Rightarrow E = 2m_0c^2$$

$$\Rightarrow \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = 2m_0c^2 \Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{4} \Rightarrow v = \frac{\sqrt{3}}{2}c$$

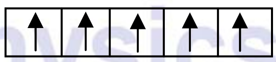
Ans. 25: (c)

Solution: $|\psi(0)\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $\quad |\psi(t)\rangle = \dfrac{1}{\sqrt{2}}\begin{pmatrix}\exp-\dfrac{iE_0t}{\hbar}\\ \exp\dfrac{iE_0t}{\hbar}\end{pmatrix}$

$$\left|\langle\psi(0)|\psi(t)\rangle\right|^2 = \frac{1}{4}\left|\exp-\frac{iE_0t}{\hbar} + \exp\frac{iE_0t}{\hbar}\right|^2 = \cos^2\left(\frac{E_0t}{\hbar}\right)$$

Ans. 26: (a)

Solution: For $3d^5$: $\quad M_L = -2 \quad -1 \quad 0 \quad +1 \quad +2$



Highest $S = \sum m_s = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = \frac{5}{2}$

Highest $L = \left|\sum m_l\right| = -2-1+0+1+2 = 0$

$\therefore J = |L-S| = \frac{5}{2}$

The spectral terms $= {}^{2S+1}L_J = {}^6S_{5/2}$

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Ans. 27: (b)

Solution: $\vec{s}=\vec{s}_1+\vec{s}_2$, $s_1=\frac{1}{2}, s_1=\frac{1}{2}, s=0,1$, $\langle \vec{s}_1.\vec{s}_2 \rangle = \frac{s(s+1)\hbar^2 - s_1(s_1+1)\hbar^2 - s_2(s_2+1)\hbar^2}{2}$

For $s=1, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{2\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = \frac{3}{4}\hbar^2$ for triplet

$s=0, \langle \vec{s}_1 \cdot \vec{s}_2 \rangle = \frac{0\hbar^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2}{2} = -\frac{3}{4}\hbar^2$ for singlet

Ans.28: (b) and (c)

Solution:

The capacitance of the parallel-plate capacitor filled with dielectric exceeds the vacuum value by a factor of the dielectric constant: $C = kC_{vac}$. Thus, the electromagnetic energy $U = \frac{1}{2}CV_0^2$ (if V_0 is unchanged) is increased by the same factor k: $U = \frac{1}{2}kC_{vac}V_0^2 = kU_0$.

We are left with (b) and (d), but do not rush to choose the second one.

You might think that, when you insert a dielectric between the plates, the electric field between them (that is, inside the dielectric) is reduced by the factor of k in comparison with the vacuum value $\frac{V_0}{d}$. But you forgot about the battery!

In fact, when the dielectric is inserted, the surface charge density on the plates of the capacitor is changed due to the battery.

Let's apply $\oint_S \vec{D}\cdot d\vec{a} = Q_{fenc}$ to the Gaussian pillbox one surface of which is inside the positive plate of the capacitor and the other one is inside the dielectric material (Q_{fenc} here is a free charge enclosed by the Gaussian surface; pillbox's surfaces are parallel to the plate's surface).

Noting that $D = 0$ inside the metal plate, one has $DA = \sigma A \Rightarrow D = \sigma$.

The electric field inside the dielectric is $E = \frac{D}{\in} = \frac{D}{\in_0 k} \Rightarrow E = \frac{\sigma}{\in_0 k}$.

We can find σ using the fact that V_0 is unchanged during the insertion process. $C_{vac} = \frac{Q_{vac}}{V_0} = \frac{\in_0 A}{d}$, $\quad C = \frac{Q}{V_0} = \frac{\in_0 kA}{d}$, so $\sigma = \frac{Q}{A} = k\frac{Q_{vac}}{A} = k\sigma_{vac}$.

Finally, $E = \frac{\sigma}{\in_0 k} = \frac{\sigma_{vac}}{\in_0} \equiv \frac{V_0}{d}$.

