## Solution
## IIT-JAM Full Length Test

**Ans. 1: (c)**

**Solution:** $V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)=0\Rightarrow\frac{q_1}{q_2}=-\frac{R_1}{R_2}\Rightarrow\frac{\sigma_1\times4\pi R_1^2}{\sigma_2\times4\pi R_2^2}=-\frac{R_1}{R_2}\Rightarrow\frac{R_2}{R1}=-\frac{\sigma_1}{\sigma_2}$

**Ans. 2: (c)**

**Solution:** $D^2-1=0\Rightarrow D=\pm1\Rightarrow y(t)=c_1e^t+c_2e^{-t}$

Applying boundary condition

$y(0)=1\Rightarrow1=c_1+c_2$ and $y(\infty)=0\Rightarrow0=c_1e^{\infty}+c_2e^{-\infty}$

$\Rightarrow c_1=0,c_2=1\ \Rightarrow y(t)=e^{-t}$

**Ans. 3: (a)**

**Solution:** $Y=A.B(C+D)$

**Ans. 4: (a)**

**Ans. 5: (c)**

**Ans. 6: (d)**

**Solution:**

(a) $\vec{\nabla}\times\vec{A}=0$ (b) $\vec{\nabla}\times\vec{A}=0$

(c) $\vec{\nabla}\times\vec{A}=B_0\hat{k}$ (d) $\vec{\nabla}\times\vec{A}=-B_0\hat{k}$

**Ans. 7: (d)**

**Solution:** After transferring balls from basket I into II, there can be either (a) 5 red and 3 blue or (b) 4 red and 4 blue balls in the second basket.

Probability that the ball transferred is blue $=\frac{4}{7}$

Probability that the ball transferred is red $=\frac{3}{7}$

Hence probability of drawing a blue ball from basket II $=\frac{4}{7}\times\frac{4}{8}+\frac{3}{7}\times\frac{3}{8}=\frac{25}{56}$

**Ans. 8: (d)**

**Solution:** $T^2\propto R^3\Rightarrow\frac{T_2}{T}=\left(\frac{R}{4R}\right)^{\frac{3}{2}}=\frac{1}{8}\Rightarrow T_2=\frac{T}{8}$

**Ans. 9: (b)**

**Ans. 10: (d)**

**Solution:** $R+T=1\Rightarrow R=1-T=1-\frac{4\rho_1\rho_2c_1c_2}{(\rho_1c_1+\rho_2c_2)^2}=\left[\frac{\rho_1c_1-\rho_2c_2}{\rho_1c_1+\rho_2c_2}\right]^2$

**Ans. 11: (d)**

**Solution:** For discharging of an $RC$ circuit, $V=V_0e^{-t/\tau}$

So, when $\frac{V_0}{2}=V_0e^{-t/\tau}\Rightarrow\ln\frac{1}{2}=-\frac{t}{\tau}\Rightarrow\tau=\frac{t}{\ln2}$

From graph when $V=\frac{V_0}{2},t=100\,s$ $\quad\therefore\frac{100}{\ln2}=144.3\sec$

**Ans. 12: (d)**

**Solution:** For C.F $\left(D^2-1\right)y=0 \Rightarrow m=\pm 1 \Rightarrow C.F.=C_1e^t+C_2e^{-t}$

$$P.I.=\frac{1}{D^2-1}(2\cosh t)=\frac{1}{D^2-1}2\left(\frac{e^t+e^{-t}}{2}\right)=\frac{1}{D^2-1}\left(e^t\right)+\frac{1}{D^2-1}\left(e^{-t}\right)=\frac{t}{2}e^t+\frac{t}{2}\left(-e^{-t}\right)$$

$$\Rightarrow y=C_1e^t+C_2e^{-t}+\frac{t}{2}e^t-\frac{t}{2}e^{-t}$$

As, $y(0)=0 \Rightarrow C_1+C_2=0............(1)$

$$\frac{dy}{dt}=C_1e^t-C_2e^{-t}+\frac{t}{2}e^t+\frac{1}{2}e^t+\frac{t}{2}e^{-t}-\frac{1}{2}e^{-t}$$

Also, $\left.\frac{dy}{dt}\right|_{t=0}=0 \Rightarrow C_1-C_2+0+\frac{1}{2}+0-\frac{1}{2}=0 \Rightarrow C_1-C_2=0...........(2)$

