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JEST Physics-2023

Solution

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Section A

Correct answer: +3, wrong answer: −1.

Q14. Calculate the contour integral

$$I=\oint_C \frac{\cos^2(z)-z^2}{(z-a)^3}dz$$

where the clockwise contour C is encircling the point $z=a$ in the complex plane.

(a) $-(\sin 2a+1)2\pi i$ (b) $(\cos 2a+1)2\pi i$

(c) $-(\cos 2a+1)2\pi i$ (d) $(\sin 2a+1)2\pi i$

Ans: 14. (b)

Solution.: $I=\oint_C \frac{\cos^2(z)-z^2}{(z-a)^3}dz$. Let $f(z)=\frac{\cos^2(z)-z^2}{(z-a)^3}$.

$f(z)$ has pole of order 3 at $z=a$.

Residue at $z=a$:

$$a_{-1}=\lim_{z\to a}\frac{1}{2!}\frac{d^2}{dz^2}\left[(z-a)^3\frac{\cos^2(z)-z^2}{(z-a)^3}\right]=\lim_{z\to a}\frac{1}{2}\frac{d}{dz}\left[2\cos z(-\sin z)-2z\right]$$

$$a_{-1}=\lim_{z\to a}\frac{1}{2}\frac{d}{dz}\left[-\sin 2z-2z\right]=\lim_{z\to a}\frac{1}{2}\left[-2\cos 2z-2\right]=-(\cos 2a+1)$$

$I=-2\pi i\sum \text{Res at}(z=a)=2\pi i(\cos 2a+1)$ where – sign for clockwise contour C.

Q19. Which of the following vanishes identically?

(a) $\vec{\nabla}\times\frac{((y+x)\hat{i}+(y-x)\hat{j})}{x^2+y^2}$ (b) $\vec{\nabla}\times\frac{(y\hat{i}-x\hat{j})}{x^2+y^2}$

(c) $\vec{\nabla}\times\frac{(x\hat{i}+y\hat{j})}{x^2+y^2}$ (d) $\vec{\nabla}\cdot\left[\frac{(x\hat{i}+y\hat{j}+z\hat{k})}{(x^2+y^2+z^2)^{3/2}}\right]$

Ans: 19. (c)

Solution.:

(a)

$$\vec{\nabla}\times\frac{((y+x)\hat{i}+(y-x)\hat{j})}{x^2+y^2}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ \frac{y+x}{x^2+y^2} & \frac{y-x}{x^2+y^2} & 0\end{vmatrix}=\hat{z}\left[\frac{\partial}{\partial x}\left(\frac{y-x}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{y+x}{x^2+y^2}\right)\right]$$

$$\Rightarrow \vec{\nabla}\times\frac{\left((y+x)\hat{i}+(y-x)\hat{j}\right)}{x^2+y^2}=\hat{z}\left[\frac{-\left(x^2+y^2\right)-(y-x)2x}{\left(x^2+y^2\right)^2}-\frac{\left(x^2+y^2\right)-(y+x)2y}{\left(x^2+y^2\right)^2}\right]$$

$$\Rightarrow \vec{\nabla}\times\frac{\left((y+x)\hat{i}+(y-x)\hat{j}\right)}{x^2+y^2}=\hat{z}\left[\frac{-2\left(x^2+y^2\right)-2xy+2xy+2x^2+2y^2}{\left(x^2+y^2\right)^2}\right]=0$$

$$\text{(b)}\ \vec{\nabla}\times\frac{\left(y\hat{i}-x\hat{j}\right)}{x^2+y^2}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ \frac{y}{x^2+y^2} & \frac{-x}{x^2+y^2} & 0\end{vmatrix}=\hat{z}\left[\frac{\partial}{\partial x}\left(\frac{-x}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right)\right]$$

$$=\hat{z}\left[\frac{-\left(x^2+y^2\right)-(-x)2x}{\left(x^2+y^2\right)^2}-\frac{\left(x^2+y^2\right)-y\times 2y}{\left(x^2+y^2\right)^2}\right]=\hat{z}\left[\frac{-2\left(x^2+y^2\right)+2x^2+2y^2}{\left(x^2+y^2\right)^2}\right]=0$$

$$\text{(c)}\ \vec{\nabla}\times\frac{\left(x\hat{i}+y\hat{j}\right)}{x^2+y^2}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ \frac{x}{x^2+y^2} & \frac{y}{x^2+y^2} & 0\end{vmatrix}=\hat{z}\left[\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)\right]$$

$$=\hat{z}\left[\frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)\right]=\hat{z}\left[\frac{0-(y)2x}{\left(x^2+y^2\right)^2}-\frac{0-(x)2y}{\left(x^2+y^2\right)^2}\right]=\hat{z}\left[\frac{-2xy+2xy}{\left(x^2+y^2\right)^2}\right]=0$$

$$\text{(d)}\ \vec{\nabla}\cdot\left[\frac{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)}{\left(x^2+y^2+z^2\right)^{3/2}}\right]=\frac{\partial}{\partial x}\left(\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}\right)+\frac{\partial}{\partial y}\left(\frac{y}{\left(x^2+y^2+z^2\right)^{3/2}}\right)+\frac{\partial}{\partial z}\left(\frac{z}{\left(x^2+y^2+z^2\right)^{3/2}}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}\right)=\frac{\left(x^2+y^2+z^2\right)^{3/2}-x\frac{3}{2}\left(x^2+y^2+z^2\right)^{1/2}2x}{\left(x^2+y^2+z^2\right)^3}$$

$$=\frac{\left(x^2+y^2+z^2\right)^{3/2}-3x^2\left(x^2+y^2+z^2\right)^{1/2}}{\left(x^2+y^2+z^2\right)^3}$$

$$\vec{\nabla}\cdot\left[\frac{\left(x\hat{i}+y\hat{j}+z\hat{k}\right)}{\left(x^2+y^2+z^2\right)^{3/2}}\right]=\frac{3\left(x^2+y^2+z^2\right)^{3/2}-3\left(x^2+y^2+z^2\right)\left(x^2+y^2+z^2\right)^{1/2}}{\left(x^2+y^2+z^2\right)^3}=0$$

Q22. How many independent real parameters are required to describe an arbitrary $N \times N$ Hermitian matrix?

(a) $N^2 - N$ (b) N^2

(c) $2N$ (d) $N^2 - 1$

Ans: 22. (b)

Q23. Two fair six-faced dice are thrown simultaneously. The probability that one of the dice yields an outcome that is a multiple of 2 and the other yields a multiple of 3 is:

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{13}{36}$ (d) $\frac{11}{36}$

Ans: 23. (d)

Solution.:

Possible combinations are

$(2,3);\ (2,6);\ (4,3);\ (4,6);\ (6,3)$ and $(3,2);\ (6,2);\ (3,4);\ (6,4);\ (3,6)$ and $(6,6)$

$\text{probability} = \frac{11}{36}$

Section B

Correct answer: +9, wrong answer: −3.

Q28. Compute the contour integral:

$$I = \oint \frac{z\,dz}{\sinh(2\pi z)}$$

where the contour is a circle of radius $\frac{3}{4}$ centred around the origin and the direction is counterclockwise.

(a) 0 (b) -1

(c) π (d) 1

Ans: 28. (a)

Solution.: $I = \oint \frac{z\,dz}{\sinh(2\pi z)}$.

Let $f(z) = \frac{z}{\sinh(2\pi z)}$. For poles of $f(z)$,

$$\sinh(2\pi z) = 0 \Rightarrow \frac{e^{2\pi z} - e^{-2\pi z}}{2} = 0 \Rightarrow e^{4\pi z} = 1 = e^{2n\pi i}$$

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$\Rightarrow z=\frac{ni}{2}=0,\pm\frac{i}{2}$ are simple pole that lies inside contour.

Residue at $z=0$: $a_{-1}=\lim_{z\to 0}\left[(z-0)\frac{z}{\sinh(2\pi z)}\right]=\lim_{z\to 0}\frac{2z}{2\pi\cosh(2\pi z)}=0$

Residue at $z=\frac{i}{2}$: $a_{-1}=\lim_{z\to\frac{i}{2}}\left[\left(z-\frac{i}{2}\right)\frac{z}{\sinh(2\pi z)}\right]=\lim_{z\to\frac{i}{2}}\frac{2z-\frac{i}{2}}{2\pi\cosh(2\pi z)}=\frac{\frac{i}{2}}{2\pi\cosh(\pi i)}$

Residue at $z=-\frac{i}{2}$:

$$a_{-1}=\lim_{z\to-\frac{i}{2}}\left[\left(z+\frac{i}{2}\right)\frac{z}{\sinh(2\pi z)}\right]=\lim_{z\to-\frac{i}{2}}\frac{2z+\frac{i}{2}}{2\pi\cosh(2\pi z)}=\frac{-\frac{i}{2}}{2\pi\cosh(\pi i)}$$

$I=2\pi i\sum \text{Res}\ =0$

Q29. Solve the differential equation,

$$\frac{dy}{dx}=xy+xy^2$$

If $y\left(x=\sqrt{2}\right)=\frac{e}{2-e}$ where e is the base of natural logarithms, compute $y(x=0)$

(a) -1 (b) 1

(c) e (d) 0

Ans: 29. (b)

Solution.:

$$\frac{dy}{dx}=xy+xy^2\Rightarrow\frac{dy}{y+y^2}=xdx\Rightarrow\frac{dy}{y(1+y)}=xdx\Rightarrow\int\left(\frac{1}{y}-\frac{1}{1+y}\right)dy=\int xdx$$

$$\Rightarrow \ln y-\ln(1+y)=\frac{x^2}{2}+\ln C\Rightarrow\frac{y}{1+y}=Ce^{\frac{x^2}{2}}$$

$$\because y\left(x=\sqrt{2}\right)=\frac{e}{2-e}\Rightarrow\frac{\frac{e}{2-e}}{1+\frac{e}{2-e}}=Ce^{\frac{2}{2}}\Rightarrow\frac{e}{2}=Ce^{1}\Rightarrow C=\frac{1}{2}.$$

$$y(x=0):\ \frac{y}{1+y}=\frac{1}{2}e^{\frac{0}{2}}\Rightarrow\frac{y}{1+y}=\frac{1}{2}\Rightarrow y=1$$

Q33. If a power series $y=\sum_{j=0}^{\infty}a_jx^j$ analysis is carried out of the following differential equation

$$\frac{d^2y}{dx^2}+\frac{1}{x^2}\frac{dy}{dx}-\frac{4}{x^2}y=0$$

which of the following recurrence relations results?