Ans.29: (a), (b), (c)

Solution.:

$f(x) = x^2$ for $-\pi < x < \pi$

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

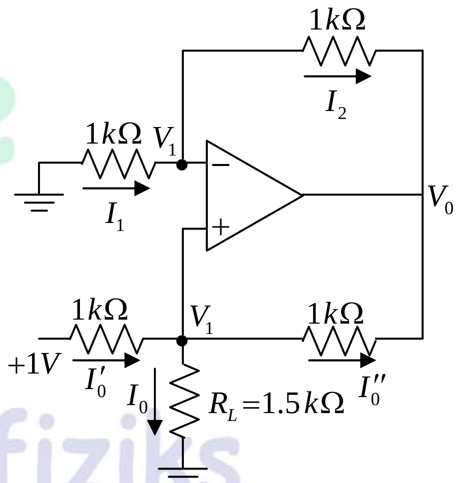
$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx dx = \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi} = \frac{4}{n^2}\cos n\pi = \frac{4}{n^2}(-1)^n$$

For $n = 1 \Rightarrow a_1 = -4$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx dx = \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi} = 0$$

Ans. 30: (a), (b), (d)



Solution: Apply KCL at inverting terminal

$$I_1 \approx I_2 \Rightarrow \frac{0 - V_1}{1k} \approx \frac{V_1 - V_0}{1k} \Rightarrow V_0 = 2V_1$$

Apply KCL at Non-inverting terminal

$$I_0' = I_0 + I_0'' \Rightarrow \frac{1 - V_1}{1k} = \frac{V_1}{R_L} + \frac{V_1 - V_0}{1k} \Rightarrow V_1 = R_L \text{ Volts}$$

(a) $I_0 = \frac{V_1}{R_L} = \frac{1.5\,V}{1.5k} = 1mA$

(b) $V_0 = 2V_1 = 2 \times 1.5 = 3V$

(c) $I_0 = \frac{V_1}{R_L} = \frac{3\,V}{3k} = 1mA$

(d) $V_0 = 2V_1 = 6V$

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Ans. 31: (a), (b) & (c)

Solution:

$$|\psi|^2 = |A|^2 e^{-\frac{8x^2}{x_0^2}}$$

Compare $|\psi|^2$ with $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x-\langle x\rangle)^2}{2\sigma^2}\right]$

$$\therefore \ \langle x\rangle = 0$$

and $(\Delta x)^2 = \frac{x_0^2}{16} \Rightarrow \Delta x = \frac{x_0}{4}$

Therefore,

$$\langle x^2\rangle = (\Delta x)^2 + \langle x\rangle^2 = \frac{x_0^2}{16}$$

Now, $\Delta x \Delta P = \frac{\hbar}{2}$

$$\Rightarrow (\Delta P)^2 = \frac{\hbar^2}{4(\Delta x)^2} = \frac{\hbar^2}{4\times\frac{x_0^2}{16}} = \frac{4\hbar^2}{x_0^2}$$

Now, $\psi(x) = A\exp\left[-\frac{4x^2}{x_0^2} - 2ik_0 x\right]$

$$\therefore \ \langle p\rangle = -2\hbar k_0$$

Ans. 32: (a) & (b)

Solution:

Triangle is not a repeatable unit cell while Pentagon does not follow symmetry rule, therefore (c) and (d) are not allowed Bravais lattice.

Thus, correct options are (a) & (b).

Ans. 33: 12

Solution:

$$V_{TH} = 3\times\frac{4\times 12}{9} = 16V \text{ and } R_N = \frac{6\times 3}{9} = 2\Omega$$

$$\Rightarrow V_L = 16\times\frac{6}{8} = 12\,V$$

Ans. 34: 0.45

Solution:

The probability that the random variable takes the value between 2 and 4 is

$$\int_2^4 f(x)dx = \int_2^4 kx^2 dx = \frac{k}{3}\left[x^3\right]_2^4 = \frac{k}{3}\times 56 = \frac{56k}{3}$$

since $\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_1^5 kx^2 dx = 1$

Thus $\frac{k}{3}\left[x^3\right]_1^5 = 1 \Rightarrow \frac{124k}{3} = 1 \Rightarrow k = \frac{3}{124}$

Hence required probability $= \frac{56}{3} \times \frac{3}{124} = \frac{14}{31}$

Ans. 35: 3.14

Solution: $V(x) = x(x-2)^2 \Rightarrow \frac{\partial V}{\partial x} = (x-2)^2 + 2x(x-2) = 0 \Rightarrow x = 2, x = \frac{2}{3}$

$$\frac{\partial^2 V}{\partial x^2} = 2(x-2) + 2(x-2) + 2x \Rightarrow \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=2} = 2\times 2 = 4$$

$$\Rightarrow \omega = \sqrt{\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=2}} \Rightarrow \omega = \frac{2\pi}{T} = 2 \Rightarrow T = \pi$$