From equation (1) and (2),

$$C_1=0, C_2=0.$$

Thus $y=\frac{t}{2}e^t-\frac{t}{2}e^{-t} \Rightarrow y=t\sinh t$

**Ans. 13: (b)**

**Solution:** $dH=TdS+VdP \Rightarrow \left(\frac{\partial H}{\partial S}\right)_P=T, \left(\frac{\partial H}{\partial P}\right)_S=V$

**Ans. 14: (b)**

**Solution:**

$$\psi(x)=\begin{cases} k, & -\frac{a}{2}-\frac{d}{2}<x<-\frac{a}{2}+\frac{d}{2} \\ 0, & -\frac{a}{2}+\frac{d}{2}<x<\frac{a}{2}-\frac{d}{2} \\ k, & \frac{a}{2}-\frac{d}{2}<x<\frac{a}{2}+\frac{d}{2} \\ 0, & \frac{a}{2}+\frac{d}{2}>0 \end{cases}$$

$$\langle\psi|\psi\rangle=1$$

$$k^2\int_{-\frac{a}{2}-\frac{d}{2}}^{-\frac{a}{2}+\frac{d}{2}}dx+k^2\int_{\frac{a}{2}-\frac{d}{2}}^{\frac{a}{2}+\frac{d}{2}}dx=1$$

$$k^2\left[\left(-\frac{a}{2}+\frac{d}{2}\right)-\left(-\frac{a}{2}-\frac{d}{2}\right)\right]+k^2\left[\left(\frac{a}{2}+\frac{d}{2}\right)-\left(\frac{a}{2}-\frac{d}{2}\right)\right]=1$$

$$k^2\left[\frac{d}{2}+\frac{d}{2}+\frac{d}{2}+\frac{d}{2}\right]=1 \Rightarrow k=\frac{1}{\sqrt{2d}}$$


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**Ans. 15: (b)**

**Solution:** $T_1 = ka^{3/2} = T$ , $T_2 = k(4a)^{3/2} = 8T$ , $T_3 = k(64a)^{3/2} = 8^3T = 512T$

Common time that all three star will meet again is, which is LCM of all time period ie $8^3 = 512T$

**Ans. 16: (b)**

**Solution:** $v_\perp = 3\ m/s$ and $v_{||} = 2\ m/s$ , thus $t = \frac{2m}{v_{||}} = 1$ sec

**Ans. 17: (b)**

**Solution:** Volume occupied by atoms in DC crystal $= n_{eff} \times \frac{4\pi}{3} r^3$

Volume of the unit cell $= a^3$

Fraction of unoccupied volume $= \frac{a^3 - n_{eff} \times \frac{4\pi}{3} r^3}{a^3}$

where, $n_{eff} = 8$, $r = \frac{\sqrt{3}a}{8}$

$\therefore$ fraction of unoccupied volume $= \frac{a^3 - 8\times\frac{4\pi}{3}\left(\frac{\sqrt{3}}{8}a\right)^3}{a^3} = \frac{16-\sqrt{3}\pi}{16}$

**Ans. 18: (a)**

**Solution:** $P(\varepsilon) = \frac{\exp-\frac{\varepsilon}{kT}}{1+\exp-\frac{\varepsilon}{kT}}$, population of particle in the level with energy $\varepsilon$ is

$$NP(\varepsilon) = N\frac{\exp-\frac{\varepsilon}{kT}}{1+\exp-\frac{\varepsilon}{kT}},\ \text{for } (k_BT > \varepsilon),\ NP(\varepsilon) = N\frac{\exp-\frac{\varepsilon}{kT}}{1+\exp-\frac{\varepsilon}{kT}} = N\frac{1}{1+1} = \frac{N}{2}$$

**Ans. 19: (d)**

**Solution:** We know that for any matrix

1. The product of eigenvalues is equals to the determinant of that matrix.

2. $\lambda_1 + \lambda_2 + \lambda_3 + ....... =$ Trace of matrix

$\lambda_1 + \lambda_2 + \lambda_3 = 11$ and $\lambda_1\lambda_2\lambda_3 = 36$. Hence, the largest eigen value of the matrix is 6.