(a) $a_{j+1}=a_j\frac{4-j(j+1)}{j+1},\ j=0,1,2,....$

(b) $a_{j+2}=a_j\frac{4-j(j-1)}{j+1},\ j=0,1,2,....$

(c) $a_{j+2}=a_j\frac{4-j(j+1)}{j+1},\ j=0,1,2,....$

(d) $a_{j+1}=a_j\frac{4-j(j-1)}{j+1},\ j=0,1,2,....$

Ans: 33. (d)

Solution.:

Let $y=\sum_{j=0}^{\infty}a_jx^j$, $\frac{dy}{dx}=\sum_{j=0}^{\infty}ja_jx^{j-1}$ and $\frac{d^2y}{dx^2}=\sum_{j=0}^{\infty}j(j-1)a_jx^{j-2}$

$$\because\frac{d^2y}{dx^2}+\frac{1}{x^2}\frac{dy}{dx}-\frac{4}{x^2}y=0\Rightarrow x^2\frac{d^2y}{dx^2}+\frac{dy}{dx}-4y=0$$

$$\Rightarrow\sum_{j=0}^{\infty}j(j-1)a_jx^j+\sum_{j=0}^{\infty}ja_jx^{j-1}-4\sum_{j=0}^{\infty}a_jx^j=0$$

Equating coefficient of x^j to zero; $j(j-1)a_j+(j+1)a_{j+1}-4a_j=0$

$$\Rightarrow(j+1)a_{j+1}=4a_j-j(j-1)a_j\Rightarrow a_{j+1}=\frac{4-j(j-1)}{j+1}a_j$$

Q46. Given the vector $\vec{v}=y\hat{i}+3x\hat{j}$, what is the value of the line integral $\oint\vec{v}\cdot d\vec{r}$ along the unit circle (centered at the origin) in an anti-clockwise direction?

(a) $\frac{2\pi}{3}$ (b) π (c) 0 (d) 2π

Ans: 46. (d)

Solution.: $\oint\vec{v}\cdot d\vec{r}=\int_S(\vec{\nabla}\times\vec{v}).d\vec{a}=\int_{r=0}^{1}\int_{\phi=0}^{2\pi}(2\hat{k}).(rdrd\phi\hat{k})=2\times\frac{1}{2}\times2\pi=2\pi$

Q47. Choose the largest eigenvalue of the matrix $\begin{pmatrix}1&1&1&2&3\\1&22&3&2&1\\0&0&0&2&2\\0&0&3&0&3\\0&0&0&0&1\end{pmatrix}$

(a) 3 (b) 5

(c) 8 (d) 10

Ans: 47. (b)

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Solution.: Characteristic equation $|A-\lambda I|=0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 & 2 & 3 \\ 12 & 2-\lambda & 3 & 2 & 1 \\ 0 & 0 & 0-\lambda & 2 & 2 \\ 0 & 0 & 3 & 0-\lambda & 3 \\ 0 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}=0$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 1-\lambda & 1 & 1 & 2 \\ 12 & 2-\lambda & 3 & 2 \\ 0 & 0 & -\lambda & 2 \\ 0 & 0 & 3 & -\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)(-3)\begin{vmatrix} 1-\lambda & 1 & 1 \\ 12 & 2-\lambda & 2 \\ 0 & 0 & 2 \end{vmatrix}-(1-\lambda)(\lambda)\begin{vmatrix} 1-\lambda & 1 & 1 \\ 12 & 2-\lambda & 3 \\ 0 & 0 & -\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)(-3\times 2)\begin{vmatrix} 1-\lambda & 1 \\ 12 & 2-\lambda \end{vmatrix}-(1-\lambda)(\lambda)(-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 12 & 2-\lambda \end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)\left[-6\{(1-\lambda)(2-\lambda)-12\}+\lambda^2\{(1-\lambda)(2-\lambda)-12\}\right]=0$$

$$\Rightarrow (1-\lambda)\{(1-\lambda)(2-\lambda)-12\}\left[-6+\lambda^2\right]=0$$

$$\Rightarrow (1-\lambda)=0; (-6+\lambda^2)=0; \{(1-\lambda)(2-\lambda)-12\}=0$$

Thus $\Rightarrow \lambda_1=1, \lambda_2=+\sqrt{6}, \lambda_3=-\sqrt{6}$

and $(1-\lambda)(2-\lambda)-12=0 \Rightarrow \lambda^2-3\lambda-10=0 \quad \Rightarrow \lambda_4=5, \lambda_5=-2$

Section A

Correct answer: +3, wrong answer: −1.

Q1. The position and velocity vectors of a particle changes from $\vec{R}_1$ to $\vec{R}_2$ and $\vec{v}_1$ to $\vec{v}_2$ respectively as time flows from t_1 to t_2. If $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are the instantaneous position, velocity and acceleration vectors of the particle, compute the integral:

$$\vec{I} = \int_{t_1}^{t_2} \vec{r} \times \vec{a}\, dt$$

Mark the correct answer.

(a) $\vec{I} = \vec{R}_2 \times \vec{v}_2 - \vec{R}_1 \times \vec{v}_1$ (b) $\vec{I} = \vec{R}_1 \times \vec{v}_1 - \vec{R}_2 \times \vec{v}_2$

(c) $\vec{I} = 0$ (d) $\left|\vec{I}\right| = \left|\vec{R}_1 \times \vec{v}_1\right| + \left|\vec{R}_2 \times \vec{v}_2\right|$

Ans: 1. (a)

Solution:

$$\frac{d}{dt}(\vec{r} \times \vec{v}) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} \qquad \because \frac{d\vec{r}}{dt} \times \vec{v} = 0$$

$$\Rightarrow (\vec{r} \times \vec{a}) = \frac{d}{dt}(\vec{r} \times \vec{v}) \qquad \because \vec{v} = \frac{d\vec{r}}{dt}, \vec{a} = \frac{d\vec{v}}{dt}$$

$$I = \int_{t_1}^{t_2} (\vec{r} \times \vec{a})\, dt = \int_{t_1}^{t_2} \frac{d}{dt}(\vec{r} \times \vec{v}) dt \Rightarrow I = [\vec{r} \times \vec{v}]_{t_1}^{t_2} \Rightarrow I = \vec{r}_2 \times \vec{v}_2 - \vec{r}_1 \times \vec{v}_1$$

Q10. A rocket is moving in free space with speed $\frac{c}{2}$. After a fuel tank is gently detached, the rocket is found to be moving with a speed $\frac{c}{4}$ with respect to the detached fuel tank. What is the final speed of the rocket in the original frame of reference?

(a) $\frac{2}{7}c$ (b) $\frac{2}{3}c$ (c) $\frac{3}{4}c$ (d) $\frac{4}{5}c$

Ans: 10. (b)

Solution:

$$v_{RE} = v \text{ [Let]}, \; v_{RF} = \frac{c}{4}, \; v_{FE} = \frac{c}{2}$$

$$\because v_{RF} = \frac{v_{RE} - v_{FE}}{1 - \frac{v_{RE} v_{FE}}{c^2}} \Rightarrow \frac{c}{4} = \frac{v - \frac{c}{2}}{1 - \frac{v\left(\frac{c}{2}\right)}{c^2}} \Rightarrow v = \frac{2c}{3}$$

Q12. The force experienced by a mass confined to move along the x-axis is of the form $F(x) = -k_1x - k_2x^n$ where x is the displacement of the mass from $x = 0, k_1$ and k_2 are positive constants and n is a positive integer. For small displacements, the motion of the mass remains symmetric about $x = 0$

(a) when n is any positive integer.

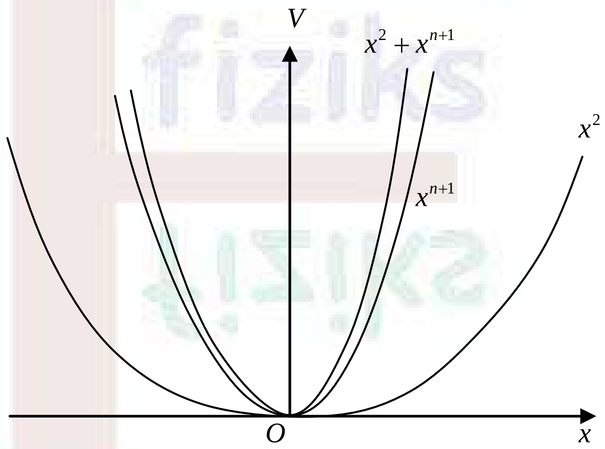
(b) when n is an odd positive integer

(c) only when $n = 1$

(d) when n is an even positive integer

Ans: 12. (b)

Solution:



Here n will be odd positive integer.

$$V = -\int F dx \quad \Rightarrow V = \frac{1}{2}k_1x^2 + \frac{k_2}{n+1}x^{n+1}$$

For the bound small oscillations about $x = 0$, the potential should be symmetric about $x = 0$, so n should be an odd positive integer.

Q17. Two particles of mass m and $4m$ confined to move along the x-axis are subjected to the force $F(x) = -kx$. At time $t = 0$, the smaller mass m starts from rest at $x_1(t = 0) = A$ and the larger mass $4m$ starts from rest at $x_2(t = 0) = -A$. The point on the x-axis where the first collision between the two particles occurs is:

(a) $x = \frac{A}{2}$ (b) $x = -\frac{A}{2}$

(c) $x = -\frac{A}{4}$ (d) $x = 0$

Ans: 17. (b)

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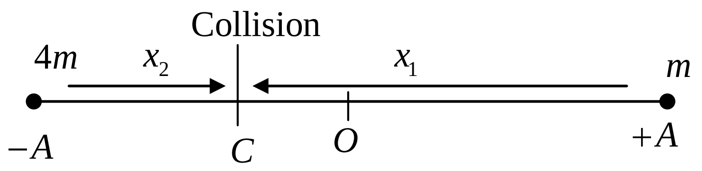
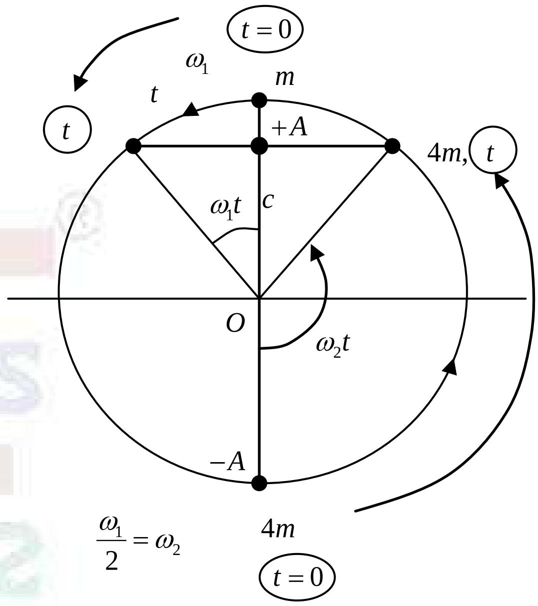
Solution:



$T_2 = 2T_1, \omega_2 = \sqrt{\frac{k}{4m}} = \frac{\omega_1}{2}, \quad \omega_1 = \sqrt{\frac{k}{m}}, T = T_1$

$x_1 = A\sin\left(\omega_1 t + \frac{\pi}{2}\right), \quad x_2 = A\sin\left(\omega_2 t + \frac{3\pi}{2}\right)$



Let they collide at point C.