Ans. 36: (a)

Solution: $L = \frac{m\dot{q}^2}{2} - \frac{1+\dot{q}}{q^2}$

Equation of motion is given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0$

$$\frac{\partial L}{\partial \dot{q}} = m\dot{q} - \frac{1}{q^2}, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = m\ddot{q} + \frac{2}{q^3}\dot{q}$$

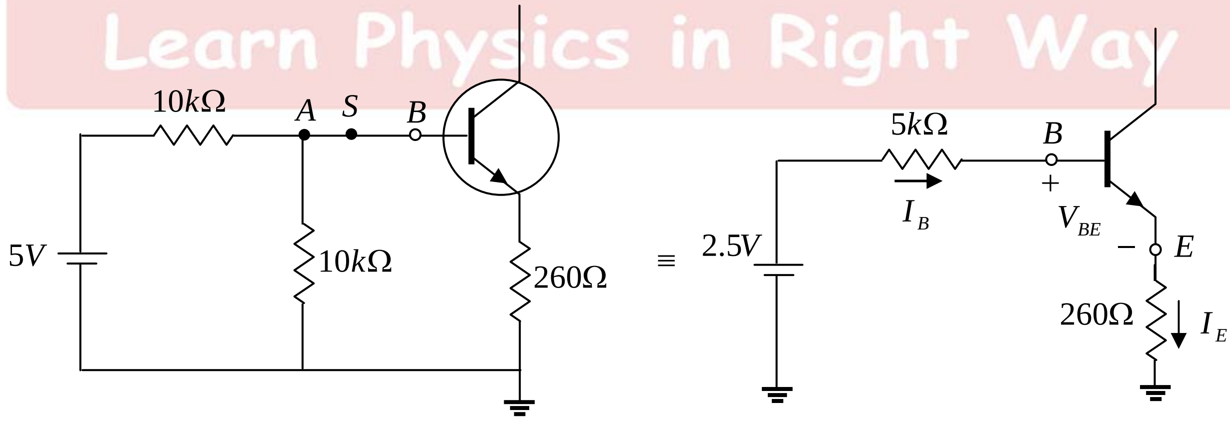
$$\left(\frac{\partial L}{\partial q}\right) = -(1+\dot{q})\left(-\frac{2\dot{q}}{q^3}\right) = (1+\dot{q})\left(\frac{2\dot{q}}{q^3}\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \Rightarrow m\ddot{q} + \frac{2}{q^3}\dot{q} - (1+\dot{q})\left(\frac{2\dot{q}}{q^3}\right) = 0$$

$$m\ddot{q} - \frac{2\dot{q}^2}{q^3} = 0$$

Ans. 37: (b)

Solution: When switch S is closed, draw Thevenin's equivalent circuit



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$$E_{Th}=\frac{10}{10+10}\times 5=2.5V\ ,\ R_{Th}=\frac{10\times 10}{10+10}=5k\Omega$$

Applying KVL to input section,

$$-2.5+5\times I_B+0.7+0.260\times I_E=0\Rightarrow -2.5+5\times I_B+0.7+0.260\times \beta I_B=0 \quad \because I_E\approx \beta I_B$$

$$\Rightarrow I_B=\frac{2.5-0.7}{5+0.260\times 50}=\frac{1.8}{18}mA=0.1\,mA\Rightarrow I_C\approx I_E=\beta I_B=50\times 0.1=5.0mA$$

$$\Rightarrow V_E=I_ER_E=5.0\times 0.260=1.3V$$

$$\Rightarrow V_A=V_B=V_{BE}+I_ER_E=0.7+1.3=2.0V$$

Ans. 38: (c)

Solution: Unperturbed: $E_n^{(0)}=\frac{\pi^2\hbar^2n^2}{8ma^2},\psi_n^{(0)}(x)=\sqrt{\frac{2}{2a}}\sin\left(\frac{\pi nx}{2a}\right)$

First order correction: $\Delta E_n^{(1)}=<\psi_n^{(0)}|W|\psi_n^{(0)}>=\frac{2a}{2a}\int_0^{2a}\sin^2\left(\frac{\pi nx}{2a}\right)\varepsilon\delta(x-a/2)dx$

$$=\begin{cases}\frac{\varepsilon}{2} & ,\quad n\quad odd\\ \varepsilon & ,\quad n=2,6,\ \ldots\\ 0 & ,\quad n=4,8\ldots\end{cases}$$