**Ans. 20: (b)**

**Solution:** $L = I\omega + mva \Rightarrow \frac{1}{2}ma^2\frac{v}{a} + mva = \frac{1}{2}mva + mva = \frac{3mva}{2}$

**Ans. 21: (b)**

**Solution:** $\delta=\frac{2\pi}{\lambda}(\mu_0-\mu_e)t=\frac{2\times3.14}{6000\times10^{-10}}(1.642-1.478)\times0.04\times10^{-3}$

$\delta=68\,rad$

**Ans. 22: (c)**

**Solution:** $B_A=\frac{\mu_0}{2}\frac{i}{(l/2\pi)}$ and $B_B=\left[\frac{\mu_0}{4\pi}\frac{i}{l/8}(\sin45^\circ+\sin45^\circ)\right]\times4\Rightarrow\frac{B_A}{B_B}=\frac{\pi^2}{8\sqrt{2}}$

**Ans. 23: (c)**

**Solution:** $\omega=\sqrt{\omega_0^2-\frac{\gamma^2}{4}}=\omega_0\sqrt{1-\frac{\gamma^2}{4\omega_0^2}}$

For lightly damped oscillator $Q=\frac{\omega_0}{\gamma}$

$$\omega=\omega_0\left(1-\frac{1}{4Q^2}\right)^{1/2}$$

After Binomial expansions

$$\omega=\omega_0\left(1-\frac{1}{8Q^2}\right)$$

$$\Rightarrow\omega_0-\omega=\frac{\omega_0}{8Q^2}$$

**Ans. 24: (c)**

**Ans. 25: (a)**

**Solution:** $\tau=mgL\sin\theta+kx\hbar\cos\theta$

For small angular displacement

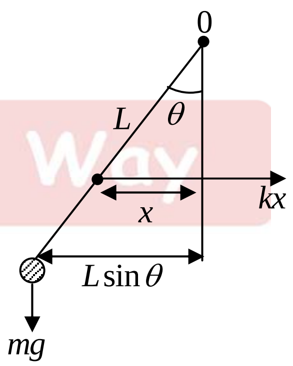
$\sin\theta\simeq\theta$ and $\cos\theta\cong1$

$\therefore\tau=mgL\theta+kxh$

$$\frac{Id^2\theta}{dt^2}=mgL\theta+k\hbar^2\theta=(mgL+kh^2)\theta$$

$$\frac{d^2\theta}{dt^2}+\frac{(mgL+kh^2)\theta}{I}=0$$

$$\therefore T=2\pi\sqrt{\frac{mL^2}{mgL+kh^2}}$$




**Ans. 26: (c)**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**Ans. 27: (d)**

**Solution:** $\dfrac{\omega_1}{\omega_2}=\dfrac{\text{number of intercept on y-axis}}{\text{number of intercept on x-axis}}=\dfrac{6}{2}=\dfrac{3}{1}$

And this shape appears for $\pi/2$ phase difference. Thus, the correct option is (d).

**Ans. 28: (d)**

**Solution:** $\vec{r}(t)=3\cos\omega t\,\hat{i}+4\sin\omega t\hat{j}$

$$\vec{v}=\frac{d\vec{r}}{dt}=-3\omega\sin\omega t\,\hat{i}+4\omega\cos\omega t\hat{j}\Rightarrow\vec{p}=m\left(-3\omega\sin\omega t\,\hat{i}+4\omega\cos\omega t\hat{j}\right)$$

$$L=(\vec{r}\times\vec{p})=m\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3\cos\omega t & 4\sin\omega t & 0\\ -3\omega\sin\omega t & 4\omega\cos\omega t & 0\end{pmatrix}\Rightarrow L=12m\omega\hat{k}$$

**Ans. 29: (b)**

**Solution:** Pressure at the base $=\rho gh\qquad=900\times10\times0.4\,N/m^2=3600\,N/m^2$

Force acting on the base $=p\times A=3600\times2\times10^{-3}\,N=7.2\,N$

**Ans. 30: (a)**

**Solution:** The loss in kinetic energy which is transformed into heat

$$=\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2$$

Here, $m_1=m,\ m_2=\dfrac{m}{9},\ u_1=u,\ u_2=0$

Loss of energy $=\dfrac{1}{2}\left(\dfrac{m\times\frac{m}{9}}{m+\frac{m}{9}}\right)\times(u+0)^2=\dfrac{1}{2}\dfrac{m}{10}u^2$