$\omega_1 t + \omega_2 t = \pi \Rightarrow \omega_1 t + \frac{\omega_1}{2} t = \pi \Rightarrow \frac{3}{2}\frac{2\pi}{T} t = \pi$

$\Rightarrow t = \frac{T}{3}$

For particle of mass m; $x = A\sin\left(\omega_1 t + \frac{\pi}{2}\right)$

$x = A\sin\left(\frac{2\pi}{T}\cdot\frac{T}{3} + \frac{\pi}{2}\right) = A\cos\left(\frac{2\pi}{3}\right) \Rightarrow x = -\frac{A}{2}$

Section B

Correct answer: +9, wrong answer: −3.

Q32. A particle of mass 1 kg, angular momentum $L = \sqrt{2}$ kg m^2/s and total energy $E = 3\,J$ is subjected to a central force field $\vec{F} = -k\vec{r}$ where $k = 2$ kg/s^2. Which of the following statements is true? [Note: The centres of all the circles in the options below are at the origin.] =

(a) The particle is constrained to be in the region outside the circle with radius $R = \sqrt{\frac{3+\sqrt{5}}{2}}$

(b) The particle is bounded within the annular region described by the two circles with radii $r_1 = \sqrt{\frac{5-\sqrt{3}}{2}}$ and $r_2 = \sqrt{\frac{5+\sqrt{3}}{2}}$.

(c) The particle is bounded within the annular region described by the two circles with radii $r_1 = \sqrt{\frac{3-\sqrt{5}}{2}}$ and $r_2 = \sqrt{\frac{3+\sqrt{5}}{2}}$.

(d) The particle is constrained to be in the region outside the circle with radius $R=\sqrt{\frac{5+\sqrt{3}}{2}}$.

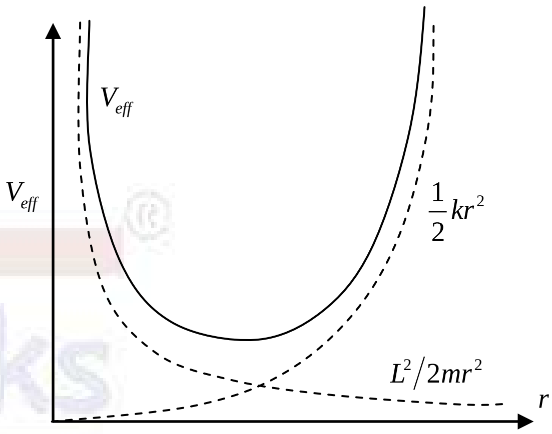
Ans: 32. (c)

Solution:

$\vec{F}=-k\vec{r}=-kr\hat{r}$ (Central force)

$V=-\int \vec{F}\cdot d\vec{r}=\frac{1}{2}kr^2$

$E=\frac{1}{2}m\dot{r}^2+V_{eff}$; $V_{eff}=\frac{L^2}{2mr^2}+\frac{1}{2}kr^2$



In case of bounded motion, the radial velocity $\dot{r}=0$ at turning points. So, total energy

$E=\frac{L^2}{2mr^2}+\frac{1}{2}kr^2$ (At turning point)

$3=\frac{2}{2\cdot 1\cdot r^2}+\frac{1}{2}\cdot 2\cdot r^2 \Rightarrow r^4-3r^2+1=0 \Rightarrow r=\sqrt{\frac{3\pm\sqrt{5}}{2}}$

$r_{\min}=\sqrt{\frac{3-\sqrt{5}}{2}}$ and $r_{\max}=\sqrt{\frac{3+\sqrt{5}}{2}}$

So, the particle is bounded within the annular region described by the two circles with radii

$r_1=\sqrt{\frac{3-\sqrt{5}}{2}}$ and $r_2=\sqrt{\frac{3+\sqrt{5}}{2}}$

Q35. The action corresponding to the motion of a particle in one dimension is:

$$S=\int_{t_i}^{t_f} dt\left[\frac{1}{2}m\dot{x}^2-V(x)+\alpha x\ddot{x}+\beta x\dot{x}\right]$$

where m is the mass of the particle, α,β are constants, and $V(x)$ is a potential which is a function of x. The position and velocity are held fixed at the end points of the trajectory. The equation of motion of the particle is

(a) $(2\alpha+m)\ddot{x}-\frac{dV}{dx}=0$ (b) $(2\alpha-m)\ddot{x}+\beta\dot{x}-\frac{dV}{dx}=0$

(c) $(2\alpha-m)\ddot{x}-\beta\dot{x}-\frac{dV}{dx}=0$ (d) $(2\alpha-m)\ddot{x}-\frac{dV}{dx}=0$

Ans: 35. (d)

Solution: $L(x,\dot{x},\ddot{x},t)=\frac{1}{2}m\dot{x}^2-V(x)+\alpha x\ddot{x}+\beta x\dot{x}$

LEM: $\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{x}}\right)-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)+\frac{\partial L}{\partial x}=0 \Rightarrow \frac{d^2}{dt^2}(\alpha x)-\frac{d}{dt}(m\dot{x}+\beta x)+\left[-\frac{\partial V}{\partial x}+\alpha\ddot{x}+\beta\dot{x}\right]=0$

$(2\alpha-m)\ddot{x}-\frac{\partial V}{\partial x}=0$

Q36. A stationary body explodes into two fragments, each of rest mass m. The two fragments move apart at speeds ηc (where c is the speed of light and $0<\eta<1$) relative to the original body. The rest mass of the original body is:

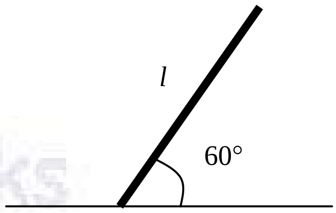
(a) $2m\sqrt{1-\eta^2}$ (b) $2m(1-\eta^2)$ (c) $\frac{2m}{\sqrt{1-\eta^2}}$ (d) $2m$

Ans: 36. (c)

Solution: Using conservation of energy principle

$$Mc^2=\frac{2mc^2}{\sqrt{1-\frac{(\eta c)^2}{c^2}}} \Rightarrow M=\frac{2m}{\sqrt{1-\eta^2}}$$

Q42. A rod of length $l=1$ meter is held on a frictionless horizontal surface at an angle of $\theta=60^\circ$ with the horizontal, as shown in the figure. Take the point of contact of the rod with the horizontal plane as the origin $(x=0)$. As the support holding the rod is suddenly removed, the rod comes in contact with the horizontal surface. What will be the coordinate of the left end of the rod at the moment of contact?

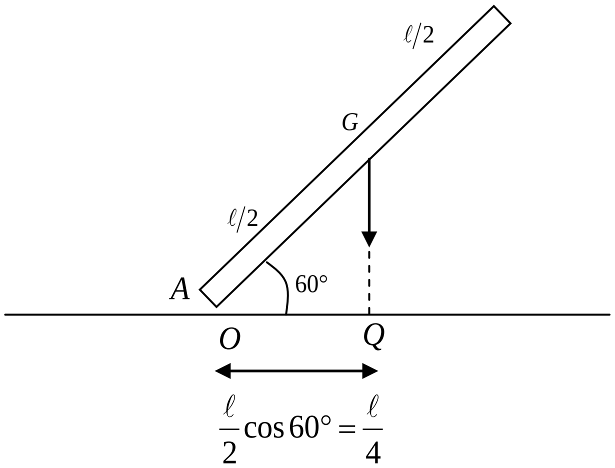


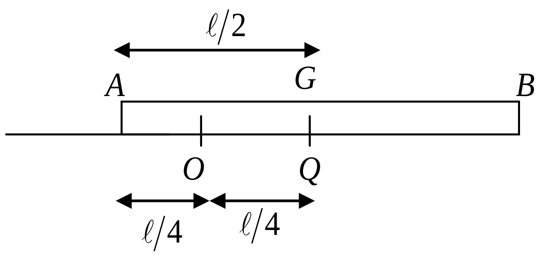
(a) -0.15 m (b) -0.5 m

(c) -0.2 m (d) -0.25 m

Ans: 42. (d)

Solution:





Center of mass of the rod will move in vertically downward direction under gratation.

$$|AO| = AG - OQ = \frac{\ell}{2} - \frac{\ell}{2}\cos 60° = \frac{\ell}{4} = 0.25\,m$$

Coordinate of left end A

$$x_A = -0.25m$$

Section A

Correct answer: +3, wrong answer: −1.