Ans. 39: (b)

Solution: $F[f(x)]=\frac{1}{\sqrt{2\pi}}\left[\int_{-1}^{0}-xe^{-i\omega x}dx+\int_0^1 xe^{-i\omega x}dx\right]$

$$=\frac{1}{\sqrt{2\pi}}\left[-\left\{\left(\frac{xe^{-i\omega x}}{-i\omega}\right)_{-1}^{0}-\left(\frac{e^{-i\omega x}}{-\omega^2}\right)_{-1}^{0}\right\}+\left(\frac{xe^{-i\omega x}}{-i\omega}\right)_0^1-\left(\frac{e^{-i\omega x}}{-\omega^2}\right)_0^1\right]$$

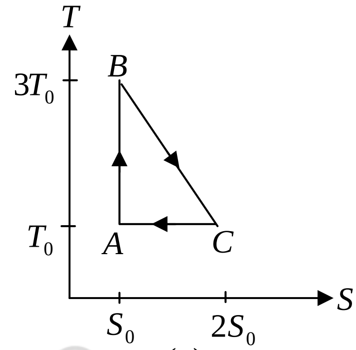
$$=\frac{1}{\sqrt{2\pi}}\left[\frac{-e^{i\omega}}{i\omega}+\frac{1-e^{i\omega}}{-\omega^2}+\frac{e^{-i\omega}}{-i\omega}-\frac{\left(e^{-i\omega-1}\right)}{-\omega^2}\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{i\omega}-e^{-i\omega}}{i\omega}+\frac{1-e^{i\omega}-e^{-i\omega}+1}{-\omega 2}\right]=\frac{1}{\sqrt{2\pi}}\left[\frac{2i\sin\omega}{i\omega}+\frac{2-2\cos\omega}{-\omega^2}\right]$$

$$=\sqrt{\frac{2}{\pi}}\left[\frac{\omega\sin\omega+\cos\omega-1}{\omega^2}\right]=\sqrt{\frac{2}{\pi}}\left[\frac{\cos\omega+\omega\sin\omega-1}{\omega^2}\right]$$

Ans. 40: (d)

Solution: Efficiency, $\eta = \frac{W}{Q_1}$



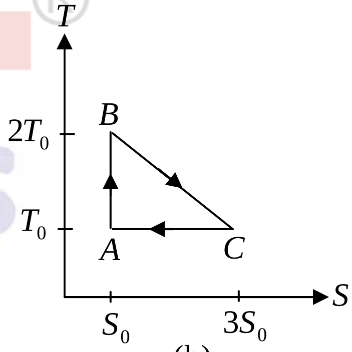
(a)

In (a)

$$W = \frac{1}{2}(3T_0 - T_0)(2S_0 - S_0) = T_0 S_0$$

$$Q_1 = T_0 S_0 + T_0 S_0 = 2T_0 S_0$$

$$\eta = \frac{W}{Q_1} = 50\%$$



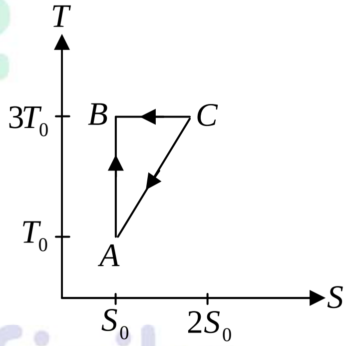
(b)

In (b)

$$W = \frac{1}{2}(2T_0 - T_0)(3S_0 - S_0) = T_0 S_0$$

$$Q_1 = T_0 S_0 + T_0 \times 2S_0 = 3T_0 S_0$$

$$\eta = \frac{W}{Q_1} = 33\%$$



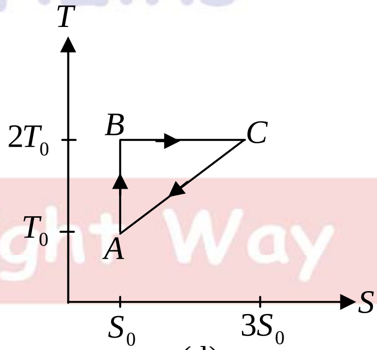
(c)

In (c)

$$W = \frac{1}{2}(3T_0 - T_0)(2S_0 - S_0) = T_0 S_0$$

$$Q_1 = (3T_0)(2S_0 - S_0) = 3T_0 S_0$$

$$\eta = \frac{W}{Q_1} = 33\%$$



(d)