Now, initial kinetic energy $=\dfrac{1}{2}mu^2$

Required fraction $=\dfrac{\text{loss in kinetic energy}}{\text{initial kinetic energy}}=\dfrac{\frac{1}{2}\frac{m}{10}u^2}{\frac{1}{2}mu^2}=\dfrac{1}{10}=0.1$

**Ans. 31: (b), (d)**

**Solution:** (a) At resonance $\omega'=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{\sqrt{0.1\times10\times10^{-6}}}=10^3\ rad/s$, so $\omega=\dfrac{\omega'}{2}=5\times10^2\ rad/s$

(b) Given that peak voltage $=300\sqrt{2}\ V$ and frequency $\omega=5\times10^2\ rad/s$.

$$R=150,\ X_C=\frac{1}{\omega C}=\frac{1}{5\times10^2\times10^{-5}}=200\Omega,\ X_L=\omega L=5\times10^2\times0.1=50\Omega$$

$$Z=\sqrt{R^2+(X_C-X_L)^2}=\sqrt{150^2+(200-50)^2}=\sqrt{2\times150^2}=150\sqrt{2}\ \Omega.$$

Peak current $I_M = \frac{300\sqrt{2}}{150\sqrt{2}} = 2\ A$

(c) $\theta = \arctan\left(-\frac{X_C - X_L}{R}\right) = \arctan\left(-\frac{200-50}{150}\right) = -45^0$.

(d) Peak voltage across the inductor $= I_M X_L = 2\times 50 = 100\ V$

**Ans. 32: (a),(c) and (d)**

**Solution:** $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$

$P(\in) = \frac{1}{2}$ $\quad P(-\in) = \frac{1}{2}$ so $\langle E\rangle = 0$

$\Delta E = \sqrt{\langle E^2\rangle - \langle E\rangle} = \in$

And $|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-\frac{i\in t}{\hbar}} + |\psi_-\rangle e^{\frac{i\in t}{\hbar}}\right]$ At $t = \frac{h}{6\in}$,

$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-\frac{i\in h\times 2\pi}{6\in h}} + |\psi\rangle e^{\frac{i\in h\times 2\pi}{6\in h}}\right] = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{\frac{-i\pi}{3}} + |\psi_-\rangle e^{\frac{i\pi}{3}}\right]$

**Ans. 33: (a), (c)**

**Solution:** The eccentricity of curve $e = \sqrt{1 + \frac{2El^2}{mk^2}}$ where $k = Gm_1m_2$ and $E$ is energy.

If total energy is negative then orbit can be either elliptical or circular so (a) is correct

In two body central force problem motion is confined in a plane and angular momentum is conserved so (b) is wrong

If the total energy of the system is $0$, then the orbit is a parabola one can calculate $e = 1$ so (c) is correct

From second law of kepler $\frac{dA}{dt} = \frac{J}{2m} \Rightarrow \frac{S}{T} = \frac{J}{2m} \Rightarrow T = \frac{2mS}{J}$ so (d) wrong

**Ans. 34: (a), (b), (c)**

**Ans. 35: (a), (c) and (d)**

**Solution:** $V_{dip}(r,\theta) = \frac{\hat{r}.\vec{p}}{4\pi\varepsilon_o r^2} = \frac{p\cos\theta}{4\pi\varepsilon_o r^2}$.

$\vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\hat{\theta}\right)$

$\because \theta = 90^0,\ E_r = 0$

**Ans. 36: (b) and (d)**

**Solution:** $J = \left|m\left(\vec{r}\times\left(\vec{\omega}\times\vec{r}\right)\right)\right| + m\left|\left(\vec{r}\times\left(\vec{\omega}\times\vec{r}\right)\right)\right| = 2m\left|\left(\vec{r}\times l\omega\sin\theta\right)\right| = 2m\omega l^2\sin\theta$

$Torque = \frac{dL}{dt} = \vec{\omega}\times\vec{J} = 2m\omega^2 l^2\sin(90-\theta)\sin\theta = 2m\omega^2 l^2\cos\theta\sin\theta$


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**Ans. 37: (a) and (c)**

**Ans. 38: (a), (b), (c) and (d)**

**Solution:** AC Analysis:

$R_E$ is "shorted out" by $C_E$ for the ac analysis. Therefore

$$Z_i = R_B \,||\, \beta r_e = 470k\Omega \,||\, (120)6\Omega \approx 717\,\Omega,\ Z_o = R_C = 2.2\,k\Omega$$

$$A_v = -\frac{R_C}{r_e} = -\frac{2.2\,k\Omega}{5.99\,\Omega} \approx -367$$

$$A_i = -A_v\frac{Z_i}{R_L} = -(-367)\frac{717\,\Omega}{2.2\,k\Omega} \approx 120$$

**Ans. 39: (a),(b) and (c)**

**Solution:** (a) ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$ and ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^-$

Change in mass number $= 232 - 208 = 24 \Rightarrow \frac{24}{4} = 6$ $\alpha$ -decay

Change in atomic number after $6\alpha$ -decay $= 90 - 12 = 78$.