Q11. Two identical magnetic dipoles of length ℓ, which are free to rotate, are kept fixed at a distance $d\left(d \gg \ell\right)$. In their minimum energy configuration, they will orient themselves

(a) anti-parallel to each other and perpendicular to the line joining them

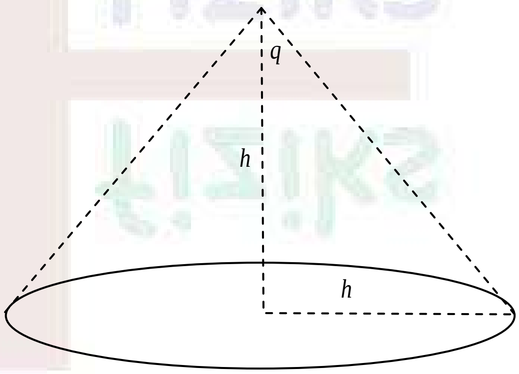
(b) parallel to each other and aligned to the line joining them

(c) anti-parallel to each other and aligned to the line joining them

(d) parallel to each other and perpendicular to the line joining them

Ans: 11. (b)

Q18. A point charge q is located at the apex of a cone of height h and base radius h. The flux of the electric field through the cone due to the point charge is



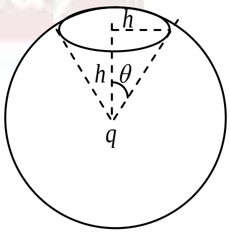
(a) $\left(1-\frac{1}{\sqrt{2}}\right)\frac{q}{2\pi \in_0}$ (b) $\left(1-\frac{1}{\sqrt{2}}\right)\frac{\pi q}{2 \in_0}$

(c) $\frac{1}{\sqrt{2}}\frac{q}{2 \in_0}$ (d) $\left(1-\frac{1}{\sqrt{2}}\right)\frac{q}{2 \in_0}$

Ans: 18. (d)

Solution:

The flux of the electric field through the cone due to the point charge is



$$\phi_E = \int_S \vec{E}.d\vec{a} = \int_{\theta'=0}^{\theta}\int_{\phi=0}^{2\pi}\left(\frac{1}{4\pi \in_0}\frac{q}{r^2}\hat{r}\right).\left(r^2 \sin\theta' d\theta' d\phi\right) = \frac{q}{2 \in_0}\left(1-\cos\theta\right)$$

$$\Rightarrow \phi_E = \left(1-\frac{1}{\sqrt{2}}\right)\frac{q}{2 \in_0} \quad \text{where } \cos\theta = \frac{h}{h\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Section B

Correct answer: +9, wrong answer: −3.

Q34. Two semi-infinite wires are placed on the x- axis, one from $-\infty$ to the $-d$, and the other from d to ∞. Both wires carry a steady current I in the same direction. The magnitude of the magnetic field at a distance d away from the center of this gap in the $y-z$ plane (ignore the charge accumulation) is:

(a) $\frac{\mu_0 I}{\pi d}\sqrt{2}$

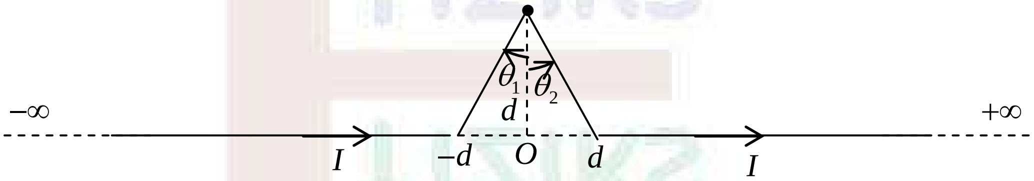
(b) $\frac{\mu_0 I}{2\pi d}\left(1-\frac{1}{\sqrt{2}}\right)$

(c) $\frac{\mu_0 I}{\pi d}\frac{1}{\sqrt{2}}$

(d) $\frac{\mu_0 I}{\pi d}\frac{1}{2}$

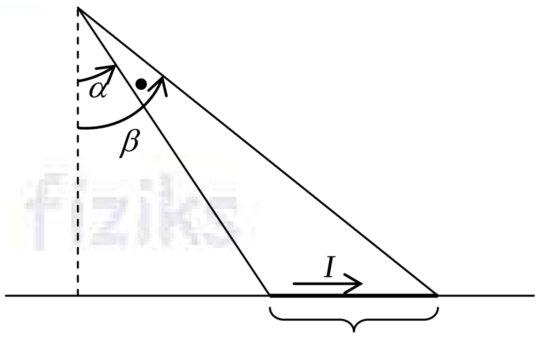
Ans: 34. (b)

Solution:



Here $\theta_1 = \theta_2 = 45^0 = \frac{\pi}{4}$

$\because B = \frac{\mu_0 I}{4\pi d}(\sin\beta - \sin\alpha)$



Magnitude of field due to each wire is same.

So

$$B_1 = B_2 = \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{4} - \sin\frac{\pi}{2}\right) = \frac{\mu_0 I}{4\pi d}\left(1-\frac{1}{\sqrt{2}}\right)$$

Thus $B = 2B_1 = \frac{\mu_0 I}{2\pi d}\left(1-\frac{1}{\sqrt{2}}\right)$

Q37. Calculate the magnitude of the force experienced by a point charge $+q$ placed at a distance d from the center of a grounded conducting sphere of radius $a(<d)$.

(a) $\frac{q^2 ad}{4\pi\epsilon_0\left(d^2-a^2\right)^2}$

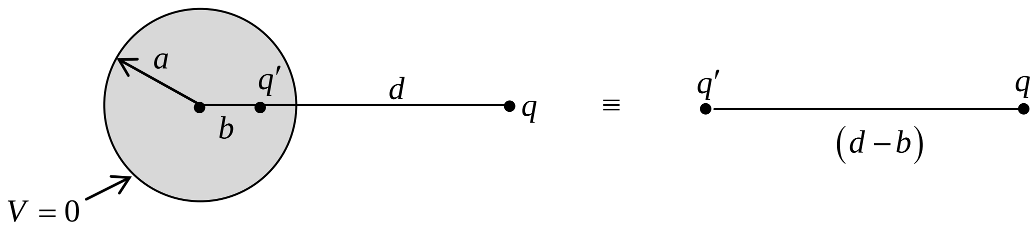
(b) $\frac{q^2}{4\pi\epsilon_0\left(d-a\right)^2}$

(c) $\frac{q^2}{4\pi\epsilon_0 d^2}$

(d) 0

Ans: 37. (a)

Solution:



Let $q' = -\frac{a}{d}q$ be image charge that develops $b = \frac{a^2}{d}$ distance away from center of sphere.

The magnitude of the force experienced by a point charge $+q$ is

$$F = \frac{1}{4\pi \in_0} \frac{qq'}{(d-b)^2} = \frac{q^2 ad}{4\pi \in_0 (d^2 - a^2)^2}$$

Q44. A conducting spherical soap bubble of radius R with a wall thickness of $W(\ll R)$ is charged to a potential of V_0. The bubble bursts and becomes a spherical drop with potential V_d. Select the correct value of the ratio $\eta = \frac{V_d^3 W}{V_0^3 R}$

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

Ans: 44. (a)

Solution:

Potential of spherical soap bubble spherical drop; Potential of spherical drop $V_d = k\frac{q}{r}$

Volume of spherical soap bubble = Volume of spherical drop

$$\frac{4}{3}\pi(R+W)^3 - \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \Rightarrow r = (3R^2W)^{1/3}.$$

$$\Rightarrow \frac{V_d}{V_0} = \frac{R}{r} = \frac{R}{(3R^2W)^{1/3}} \Rightarrow \eta = \frac{V_d^3 W}{V_0^3 R} = \frac{1}{3}$$

Section A

Correct answer: +3, wrong answer: −1.

Q4. Consider a spin-$1/2$ particle in the quantum state

$$\left|\psi(\beta,\alpha)\right\rangle = \cos\left(\frac{\beta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}\left|\downarrow\right\rangle$$

where $0 \le \beta \le \pi$ and $0 \le \alpha \le 2\pi$. For which values of (δ,γ) is the state $\left|\psi(\delta,\gamma)\right\rangle$ orthogonal to $\left|\psi(\beta,\alpha)\right\rangle$?

(a) $(\pi+\beta,\pi-\alpha)$ (b) $(\pi-\beta,\pi-\alpha)$

(c) $(\pi+\beta,\pi+\alpha)$ (d) $(\pi-\beta,\pi+\alpha)$

Ans: 4. (d)

Solution:

Given $\left|\psi(\beta,\alpha)\right\rangle = \cos\left(\frac{\beta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}\left|\downarrow\right\rangle$

Consider another state $\left|\psi(\delta,\gamma)\right\rangle = \cos\left(\frac{\delta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\delta}{2}\right)e^{i\gamma}\left|\downarrow\right\rangle$

Orthogonal to $\left|\psi(\beta,\alpha)\right\rangle$, Therefore $\left\langle\psi(\delta,\gamma)\middle|\psi(\beta,\alpha)\right\rangle = 0$

$$\Rightarrow \left(\cos\left(\frac{\delta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\delta}{2}\right)e^{-i\gamma}\left|\downarrow\right\rangle\right)\left(\cos\left(\frac{\beta}{2}\right)\left|\uparrow\right\rangle + \sin\left(\frac{\beta}{2}\right)e^{i\alpha}\left|\downarrow\right\rangle\right) = 0$$

$$\Rightarrow \cos\left(\frac{\delta}{2}\right)\cos\left(\frac{\beta}{2}\right) + \sin\left(\frac{\delta}{2}\right)\sin\left(\frac{\beta}{2}\right)e^{i(\alpha-\gamma)} = 0$$

For $\alpha-\gamma=-\pi \quad \Rightarrow \gamma=\alpha+\pi$ and $\delta=\pi-\beta$

$$\therefore \cos\left(\frac{\pi}{2}-\frac{\beta}{2}\right)\cos\left(\frac{\beta}{2}\right) + \sin\left(\frac{\pi}{2}-\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)e^{-i\pi} = 0$$

$$\Rightarrow \sin\left(\frac{\beta}{2}\right)\cos\left(\frac{\beta}{2}\right) + \cos\left(\frac{\beta}{2}\right)\sin\left(\frac{\beta}{2}\right)(-1) = 0 \ \Rightarrow 0 = 0$$

Thus option (d) is correct answer.

Q7. Consider a free particle in one dimension described by the wavefunction:

$$\psi\left(x,t=0\right)=A\exp\frac{ipx}{\hbar}+B\exp\frac{-ipx}{\hbar}$$

The probability current density corresponding to $\psi\left(x,t\right)$ at a later time t is:

(a) $\dfrac{p\left(|A|^2-|B|^2\right)}{m}\cos\left(\dfrac{p^2}{2m\hbar}t\right)$ (b) $\dfrac{p\left(|A|^2-|B|^2\right)}{m}$

(c) $\dfrac{p\left(|A|^2+|B|^2\right)}{m}$ (d) $\dfrac{p\left(|A|^2+|B|^2\right)}{m}\cos\left(\dfrac{p^2}{2m\hbar}t\right)$

Ans: 7. (b)

Solution:

$\psi\left(x,t=0\right)=Ae^{ipx/\hbar}+Be^{-ipx/\hbar}=\psi_1\left(x,0\right)+\psi_2\left(x,0\right)$

The probability current density is

$$P=P_1+P_2=v_1|\psi_1|^2+v_2|\psi_2|^2=\frac{p}{m}|A|^2-\frac{p}{m}|B|^2=\frac{p}{m}\left(|A|^2-|B|^2\right)$$

Thus, correct option is (b).