In (d)

$$W = \frac{1}{2}(2T_0 - T_0)(3S_0 - S_0) = T_0 S_0$$

$$Q_1 = (2T_0)(3S_0 - S_0) = 4T_0 S_0$$

$$\eta = \frac{W}{Q_1} = 25\%$$

Ans. 41: (a)

Solution: Path difference due to slab $(\mu - 1)t$. For maximum intensity path difference should be equal $n\lambda$. Thus $(\mu - 1)t = n\lambda$

For minimum thickness t of plate, n should be minimum i.e. $n = 1$

or, $t = \frac{\lambda}{\mu - 1}$ or $t = \frac{\lambda}{1.5 - 1}$ or $t = 2\lambda$

Ans. 42: (a)

Solution:

$$\begin{vmatrix} x-\frac{1}{2} & y-\frac{1}{2} & z-\frac{1}{2} \\ \frac{1}{4}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \\ \frac{1}{3}-\frac{1}{2} & \frac{1}{4}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-\frac{1}{2} & y-\frac{1}{2} & z-\frac{1}{2} \\ -\frac{1}{2} & 0 & 0 \\ -\frac{1}{6} & -\frac{1}{2} & 0 \end{vmatrix} = 0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)(0+0)-\left(y-\frac{1}{2}\right)(0-0)+\left(z-\frac{1}{2}\right)\left(\frac{1}{4}+0\right)=0$$

$$\Rightarrow 0x+0y+\frac{1}{4}z-\frac{1}{8}=0 \Rightarrow 0x+0y+1z-\frac{1}{2}=0$$

$\therefore$ Miller indices is (001)

Ans.43: (c)

Solution: $I_i = \frac{5}{2}$ and Parity = even

$I_f = \frac{1}{2}$ and Parity = even

$\therefore \Delta I = |I_i - I_f| .... |I_i + I_f| = 2, 3$

Thus, the most probable transition is corresponds to $\Delta I = 2$ without change in parity is $E2$

Ans. 44: (d)

Solution: $S = S(T,V) \Rightarrow dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

$$T\left(\frac{\partial S}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_V + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \Rightarrow C_P - C_V = T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P$$

It is given $V = \frac{RT}{P} - \frac{b}{T}$

$$\left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} + \frac{b}{T^2}$$

$$P=\frac{RT}{V+\frac{b}{T}}\Rightarrow\left(\frac{\partial P}{\partial T}\right)_V=\frac{R}{V+\frac{b}{T}}+RT.\frac{-1}{\left(V+\frac{b}{T}\right)^2}.\frac{-b}{T^2}=\frac{R}{\left(V+\frac{b}{T}\right)}\left(1+\frac{Tb}{\left(V+\frac{b}{T}\right)T^2}.\right)$$

$$\frac{TR}{T\left(V+\frac{b}{T}\right)}\left(1+\frac{RTb}{R\left(V+\frac{b}{T}\right)T^2}.\right)=\frac{P}{T}\left(1+\frac{bP}{RT^2}\right)$$

$$C_P-C_V=T\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_P=T.\frac{P}{T}\left(1+\frac{bP}{RT^2}\right).\left(\frac{R}{P}+\frac{b}{T^2}\right)=R\left(1+\frac{bP}{RT^2}\right)^2$$

Ans. 45: (d)

Solution:

(a) $\pi^- + p \rightarrow \Sigma^+ + K^-$

$q:$	-1	$+1$	$+1$	-1: conserved	
spin:	0	$\frac{1}{2}$	$\frac{1}{2}$	0 : conserved	
$B:$	0	$+1$	$+1$	0 : conserved	
$I:$	1	$\frac{1}{2}$	1	$\frac{1}{2}$: conserved	
$I_3:$	-1	$+\frac{1}{2}$	$+1$	$-\frac{1}{2}$: Not conserved	
$S:$	0	-1	0	-1: Not conserved ($\Delta S=-2$)	

This is not allowed interaction

(b) $\pi^+ \rightarrow e^+ + \gamma$

$q:$	$+1$	$+1$	0 : conserved
$Le:$	0	-1	0 : Not conserved

This is not allowed interaction

(c) $\Omega^- \rightarrow \Sigma^- + \pi^0$

$q:$	-1	-1	0 : conserved
spin:	$\frac{3}{2}$	$\frac{1}{2}$	0 : conserved
$B:$	$+1$	$+1$	0 : conserved
$I:$	0	1	1: conserved
$I_3:$	0	-1	0 : Not conserved
$S:$	-3	-1	0 : Not conserved ($\Delta S=-2$)