Final products mass number $= 82$; $4\beta^-$ -decay

(b) Number of nuclei present after 300 year $N = N_0\left(\frac{1}{2}\right)^{T/T_0}$

$$\Rightarrow N' = N_0 - N_0\left(\frac{1}{2}\right)^3 = \frac{7}{8}N_0$$

(c) $N_A\lambda_A = N_B\lambda_B \Rightarrow \frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A} = \frac{T_{1/2A}}{T_{1/2B}} = \frac{4.5\times10^9}{2.5\times10^5} = 1.8\times10^4$

(d) $R = \lambda N \Rightarrow N = \lambda N_0 \Rightarrow T_{1/2} = \frac{0.693}{\lambda} = 0.693\frac{N_0}{N}$

**Ans. 40: (b) and (d)**

**Solution:** $E_0 = 100V/m, I = \frac{P}{A} = \frac{1}{2}c\varepsilon_0 E_0^2 = \frac{1}{2}\times3\times10^8\times8.86\times10^{-12}\times(100)^2 = 13.2\ W/m^2$

$$B_0 = \frac{E_0}{c} = \frac{100}{3\times10^8} = 3.3\times10^{-7}\ \ tesla$$

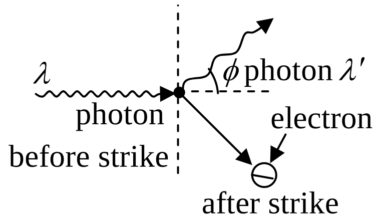
**Ans. 41: 5000**

**Solution:** $R_{L_{\max}} = \frac{15}{I_{L_{\min}}}$ where $I_{L_{\min}} = I_R - I_{ZM} = \frac{50-15}{1} - 32 = 3\ mA$

$$\Rightarrow R_{L_{\max}} = \frac{15}{3} = 5\,k\Omega = 5000\ \Omega.$$

**Ans. 42:** 2.4

**Solution:** When a photon of wavelength $\lambda$ strike a stationary electron




The wavelength of the photon increases. This effect is known as Compton effect.

The change b wavelength is given as $\Delta\lambda = \dfrac{h}{m_0 c}(1-\cos\phi)$

The change in wavelength does not depend on the energy of the incident photon.

Here, $\phi = 90^o \quad \Delta\lambda = \dfrac{h}{m_0 c}\left(1-\cos 90^o\right) = \dfrac{h}{m_0 c} = 2.4\times 10^{-12}\, m$

**Ans. 43:** 2,5,5

**Solution:**

$$\begin{bmatrix} 4-\lambda & -1 & -1 \\ -1 & 4-\lambda & -1 \\ -1 & -1 & 4-\lambda \end{bmatrix} = 0 \Rightarrow (2-\lambda)\begin{bmatrix} 1 & -1 & -1 \\ 0 & 5-\lambda & 0 \\ 0 & 0 & 5-\lambda \end{bmatrix} = (2-\lambda)(5-\lambda)^2 = 0$$

$\Rightarrow \lambda = 2,5,5$.

**Ans. 44:** 1

**Solution:** $V(x) = \dfrac{x^2}{2} + \dfrac{1}{2x^2}$

$$\frac{\partial V}{\partial x} = x - \frac{1}{x^3} = 0 \Rightarrow x_0 = \pm(1)^{1/4} = \pm 1$$

$$\frac{\partial^2 V}{\partial x^2} = 1 + \frac{3}{x^4} = 0 \quad \text{at} \quad x_0 = \pm 1 \quad \frac{\partial^2 V}{\partial x^2} = 4 > 0$$