Q16. A quantum particle moving in one dimension is in a state having the wave function

$$\psi\left(x\right)=\sinh\left(\lambda x\right)\exp\left(\frac{-ax^4+bx+ipx}{\hbar}\right)$$

where a,b,λ and p are all positive real numbers. What is the expectation value of momentum?

(a) $\hbar\lambda$ (b) p (c) b (d) $-p$

Ans: 16. (b)

Solution: $\psi\left(x\right)=\sinh\left(\lambda x\right)e^{\frac{\left(-ax^4+bx\right)}{\hbar}}e^{\frac{ipx}{\hbar}}$

The expectation value of p is $\langle p\rangle=p$

Thus correct option is (b).

Q20. Consider a two-level quantum system described by the Hamiltonian:

$$H = H_0 + H'$$

where

$$H_0 = \alpha\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \qquad H' = \in\Gamma\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

H′ is a small perturbation to the free Hamiltonian H_0. $\in$ is a small positive dimensionless number, while α, ω and Γ have dimensions of energy and are positive quantities. If we treat this problem perturbatively in the parameter ϵ, which of the following statements about the corrections to ground state energy is true?

(a) First-order correction is $\in\Gamma$; second-order correction is $-\dfrac{\in^2\Gamma^2}{2\omega}$

(b) First-order correction is $\in\Gamma$; second-order correction is 0.

(c) First-order correction is 0; second-order correction is $-\dfrac{\in^2\Gamma^2}{2\omega}$

(d) First-order correction is 0; second-order correction is $\dfrac{\in^2\Gamma^2}{2\omega}$

Ans: 20. (c)

Solution: $H_0 = \alpha\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \omega\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}\alpha+\omega & 0\\ 0 & \alpha-\omega\end{pmatrix}$

The eigenvalues are $E_1 = \alpha - \omega$ and $E_2 = \alpha + \omega$ with eigenstates $|\phi_1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ and $|\phi_2\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

The first order correction energy is

$$E_0^{(1)} = \langle H'\rangle = \langle\phi_1|H'|\phi_1\rangle = (0,1)\in\Gamma\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \in\Gamma\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 0$$

The second order correction energy is $E_0^{(2)} = \sum_{m\neq n}\dfrac{|\langle\phi_n|H'|\phi_m\rangle|^2}{E_n - E_m} = \dfrac{|\langle\phi_1|H'|\phi_2\rangle|^2}{E_1 - E_2}$

$$E_0^{(2)} = \frac{\left|\begin{pmatrix}0 & 1\end{pmatrix}\in\Gamma\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right|^2}{(\alpha-\omega)-(\alpha+\omega)} = \frac{\left|\in\Gamma\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right|^2}{-2\omega} = -\frac{(\in\Gamma)^2}{2\omega}$$

$$\therefore\; E_0^{(2)} = -\frac{\in^2\Gamma^2}{2\omega}$$

Thus correct option is (c).

Q 21 has been cancelled due to typographical error.

Q21. The spatial part of a two-electron state is anti-symmetric under exchange. If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and the spin-down states respectively of each electron, the spin part of the two-electron state cannot be:

(a) $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle+|\uparrow\rangle|\downarrow\rangle\right)$ (b) $|\uparrow\rangle|\uparrow\rangle$

(c) $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle-|\uparrow\rangle|\downarrow\rangle\right)$ (d) $|\downarrow\rangle|\downarrow\rangle$

Ans: 21. (c)

Solution:

The total wave function ψ_T of Fermions is anti-symmetric.

$$\psi_T = \psi_{space} \otimes \chi_{spin}$$

If spatial wave function (ψ_{space}) is anti-symmetric, then spin part must by symmetric.

$\therefore\ \chi_{spin} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\downarrow\rangle\right)$ is an anti-symmetric, therefore it cannot be allowed spin state. Thus correct option is (c).

Section B

Correct answer: +9, wrong answer: −3.

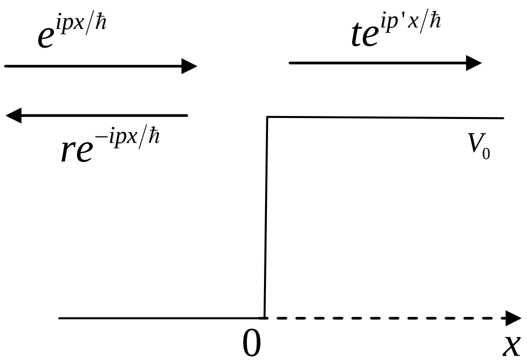
Q38. Consider a quantum particle incident from the left on a step potential given by $V_0\theta(x)$, with energy $E(>V_0)$; here $\theta(x)$ is the unit step function. The scattering state of the particle is given by
$$\psi(x)=\begin{cases}\exp\frac{ipx}{\hbar}+r\exp\frac{-ipx}{\hbar}, & x<0\\ t\exp\frac{ip'x}{\hbar} & x>0\end{cases}$$
where p and p' are the momenta of the particle corresponding to the energy *E*. Which of the following is true?

(a) $|r|^2+\frac{p'}{p}|t|^2=1$ (b) $|r|^2+|t|^2=1$

(c) $|r|^2+\frac{p}{p'}|t|^2=1$ (d) $r+t=1$

Ans: 38. (a)

Solution:



$\psi_{\text{incident}} = e^{ipx/\hbar}$, $\psi_{\text{reflected}} = re^{-ipx/\hbar}$, $\psi_{\text{transmitted}} = te^{ip'x/\hbar}$

$J_{in} = v|\psi_{in}|^2 = \frac{p}{m}$, $J_{ref} = v|\psi_{ref}|^2 = -\frac{p}{m}|r|^2$

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$$J_{tran} = v\left|\psi_{tran}\right|^2 = \frac{p'}{m}|t|^2$$

Reflection coefficient $R = \left|\frac{J_{ref}}{J_{inc}}\right| = |r|^2$, Transmitted co-efficient $T = \left|\frac{J_{Tran}}{J_{inc}}\right| = \frac{p'}{p}|t|^2$

Since $R + T = 1 \Rightarrow |r|^2 + \frac{p'}{p}|t|^2 = 1$. Thus correct option is (a).

Q39. Consider a spin-1 system whose $\hat{S}_z$ eigenstates are given by $|-1\rangle, |0\rangle, |+1\rangle$ corresponding to the eigenvalues $-\hbar, 0, \hbar$. The normalized general state $|\psi\rangle$ of the system can be expressed as $|\psi\rangle = c_{-1}|-1\rangle + c_0|0\rangle + c_{+1}|+1\rangle$ and c_{-1}, c_0, c_{+1} are complex numbers. Subjected to the condition $\langle\psi|\hat{S}_z|\psi\rangle = 0$, which of the following statements is true?

(a) $|c_{+1}|^2 + 2|c_0|^2 = 1$ (b) $|c_{-1}|^2 + 2|c_0|^2 = 1$

(c) $2|c_{-1}|^2 + |c_{+1}|^2 = 1$ (d) $2|c_{-1}|^2 + |c_0|^2 = 1$

Ans: 39. (d)

Solution: $|\psi\rangle = C_{-1}|-1\rangle + C_0|0\rangle + C_{+1}|+1\rangle$

Normalization condition $\langle\psi|\psi\rangle = 1 \Rightarrow |C_{-1}|^2 + |C_0|^2 + |C_{+1}|^2 = 1$ ...(1)

Given $\langle\psi|\hat{S}_z|\psi\rangle = 0 \Rightarrow \left(C_{-1}^*\langle -1| + C_0^*\langle 0| + C_{+1}^*\langle +1|\right)\hat{S}_z\left(C_{-1}|-1\rangle + C_0|0\rangle + C_{+1}|+1\rangle\right) = 0$

$\Rightarrow \left(C_{-1}^*\langle -1| + C_0^*\langle 0| + C_{+1}^*\langle +1|\right)\left(-\hbar C_{-1}|-1\rangle + 0 + \hbar C_{+1}|+1\rangle\right) = 0$

$\Rightarrow -\hbar|C_{-1}|^2 + \hbar|C_{+1}|^2 = 0 \Rightarrow -|C_{-1}|^2 + |C_{+1}|^2 = 0 \Rightarrow |C_{-1}|^2 = |C_{+1}|^2$ ...(2)

From equation (1), we get $|C_{-1}|^2 + |C_0|^2 + |C_{-1}|^2 = 1 \Rightarrow 2|C_{-1}|^2 + |C_0|^2 = 1$

Thus correct option is (d).