This is also not an allowed transition

(d) $K^- + p \rightarrow \Omega^- + K^+ + K^0$

$q:$	-1	$+1$	-1	$+1$		0 : conserved
spin:	0	$\frac{1}{2}$	$\frac{3}{2}$	0		0 : conserved

$B:\quad 0 \quad +1 \quad +1 \quad 0 \quad 0$: conserved

$I:\quad \frac{1}{2} \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \quad \frac{1}{2}$: conserved

$I_3:\quad -\frac{1}{2} \quad +\frac{1}{2} \quad 0 \quad +\frac{1}{2} \quad -\frac{1}{2}$: conserved

$S:\quad +1 \quad 0 \quad -3 \quad +1 \quad +1$: conserved ($\Delta S = -1$)

This is an allowed strong interaction

Ans. 46: (d)

Solution: $\Delta\lambda = \frac{\lambda^2}{c}\frac{e^B}{4\pi m}$

$$B = \frac{4\pi mc}{e}\cdot\frac{\Delta\lambda}{\lambda^2} = \frac{4\times 3.14\times 9.1\times 10^{-31}\times 3\times 10^8}{1.6\times 10^{-19}}\times\frac{0.008\times 10^{-9}}{\left(656\times 10^{-9}\right)^2} = 0.398T$$

Ans. 47: (a)

Solution: $E_J = gm_J\left(\mu_B B\right)$

For ${}^2S_{1/2}$: $g = 2,\ m_j = \pm\frac{1}{2}$

$$\therefore \Delta E = E_{+\frac{1}{2}} - E_{-\frac{1}{2}} = +\mu_B B + \mu_B B = 2\mu_B B$$

$$= 2\times 9.27\times 10^{-24}\times 0.5 = 9.27\times 10^{-24}\,J = 5.8\times 10^{-5}\,eV$$

Ans. 48: (a)

Solution: $W = \int_\infty^d \vec{F}.d\vec{l} = \frac{1}{4\pi\varepsilon_0}\int_\infty^d \frac{q^2}{4z^2}dz = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q^2}{4z}\right)\Bigg|_\infty^d = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d} = -\frac{1}{16\pi\varepsilon_0}\frac{q^2}{d} = -2\times\frac{q^2}{32\pi\varepsilon_0 d}$

Ans. 49: (b)

Solution: $a = 4.28\,A^0$

$$n = \frac{2}{a^3}$$

$$\therefore R_H = -\frac{1}{ne} = -\frac{a^3}{2e} = -\frac{\left(4.28\times 10^{-10}\right)^3}{2\times 1.6\times 10^{-19}}$$

$$= -2.45\times 10^{-10}\,m^3/C$$

Ans. 50: (a)

Solution: $\left(s^2+1\right)\frac{df(s)}{ds} = -sf(s)$

$$\int\frac{df(s)}{f(s)}\int\frac{-s}{s^2+1}$$

$$hf(s) = -\frac{1}{2}h\left(s^2+1\right) + hc$$

$$f(s) = \frac{c}{\sqrt{s^2+1}}$$

Ans. 51: (b)

Solution: $\frac{\text{Rate of heat radiation from solid sphere}(A)}{\text{Rate of heat radiation from solid sphere}(B)} = \frac{4\pi R_A^2 T_A^4}{4\pi R_B^2 T_B^4}$

$\because\ R_A = 8R_B$ and $T_A = 4T_B$

$$= \frac{4\pi R_A^2 T_A^4}{4\pi R_B^2 T_B^4} = \frac{(8R_B)^2 \times (4T_B)^4}{(R_B)^2 \times (T_B)^4} = 64 \times 16 = 1024$$

Ans. 52: (a)

Solution: $I = \frac{P}{A} = \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{P}{4\pi r^2} \Rightarrow E_0^2 = \frac{P}{2\pi\varepsilon_0 c r^2} \Rightarrow E_0 = \sqrt{\frac{P}{2\pi\varepsilon_0 c r^2}} = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2P}{cr^2}}$

$$\Rightarrow E_0 = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{2P}{cr^2}} = \sqrt{9\times 10^9 \frac{2\times 5}{3\times 10^8 \times (1)^2}} = 10\sqrt{3} = 17.32$$

Ans. 53: (b)

Solution: The Larmor formula for power radiated by an accelerated charge is related to the charge and acceleration as $P \propto a^2 q^2$.