So $x = 1, V(1) = 1$

**Ans. 45:** 8

**Solution:** 5th nearest neighbours $= \dfrac{3\times 8}{1} = 24$

**Ans. 46: 20**

**Solution:** Gain (in $dB$) $= 20\log_{10}\left|\left(\dfrac{V_0}{V_{in}}\right)\right| = 20\log_{10}\left(\dfrac{R_2}{R_1}\right) = 20\log_{10}\left(\dfrac{200}{20}\right) = 20\, dB$

**Ans. 47:** 95

**Solution:** Since the reaction is due to thermal neutron $E_a \approx 0$

$$a + A \to B + b$$

$$\Rightarrow E_0 = Q\frac{M_B}{M_B + m_\alpha} = 160\times\frac{138}{138+95} \approx 95 MeV$$

**Ans. 48:** 0.25

**Solution:** $P \propto AT^4 \Rightarrow \dfrac{P_1}{P_2} = \dfrac{R_1^2 T_1^4}{R_2^2 T_2^4} = \dfrac{R^2 T^4}{\left(\dfrac{R}{2}\right)^2 (2T)^4} = \dfrac{4}{16} = \dfrac{1}{4} = 0.25$

**Ans. 49:** $1.67\times10^{-3}$

$$t=\frac{\lambda}{4(\mu_0-\mu_E)}=1.67\times10^{-3}\,cm$$

**Ans. 50: 0.010**

**Ans. 51: 0.35**

**Solution:** From Ohm's Law $V-\varepsilon=IR,$ one can obtain the current. (Note that $V$ = 5.0 V is the voltage of the battery. The voltage induced acts to oppose this emf from the battery.)

The problem gives $\frac{dB}{dt}=150\,T/s$. The area is just $0.01\,m^2$.

Thus, the induced emf is, $\varepsilon=\frac{dB}{dt}\quad A=150\times0.01=1.5$

Thus, $V-\varepsilon=3.5=IR\Rightarrow I=0.35\,A,$ since $R=10\Omega$.

**Ans. 52:** 1.44

**Solution:** $\sqrt{p^2c^2+M^2c^4}=E_1+E_2=1.82GeV$

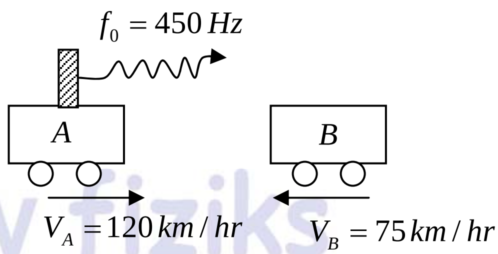
$$p=\frac{E_1}{c}\cos\theta_1+\frac{E_2}{c}\cos\theta_2=\frac{1GeV}{c}\frac{1}{\sqrt{2}}+\frac{0.82GeV}{c}\frac{1}{2}=\frac{1.11GeV}{c}$$

$$\Rightarrow p^2c^2+M^2c^4=3.312\Rightarrow M^2c^4=3.312-1.23=2.08$$

$$\Rightarrow M=\sqrt{2.076}=1.44$$

**Ans. 53:** 631.6

**Solution:** frequency observe by $B$




$$f_B=f_0\left(\frac{v_s+v_B}{v_s+v_A}\right)$$

frequency observe by $A$

$$f_A=f_B\left(\frac{v_s+v_B}{v_s-v_A}\right)$$

$$=f_0\left(\frac{v_s+v_B}{v_s-v_A}\right)\left(\frac{v_s+v_A}{v_s-v_B}\right)$$

where $f_0=450\,Hz,\ v_s=320\,m/s$

$$v_A=120\,km/hr=120\times\frac{5}{18}m/\sec=33.3\,m/s$$

$$v_B=75\,km/hr=75\times\frac{5}{18}m/s=20.8\,m/s$$

$$\therefore\ f_A=450\left(\frac{320+20.8}{320-33.3}\right)\left(\frac{320+33.3}{320-20.8}\right)$$