Q43. Consider a particle subjected to the symmetric one-dimensional infinite square well potential:

$$V(x) = \begin{cases} 0, & |x| \le \frac{L}{2} \\ \infty, & |x| > \frac{L}{2} \end{cases}$$

Find the time evolution of the wavefunction $\psi(x,t)$, if at time $t = 0$ the particle is prepared in an equal superposition of the ground and the first excited states:

$$\psi(x,0)=\frac{1}{\sqrt{2}}\left(\phi_1(x)+\phi_2(x)\right)$$

where $\phi_1(x)$ and $\phi_2(x)$ are normalized eigenfunctions of the ground state and the first excited state respectively. If τ is the smallest time at which the particle is equally likely to be in either half of the well, select the correct value of $\frac{\tau h}{mL^2}$,

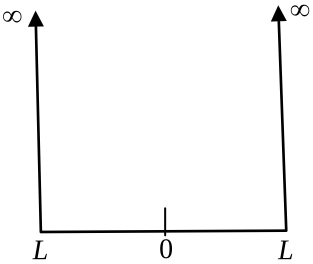
where h is the Planck's constant, m is the mass of the particle and L is the width of the well as defined above.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{4}{3}$

Ans: 43. (a)

Solution: For symmetric potential well



$\phi_1(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$: Ground state eigenstate

$\phi_2(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi}{L}x\right)$: First excited state eigenstate

$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$. Therefore, $E_1=\frac{\pi^2\hbar^2}{2mL^2}$ and $E_2=\frac{4\pi^2\hbar^2}{2mL^2}$

Given $\psi(x,0)=\frac{1}{\sqrt{2}}\left(\phi_1(x)+\phi_2(x)\right)$

The wave function at time t is $\psi(x,t)=e^{-i\hat{H}t/\hbar}\psi(x,0)=\frac{1}{\sqrt{2}}\left(e^{-iE_1t/\hbar}\phi_1(x)+e^{-iE_2t/\hbar}\phi_2(x)\right)$

According to the question $\int_0^{L/2}|\psi(x,t)|^2\,dx=\frac{1}{2}$

$$\Rightarrow\frac{1}{2}\int_0^{L/2}\left(\left(\phi_1(x)\right)^2+\left(\phi_2(x)\right)^2+e^{i(E_1-E_2)t/\hbar}\phi_1^*(x)\phi_2(x)+e^{-i(E_1-E_2)t/\hbar}\phi_2^*(x)\phi_1(x)\right)dx=\frac{1}{2}$$

$$\Rightarrow\frac{1}{2}\int_0^{L/2}\frac{2}{L}\cos^2\left(\frac{\pi}{L}x\right)dx+\frac{1}{2}\int_0^{L/2}\frac{2}{L}\sin^2\left(\frac{2\pi}{L}x\right)dx+\frac{1}{2}\cdot\frac{2}{L}\left(e^{-i(E_2-E_1)t/\hbar}+e^{i(E_2-E_1)t/\hbar}\right)$$
$$\times\int_0^{L/2}\cos\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi}{L}x\right)dx=\frac{1}{2}$$

$$\Rightarrow\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}+\frac{2}{L}\cos\left(E_2-E_1\right)t/\hbar\int_0^{L/2}\cos\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi}{L}x\right)dx=\frac{1}{2}$$

$$\Rightarrow \frac{1}{4}+\frac{1}{4}+\frac{2}{L}\cos\left(E_2-E_1\right)t/\hbar\times\frac{4}{3\pi}=\frac{1}{2} \Rightarrow \cos\left(E_2-E_1\right)t/\hbar=0 \Rightarrow \left(E_2-E_1\right)t/\hbar=\pi/2$$

$$\Rightarrow\left(\frac{4\pi^2\hbar^2}{2mL^2}-\frac{\pi^2\hbar^2}{2mL^2}\right)\frac{t}{\hbar}=\frac{\pi}{2} \Rightarrow \frac{3\pi^2\hbar^2}{2mL^2}\times\frac{t}{\hbar}=\frac{\pi}{2} \Rightarrow \frac{t\hbar}{mL^2}=\frac{2}{3}$$

Put $t=\tau$ $\Rightarrow \frac{\tau\hbar}{mL^2}=\frac{2}{3}$. Thus correct option is (a).

Q45. Consider the operator $\vec{S}\cdot\hat{n}$ with eigenkets $\left|\pm\right\rangle_{\hat{n}}$ and eigenvalues $\pm\frac{\hbar}{2}$ where $\hat{n}$ is a unit vector and $\vec{S}$ is the spin operator. A partially polarized beam of spin-$\frac{1}{2}$ particles contains a 25-75 mixture of two pure ensembles, one with $\left|+\right\rangle_{\hat{z}}$ and the other with $\left|+\right\rangle_{\hat{x}}$ respectively. What is the ensemble average of $\frac{\vec{S}\cdot\hat{x}}{\hbar}$?

(a) $\frac{1}{3}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) $\frac{3}{16}$

Ans: 45. (b)

Solution:

Given $\left|\chi\right\rangle=\frac{1}{2}\left|+\right\rangle_z+\frac{\sqrt{3}}{2}\left|+\right\rangle_x$

$$\left\langle\frac{\vec{S}\cdot\hat{x}}{\hbar}\right\rangle=\left\langle\frac{\hat{S}_x}{\hbar}\right\rangle=\left\langle\frac{\sigma_x}{2}\right\rangle=\frac{1}{2}\left\langle\sigma_x\right\rangle=\frac{1}{2}\left\langle x\middle|\sigma_x\middle|\chi\right\rangle \quad ...(1)$$

where $\sigma_x\left|\chi\right\rangle=\sigma_x\left[\frac{1}{2}\left|+\right\rangle_z+\frac{\sqrt{3}}{2}\left|+\right\rangle_x\right]=\frac{1}{2}\sigma_x\left|+\right\rangle_z+\frac{\sqrt{3}}{2}\sigma_x\left|+\right\rangle_x=\frac{1}{2}\left|-\right\rangle_z+\frac{\sqrt{3}}{2}\left|+\right\rangle_x$

Thus from equation (1)

$$\left\langle\frac{\vec{S}\cdot\hat{x}}{\hbar}\right\rangle=\frac{1}{2}\left[\frac{1}{2}\left\langle+\right|_z+\frac{\sqrt{3}}{2}\left\langle+\right|_x\right]\left[\frac{1}{2}\left|-\right\rangle_z+\frac{\sqrt{3}}{2}\left|+\right\rangle_x\right]=\frac{1}{2}\left[\frac{3}{4}\right]=\frac{3}{8}$$

Thus correct option is (b)

Section A

Correct answer: +3, wrong answer: −1.

Q5. Consider a system of classical non-interacting particles constrained to be in the *XY* plane subject to the potential: $V(x,y)=\frac{1}{2}\alpha(x-y)^2$

If they are in equilibrium with a thermal bath at temperature *T*, what is the average energy per particle? The Boltzmann constant is k_B.

(a) $\frac{5}{2}k_BT$ (b) $\frac{1}{2}k_BT$ (c) $2k_BT$ (d) $\frac{3}{2}k_BT$

Ans: 5. (c)

Solution: Hamiltonian $H=\frac{p_x^2}{2m}+\frac{p_y^2}{2m}+\frac{1}{2}\alpha\left(x^2+y^2-2xy\right)$

$$\langle H\rangle=\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT-0=2k_BT$$

Q6. There are three states of energy $E, 0, -E$ available for the population of two identical non-interacting spin less fermions. If they are in equilibrium at temperature *T*, what is the average energy of the system? The Boltzmann constant is k_B and consider β to be $\frac{1}{k_BT}$.

(a) 0

(b) $\frac{E\left(e^{\beta E}-e^{-\beta E}\right)}{1+e^{-\beta E}+e^{\beta E}}$

(c) $\frac{E\left(e^{-\beta E}-e^{\beta E}\right)}{1+e^{-\beta E}+e^{\beta E}}$

(d) $\frac{E\left(2e^{-\beta E}+e^{-\beta E}-e^{\beta E}-2e^{2\beta E}\right)}{2e^{-\beta E}+e^{-\beta E}+2+e^{\beta E}+2^{2\beta E}}$

Ans: 6. (c)

Solution: $\langle E\rangle=-\frac{\partial}{\partial\beta}(\ln Z)$ where $Z=1+e^{-\beta E}+e^{\beta E}$

$$\langle E\rangle=-\frac{\partial}{\partial\beta}\ln\left(1+e^{-\beta E}+e^{\beta E}\right)=\frac{E\left(e^{-\beta E}-e^{\beta E}\right)}{1+e^{-\beta E}+e^{\beta E}}$$

Q25. A steam engine takes steam from a boiler at 200^0C (pressure 1.5×10^6 Pa) and exhausts directly into the air at 100^0C (pressure 10^5 Pa). The maximum possible efficiency is closest to:

(a) 78% (b) 21% (c) 50% (d) 93%

Ans: 25. (b)

Solution: $\eta=\left(1-\frac{T_2}{T_1}\right)=\left(1-\frac{473}{373}\right)=0.211$ or 21%

Section B

Correct answer: +9, wrong answer: −3.

Q26. A gas is in equilibrium at temperature T. Using the kinetic theory of gasses, compute the following quantity:

$$\frac{\langle|\vec{v}|\rangle^2}{\langle|v_x|\rangle^2+\langle|v_y|\rangle^2+\langle|v_z|\rangle^2}$$

where $\vec{v}$ represents the velocity vector with the components v_x, v_y, v_z and $\langle ... \rangle$ represents the thermal average of the quantity.

(a) 0 (b) $\frac{4}{3}$ (c) 1 (d) $\frac{1}{3}$

Ans: 26. (b)

Solution:

$$\langle|\vec{v}|\rangle = 2\sqrt{\frac{k_BT}{m}} \Rightarrow \langle|\vec{v}|\rangle^2 = 4\frac{k_BT}{m} \text{ and } \langle|v_x|\rangle = \langle|v_y|\rangle = \langle|v_z|\rangle = \sqrt{\frac{k_BT}{m}}$$

Thus $\frac{\langle|\vec{v}|\rangle^2}{\langle|v_x|\rangle^2+\langle|v_y|\rangle^2+\langle|v_z|\rangle^2} = \frac{4\frac{k_BT}{m}}{3\frac{k_BT}{m}} = \frac{4}{3}$

Q27. Using the first law of thermodynamics $dU = TdS - PdV$, and the definitions of the thermodynamic potentials $H = U + PV, F = U - TS, G = H - TS$, work out the four Maxwell relations. Using these compute:

$$\chi = \left(\frac{\partial T}{\partial V}\right)_S + \left(\frac{\partial T}{\partial P}\right)_S + \left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_P$$

Which of the following does χ equal?

(a) $\left(\frac{\partial S}{\partial V}\right)_T - \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P - \left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial S}{\partial V}\right)_T - \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P + \left(\frac{\partial P}{\partial S}\right)_V$

(c) $\left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P - \left(\frac{\partial P}{\partial S}\right)_V$ (d) $\left(\frac{\partial S}{\partial V}\right)_T + \left(\frac{\partial S}{\partial P}\right)_T + \left(\frac{\partial V}{\partial S}\right)_P + \left(\frac{\partial P}{\partial S}\right)_V$

Ans: 27. (a)

Solution:

$$\because \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V, \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P, \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T, \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

Q30. One mole of an isolated ideal gas in equilibrium at pressure P_1, volume V_1, and temperature T_1 undergoes a process that changes its state. In the final state the gas is in equilibrium at pressure P_2, volume V_2, and temperature T_2. Suppose the ratios of the final and initial temperatures and pressures are:

$$\eta_T = \frac{T_2}{T_1}, \quad \eta_P = \frac{P_2}{P_1}$$

Work out the change in entropy ΔS in the process. Take the heat capacities of the gas at constant pressure and constant volume to be C_p and C_v respectively, and the ideal gas constant to be R.