The problem gives the following:

A: $q, v, a \Rightarrow P_A \propto q^2 a^2$

B: $2q, 3v, 4a \Rightarrow P_B \propto (2q)^2 (4a)^2 \propto 64 q^2 a^2$

Thus, $P_B / P_A = 64$

Ans. 54: (a)

Solution: Since $P = -2T + P_0 \Rightarrow \frac{dP}{dT} = -2$

It is given $L = T\left(\frac{dP}{dT}\right)\Delta v \Rightarrow L = -2T\Delta v \Rightarrow \frac{dL}{dT} = -2\Delta v$

Since $dS = 1.0$ Jmole^{-1} K^{-1} $1.0\ J\ mole^{-1}\ K^{-1}$

$$dS = \frac{dQ}{T} = \frac{mdL}{dT} = 1 \Rightarrow 1 = -2\Delta v \Rightarrow \Delta v = -\frac{1}{2} = 0.5$$

Ans. 55: (b) and (d)

Solution:

Magnetic field $B\hat{j}$ and an electric field $(-E)\hat{k}$

$\because \vec{F} = q\left[\vec{E} + \left(\vec{v}\times\vec{B}\right)\right] = 0 \qquad \Rightarrow |\vec{v}| = \frac{E}{B}$

For $+ve$ charge: $\vec{a} \to -\hat{k} \Rightarrow \vec{v} = \frac{E}{B}\hat{x}$

For $-ve$ charge: $\vec{a} \to \hat{k} \Rightarrow \vec{v} = \frac{E}{B}\hat{x}$

Ans. 56: (a), (b), (c)

Solution.:

$$f(x) = x^2 \text{ for } -\pi < x < \pi$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{\pi^2}{3}$$

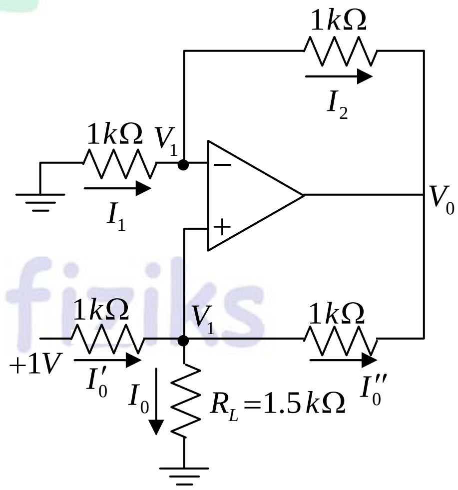
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx dx = \frac{1}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi} = \frac{4}{n^2}\cos n\pi = \frac{4}{n^2}(-1)^n$$

For $n = 1 \Rightarrow a_1 = -4$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx dx = \frac{1}{\pi}\left[x^2\left(-\frac{\cos nx}{n}\right) - 2x\left(-\frac{\sin nx}{n^2}\right) + 2\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi} = 0$$

Ans. 57: (a), (b), (d)

Solution: Apply KCL at inverting terminal



$$I_1 \approx I_2 \Rightarrow \frac{0 - V_1}{1k} \approx \frac{V_1 - V_0}{1k} \Rightarrow V_0 = 2V_1$$

Apply KCL at Non-inverting terminal

$$I_0' = I_0 + I_0'' \Rightarrow \frac{1 - V_1}{1k} = \frac{V_1}{R_L} + \frac{V_1 - V_0}{1k} \Rightarrow V_1 = R_L \text{ Volts}$$

(a) $I_0 = \frac{V_1}{R_L} = \frac{1.5\,V}{1.5k} = 1mA$

(b) $V_0 = 2V_1 = 2 \times 1.5 = 3V$

(c) $I_0 = \frac{V_1}{R_L} = \frac{3\,V}{3k} = 1mA$

(d) $V_0 = 2V_1 = 6V$

Ans. 58: (b) and (c)

$$Q_1 = e^{\beta\varepsilon} + e^{-\beta\varepsilon}, \ \varepsilon = \mu H$$

$$Q_N = [Q_1]^{\beta} = \left[e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right]^N = [2\cosh\beta\varepsilon] \quad \text{....(1)}$$