$$=450\times\frac{340.8}{286.7}\times\frac{353.3}{299.2}=631.6Hz$$

**Ans. 54:** 0.81

**Solution:** $\frac{d^2 f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0$

Auxiliary equation is,

$$(m^2 - 2m + 1) = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1,1$$

Hence, the solution is

$$f(x) = (c_1 + c_2 x)e^x$$

using boundary condition,

$$f(0) = c_1 e^0 \Rightarrow c_1 = 1 \qquad \text{(i)}$$

$$f(1) = (c_1 + c_2)e = 0 \qquad \text{(ii)}$$

From (i) and (ii), $c_2 = -1$

Hence, $f(x) = (1-x)e^x \Rightarrow f(0.5) = (1-0.5)e^{0.5} = 0.81$

**Ans. 55: 8**

**Solution:** $f_H = \frac{1}{2\pi RC} = 7.95\,kHz \approx 8\,kHz$

**Ans. 56:** 1262

**Solution:** $F(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \Rightarrow e^{(E-E_F)/k_BT} + 1 = \frac{1}{F(E)}$

$$\Rightarrow e^{-(E-E_F)/k_BT} = \frac{1-F}{F} \Rightarrow \frac{E-E_F}{k_BT} = \ln\left(\frac{1-F}{F}\right) \Rightarrow T = \frac{E-E_F}{k_B \ln\left(\frac{1-F}{F}\right)}$$

$$\therefore \quad T = \frac{0.5}{8.62\times10^{-5}\ln\left(\frac{0.99}{0.01}\right)} = \frac{0.5}{8.62\times\ln(99)} = \frac{0.5\times10^5}{8.62\times4.595} = 1262.3\ K$$

**Ans. 57: 2**

**Solution:** $V(x) = \frac{1}{2}m\omega_0^2 x^2 + \frac{a}{2mx^2}$

$$\frac{dV}{dx} = m\omega_0^2 x - \frac{a}{mx^3} = 0 \Rightarrow x^4 = \frac{a}{m^2\omega_0^2}$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=x_0} = m\omega_0^2 + \frac{3a}{mx^4} \Rightarrow m\omega_0^2 + \frac{3m^2\omega_0^2}{m} = 4m\omega_0^2$$

$$\omega = \sqrt{\frac{\left.\frac{d^2V}{dx^2}\right|_{x=x_0}}{m}} = \sqrt{\frac{4m\omega_0^2}{m}} = 2\omega_0$$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**Ans. 58:** 1.0

**Solution:** $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{1622\times365\times24\times60\times60} = 1.36\times10^{-11}\,\text{sec}^{-1}$

Number of atoms in $1.00\,g$ is $N = \dfrac{1}{226}\times6.023\times10^{23} = 2.7\times10^{21}$ atoms

Hence, activity $R = \lambda N = 1.36\times10^{-11}\times2.7\times10^{21} = 3.7\times10^{10}$ decay/sec $= 1.0\,Ci$

**Ans. 59:** 43.49

**Solution:** $2d\sin\theta = \lambda$

$$2a\sin\theta = \lambda\sqrt{h^2+k^2+l^2}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{2a}\sqrt{h^2+k^2+l^2}\right)$$

The first peak in FCC appears for plane $(1\,1\,1)$

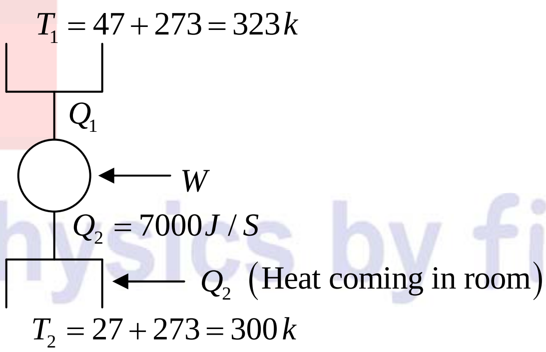
$$\therefore\ \theta = \sin^{-1}\left(\frac{1.54\,\overset{0}{\text{A}}}{2\times3.6\,\overset{0}{\text{A}}}\sqrt{1^2+1^2+1^2}\right) = \sin^{-1}(0.37) = 21.74^0$$

The first XRD peak will appear at angle $2\theta$

$$\therefore\ 2\theta = 43.49^0$$

**Ans. 60:** 466.67

**Solution:**




$$Q_2 + W = Q_1$$

Coefficient of performance of refrigerator $(AC) = \dfrac{Q_2}{W}$

Also, coefficient of performance of refrigerator, $= \dfrac{T_2}{T_1 - T_2}$

$$\Rightarrow \frac{300}{47-27} = \frac{7000}{W}$$

$$\Rightarrow W = \frac{7000\times20}{300}\,J/s = \frac{1400}{3} = 466.67\,W$$

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