(a) $C_p \ln(\eta_T) - R\ln(\eta_P)$ (b) $C_p \ln(\eta_T) + R\ln(\eta_P)$

(c) $C_v \ln(\eta_T) - R\ln(\eta_P)$ (d) $C_v \ln(\eta_T) + R\ln(\eta_P)$

Ans: 30. (a)

Solution:

$dS = \frac{dQ}{T}$ where $dQ = C_V dT + PdV$

$\because PV = RT \Rightarrow PdV + VdP = RdT \Rightarrow PdV = RdT - VdP$

$\Rightarrow dQ = C_V dT + RdT - VdP = (C_V + R)dT - VdP = C_P dT - VdP$

Thus $\Delta S = \int \frac{dQ}{T} = \int_{T_1}^{T_2} C_P \frac{dT}{T} - \int_{P_1}^{P_2} \frac{V}{T} dP = \int_{T_1}^{T_2} C_P \frac{dT}{T} - \int_{P_1}^{P_2} \frac{R}{P} dP = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$

Q48. A cylinder of height L and cross-section A placed vertically along the central axis is filled with non-interacting particles each of mass m which are acted upon by a gravitational force of magnitude mg in the downward direction. The system is maintained at a temperature T. What is the ratio $\frac{C_v(T\to 0)}{C_v(T\to\infty)}$, where C_v is the specific heat at constant volume.

(a) $\frac{3}{5}$ (b) $\frac{3}{2}$

(c) $\frac{1}{3}$ (d) $\frac{5}{3}$

Ans: 48. (d)

Solution:

Here, $H=\frac{p^2}{2m}+mgz=\frac{p_x^2+p_y^2+p_z^2}{2m}+mgz$

Single particle P.F. is

$$Q_1=\frac{1}{h^3}\int e^{-\beta H}d\omega,\ d\omega=dp_x dp_y dp_z dx\,dy\,dz \quad ...(1)$$

$$Q_1=\frac{1}{h^3}\int_{-\infty}^{\infty}\iint e^{-\left(\frac{p_x^2+p_y^2+p_z^2}{2mkT}\right)}dp_x dp_y dp_z\int dx\int dy\int_0^L e^{-\frac{mgz}{kT}}dz=\frac{1}{h^3}\left[2\pi mkT\right]^{3/2}\times\pi R^2\int_0^L e^{-\frac{mgz}{kT}}dz$$

$$Q_1=\pi R^2\left[\frac{2\pi mkT}{h^2}\right]^{3/2}\int_0^L e^{-\frac{mgz}{kT}}dz=\pi R^2\left[\frac{m}{2\pi\hbar^2\beta}\right]^{3/2}\frac{1-e^{-\beta mgL}}{\beta mg} \quad ...(2)$$

$$\ln Q_1=\ln C\beta^{-5/2}\left(1-e^{-\beta mgL}\right),\quad C=\frac{\pi R^2}{mg}\left[\frac{m}{2\pi\hbar^2}\right]^{3/2}$$

$$\langle E\rangle=-\frac{\partial\ln Q_1}{\partial\beta}=\frac{5}{2}kT-\frac{mgL\,e^{-\beta mgL}}{1-e^{-\beta mgL}}\Rightarrow\langle E\rangle=\frac{5}{2}kT-\frac{mgL}{e^{\beta mgL}-1} \quad ...(3)$$

When $T\to 0$, i.e. $kT\ll mgL$ or $\beta mgL\gg 1$; $e^{\beta mgL}-1\approx e^{\beta mgL}$

$$\therefore\ \langle E\rangle_{T\to 0}=\frac{5}{2}kT-mgL\,e^{-\beta mgL}$$

$$\langle E\rangle_{T\to 0}\approx\frac{5}{2}kT,\qquad \because \text{ for } \beta mgL\gg 1,\ e^{-\beta mgL}\to 0$$

$$\therefore\ \langle E\rangle_{T\to 0}=\frac{5}{2}kT \quad ...(4)$$

For second case, i.e. $T\to\infty$, $kT\gg mgL$ or $\frac{mgL}{kT}\ll 1$ or $\beta mgL\ll 1$

$$\therefore\ e^{\beta mgL}-1\approx 1+\frac{mgL}{kT}-1=\frac{mgL}{kT}$$

$$\langle E\rangle_{T\to\infty}=\frac{5}{2}kT-\frac{mgL}{1+\frac{mgL}{kT}-1}=\frac{3}{2}kT \quad ...(5)$$

From (4) & (5)

$$C_V(T\to 0)=\frac{d}{dT}\langle E\rangle_{T\to 0}=\frac{5}{2}k;\qquad C_V(T\to\infty)=\frac{d}{dT}\langle E\rangle_{T\to\infty}=\frac{3}{2}k$$

$$\therefore\ \frac{C_V(T\to 0)}{C_V(T\to 8)}=\frac{5}{2}k\times\frac{2}{3k}=\frac{5}{3}$$

$\therefore$ (d) is correct.

Q49. System A consists of 3 identical non-interacting bosons. System B consists of 2 identical non-interacting bosons. They both have identical energy spectra - three non-degenerate energy levels $0, \in, 2\in$. The particles of A and B are distributed in various energy levels in such a way that the total energy of the combined system is 4ϵ. The average energy of the system A in units of $\in$ is

(a) 2.2 (b) 2.3

(c) 2.1 (d) 2.4

Ans: 49. (b)

Solution:

AAA, BB

0	ε	2ε	$E_{Total}=4\varepsilon$	E_A	E_B
AAA	–	BB	4ε	0	4ε
AAB	–	AB	4ε	2ε	2ε
ABB	–	AA	4ε	4ε	–
BB	AA	A	4ε	4ε	–
AB	AB	A	4ε	3ε	ε
AB	AA	B	4ε	2ε	2ε
AA	AB	B	4ε	ε	3ε
AA	BB	A	4ε	2ε	2ε
B	AAAB	–	4ε	3ε	ε
A	AABB	–	4ε	2ε	2ε

For system A, $E_{\text{A Total}}=23\varepsilon$; $\langle E_A\rangle=\frac{23\varepsilon}{10}=2.3\varepsilon$

For system B, $E_{\text{A Total}}=17\varepsilon$; $\langle E_B\rangle=\frac{17\varepsilon}{10}=1.7\varepsilon$

Note: $\Omega_A=\frac{(n_i+g_i-1)!}{n_i!(g_i-1)!}=\frac{(3+3-1)!}{3\,(3-1)!}=10$; $\Omega_B=\frac{(2+3-1)!}{2!(3-1)!}=6$

$\langle E\rangle=\langle E_A\rangle+\langle E_B\rangle=2.3\varepsilon+1.7\varepsilon=4\varepsilon$

$\therefore$ Average energy of system A is 2.3ε. So (b) is correct.

Section A

Correct answer: +3, wrong answer: −1.

Q8. Consider all possible Boolean logic gates with 2 inputs and one output. How many such gates can be constructed?

(a) 16 (b) 4 (c) 2 (d) 8

Ans: 8. (a)

Q13. A silicon crystal sample has 50 billion silicon atoms and 5 million free electrons. The silicon crystal is additionally doped with 5 million pentavalent atoms. Assume that the ambient thermal energy is much smaller than the bandgap of silicon. How many free electrons and holes are there inside the silicon crystal?

(a) Number of electrons is 30 million and number of holes is zero.

(b) Number of electrons is 10 million and number of holes is zero.

(c) Number of electrons is 10 million and number of holes is 5 million.

(d) Number of electrons is 5 million and number of holes is 5 million.

Ans: 13. (b)

Q15. What is the 2′s complement representation of 11010110?

(a) 11010101 (b) 00101001

(c) 00101010 (d) 01101011

Ans: 15. (c)

Solution.:

1′s complement representation of 00101001

2′s complement representation of 00101001+1=00101010

Q24. Convert the octal number 3720_8 to its decimal equivalent.

(a) 1000_{10} (b) 2000_{10}

(c) 2020_{10} (d) 1900_{10}

Ans: 24. (b)

Solution.:

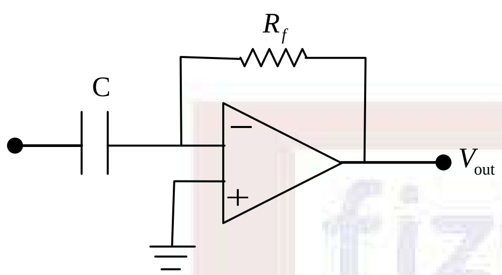
$$3720_8 = 3\times 8^3 + 7\times 8^2 + 2\times 8^1 + 0\times 8^0 = 1536 + 448 + 16 + 0 = 2000_{10}$$

Section B

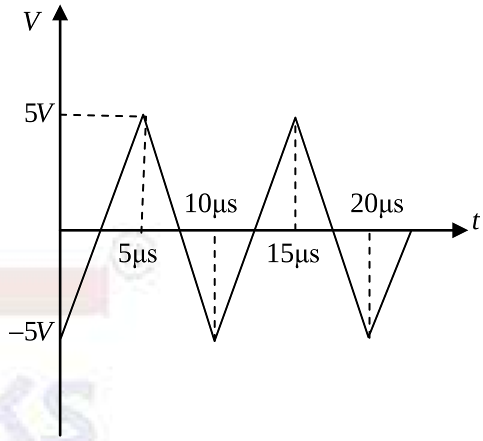
Correct answer: +9, wrong answer: −3.

Q40. Consider the Op-Amp differentiator presented in Figure (a). Take $C = 0.002\mu F$ and $R_f = 2k\Omega$. For a triangular wave input shown in the figure (b),

(a)

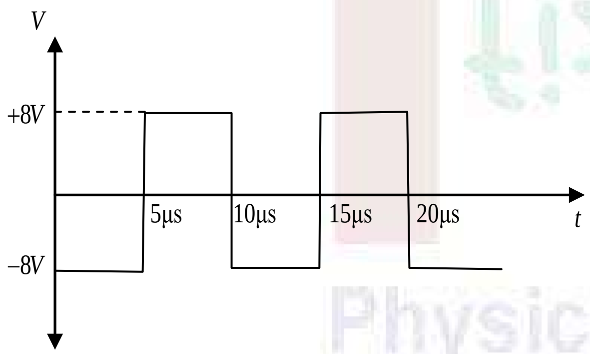


(b)

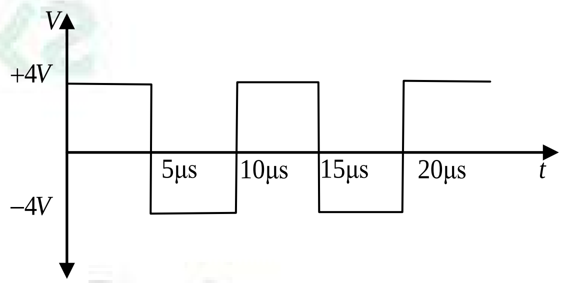


determine the output voltage waveform.