Helmholtz free energy is given by

$$A = -k_B T \ln Q_N$$

$$= -Nk_B T \ln 2\cosh\left(\varepsilon/_{k_B T}\right) \quad \text{....(2)}$$

$$U = -\frac{\partial}{\partial\beta}\ln Q_N = -N\frac{\partial}{\partial\beta}\ln\left[2\cos(\beta\varepsilon)\right]$$

$$U = -N\varepsilon\tanh(\beta\varepsilon) = -N\varepsilon\tanh\left(\varepsilon/_{k_B T}\right) \quad \text{....(3)}$$

$$M = -\left(\frac{\partial A}{\partial H}\right)_T = Nk_B T\frac{\partial}{\partial H}\left[\ln 2\cosh\left(\frac{\mu H}{kT}\right)\right]$$

$$M = N\mu\tanh\left(\varepsilon/_{kT}\right) \quad \text{....(4)}$$

The heat capacity

$$C = \left(\frac{dU}{dT}\right)_V = -\frac{\partial}{\partial T}\left\{N\varepsilon\tanh\left(\frac{\varepsilon}{k_B T}\right)\right\}$$

$$C = Nk\left(\frac{\varepsilon}{k_B T}\right)^2 \sec h^2\left(\frac{\varepsilon}{k_B T}\right) \quad \text{....(5)}$$

Ans. 59: 1.09

Solution: $\Delta E\Delta t \cong \frac{\hbar}{2}$

The interaction along is $\Delta l = c\Delta t = \frac{\hbar c}{2\Delta E}$

$$\therefore \Delta l = \frac{\hbar c}{2mc^2} = \frac{1.97\times10^{-7} eV - m}{2\times90\times10^9 eV} = 1.096\times10^{-18} m$$

Ans. 60: 20

Solution: $f = \frac{qB}{2\pi m} \Rightarrow \frac{f'}{f_\alpha} = \frac{q'}{q_\alpha}\times\frac{m_\alpha}{m'} = \frac{q}{q}\times\frac{4m_p}{3m_p} = \frac{4}{3} \Rightarrow f' = 20\ MHz$

Ans. 61: 1

Solution: $E_F = \frac{\hbar^2}{2m}(2\pi n)$

$$\therefore p = 1$$

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Ans. 62: 4.24

Solution: $\omega = 10^{15},\ \ k = \sqrt{2}\times 10^{7} \Rightarrow v = \frac{\omega}{k} = \frac{10^8}{\sqrt{2}} = \frac{3\times 10^8}{3\times\sqrt{2}} \Rightarrow v = \frac{c}{n} \Rightarrow n = 3.0\times\sqrt{2} = 4.24$

Ans. 63: **90**

Solution: $I \propto \sin^2\theta$

Ans. 64: **1.25**

Solution: $\oint_S \vec{E}.d\,\vec{a} = \frac{1}{\varepsilon_0}Q_{enc} \Rightarrow \left|\vec{E}\right|\times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$

$$\Rightarrow \left|\vec{E}\right|\times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r\left(r^2-\frac{r^3}{R}\right)r^2 dr = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{3}-\frac{r^4}{4R}\right) \Rightarrow \left|\vec{E}\right| = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$$

$$\Rightarrow E_{r=R} = \frac{\rho_0}{\varepsilon_0}\left(\frac{R}{3}-\frac{R^2}{4R}\right) = \frac{\rho_0 R}{12\varepsilon_0}$$

$$\Rightarrow E_{r=R/2} = \frac{\rho_0}{\varepsilon_0}\left(\frac{R/2}{3}-\frac{R^2/4}{4R}\right) = \frac{\rho_0}{\varepsilon_0}\left(\frac{R}{6}-\frac{R}{16}\right) = \frac{\rho_0}{2\varepsilon_0}\left(\frac{R}{3}-\frac{R}{8}\right) = \frac{5\rho_0}{48\varepsilon_0}$$

$$\Rightarrow \frac{E_{r=R/2}}{E_{r=R}} = \frac{5}{48}\times 12 = \frac{5}{4} = 1.25$$

Ans. 65: 0.45

Solution: $\langle E\rangle = \frac{2}{7}\times\frac{E_0}{1}+\frac{9}{14}\times\frac{E_0}{4}+\frac{1}{14}\times\frac{E_0}{9} = \frac{229}{504}E_0 = 0.45E_0$