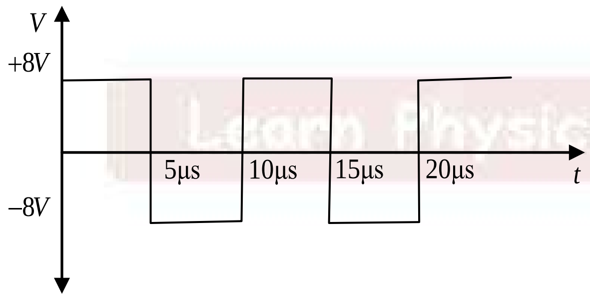
(a)



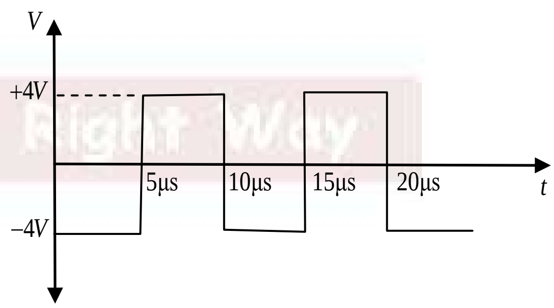
(b)



(c)



(d)

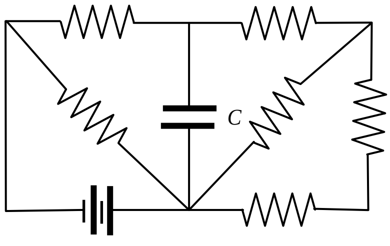


Ans: 40. (a)

Solution.:

Output voltage $= -R_f C \dfrac{dv_{in}}{dt} = -\left(2\times10^3\right)\left(2\times10^{-9}\right)\dfrac{(10-0)}{(5-0)\times10^{-6}} = -8\,volts$ (In +ve half cycle)

Q50. Consider the circuit shown in the figure below. C is the capacitance of the capacitor, E is the voltage provided by the battery and all the resistors are identical. What is the charge stored in the capacitor in units of CE, once it is fully charged.



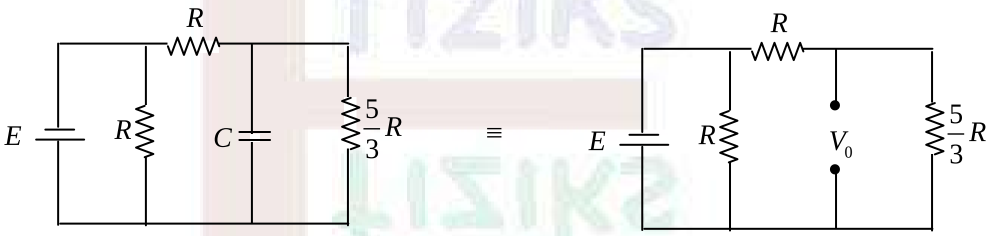
(a) $\frac{3}{4}$ (b) $\frac{5}{8}$

(c) $\frac{3}{8}$ (d) $\frac{5}{4}$

Ans: 50. (b)

Solution.:

Let resistance of each be R.



Open Circuit Voltage across capacitor is $V_0 = \dfrac{\frac{5}{3}R}{R+\frac{5}{3}R}E = \frac{5}{8}E$

Charge across capacitor is $Q_0 = CV_0 = \frac{5}{8}CE$

Section A

Correct answer: +3, wrong answer: −1.

Q2. A beam of light is propagating through a block of glass with refractive index n. The block is moving with a constant speed v in the direction opposite to that of the beam. The speed of the light beam in the block as measured by an observer in the laboratory frame is:

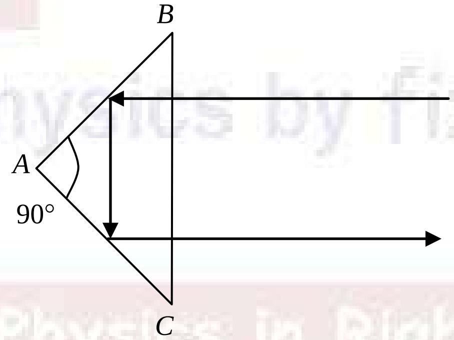
(a) $\frac{c}{2n}$ (b) $\frac{c}{n}$

(c) $\frac{c}{2n}$ (d) $\frac{c-nv}{n-\frac{v}{c}}$

Ans: 2. (d)

Solution: $v_{LB}=\frac{c}{n},\ v_{BE}=-v,\ v_{EB}=v;\quad v_{LE}=\frac{v_{LB}-v_{EB}}{1-\frac{v_{LB}v_{EB}}{c^2}}=\frac{\frac{c}{n}-v}{1-\frac{\frac{v}{n}c}{c^2}}=\frac{c-vn}{n-\frac{v}{c}}$

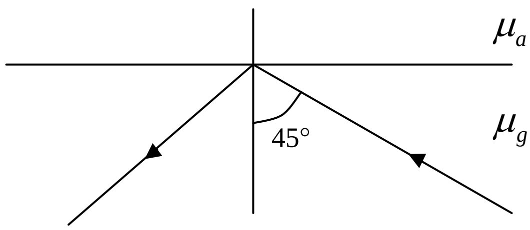
Q3. A right-angled prism is placed in air (the refractive index of air is 1) and a light beam is incident perpendicular to the base of the prism as shown in the figure. In order to get the light beam totally reflected, the minimum value of the refractive index of the prism should be



(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\pi}{2}$ (d) $\sqrt{2}$

Ans: 3. (d)

Solution:



$\mu_g \sin 45^\circ = \mu_a \sin 90^\circ$

$\mu_g \cdot \frac{1}{\sqrt{2}} = 1$

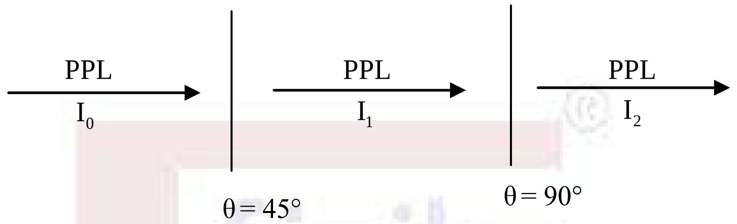
$\mu_g = \sqrt{2}$

Q9. If linearly polarized light is sent through two polarizers, the first at 45^0 to the original axis of polarization and the second at 90^0 to the original axis of polarization, what fraction of the original intensity passes through the last polarizer?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{1}{8}$

Ans: 9. (b)

Solution:



$$I_1 = I_0 \cos^2 45° = \frac{I_0}{2};\ I_2 = I_1 \cos^2 45° = \frac{I_0}{2} \cdot \frac{1}{2} = \frac{I_0}{4}$$

Section B

Correct answer: +9, wrong answer: −3.

Q31. Two linear polarizers are placed coaxially with the transmission axis of the first polarizer in the vertical orientation and the second polarizer in the horizontal orientation. A half waveplate placed coaxially between these crossed polarizers is rotating about its axis at an angular frequency ω. At $t = 0$, the fast axis of the half waveplate was oriented vertically. A beam of unpolarized light of intensity I_0 is incident along the axis of this optical system. The output intensity measured by a detector after the beam passes through this optical system is

(a) $\frac{I_0}{4}\left[1+\cos(\omega t)\right]$ (b) $\frac{I_0}{4}\left[1-\cos(2\omega t)\right]$

(c) $\frac{I_0}{4}\left[1-\cos(4\omega t)\right]$ (d) $\frac{I_0}{2}\left[1-\cos(\omega t)\right]$

Ans: 31. (c)

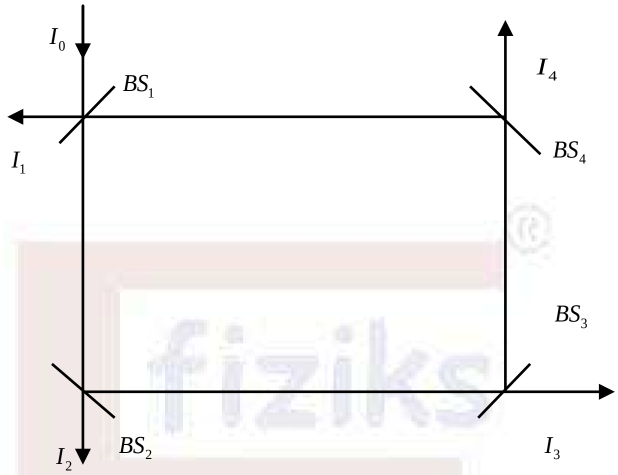
Solution:

$$I = \frac{I_0}{2}\sin^2 2\theta \sin^2 \frac{\delta}{2}$$

$\delta = \pi$ for HWP

$$I = \frac{I_0}{2}\sin^2 2\theta = \frac{I_0}{4}(1-\cos 4\theta) = \frac{I_0}{4}\left[1-\cos(4\omega t)\right]$$

Q41. A beam of light of intensity $I_0 = 120 W/m^2$ is incident on the optical system shown in the figure. BS_1, BS_2, BS_3, and BS_4 are ideal, 50/50 beamsplitters (reflects 50% and transmits 50% of incident light intensity). I_1, I_2, I_3, and I_4 are the total intensities measured by detectors. What will be the intensity I_3?



(a) 32 W/m^2 (b) 20 W/m^2

(c) 16 W/m^2 (d) 12 W/m^2

Ans: 41. (c)

Solution:

$$I_3 = \frac{I_0}{2^3} + \frac{1}{2^3}\left(\frac{I_0}{2^4}\right) + \frac{1}{2^3}\left(\frac{I_0}{2^8}\right) + \ldots = \frac{I_0}{2^3}\left[1 + \frac{1}{2^4} + \frac{1}{2^8} + \ldots\right]$$

$$I_3 = \frac{120}{8} \times \frac{1}{1 - \frac{1}{2^4}} = 16\, \text{W/m}^2$$

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