ALL INDIA TEST SERIES

FOR

CSIR - JRF (PHYSICS) June - 2024

Full Length Test – 01

PHYSICAL SCIENCES

TIME: 3 HOURS **MAXIMUM MARKS: 200**

Part 'A' This part shall carry 20 questions pertaining to *General Aptitude with emphasis, On logical reasoning, graphical, analysis, analytical and numerical ability, quantitative comparison, series formation, puzzles etc.* The candidates shall be required to answer any 15 questions. Each question shall be of two marks. The total marks allocated to this section shall be 30 out of 200.

Part ' B' This part shall contain 25 Multiple Choice Questions (MCQs) generally covering the topics given in the Part 'A' (CORE) of syllabus. All questions are compulsory. Each question shall be of 3.5 Marks. The total marks allocated to this section shall be 70 out of 200.

Part 'C' This part shall contain 30 questions from Part 'B'(Advanced) that are designed to test a candidate's knowledge of scientific concepts and/or application of the scientific concepts. The questions shall be of analytical nature where a candidate is expected to apply the scientific knowledge to arrive at the solution to the given scientific problem. A candidate shall be required to answer any 20. Each question shall be of 5 Marks. The total marks allocated to this section shall be 100 out of 200.

There will be negative marking @25% for each wrong answer.

PART A

ANSWER ANY 15 QUESTONS

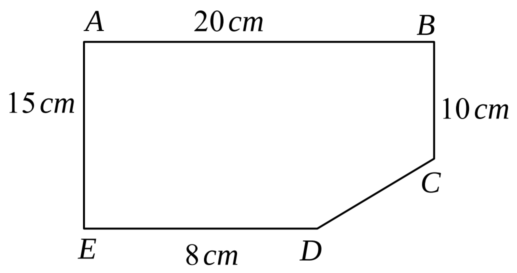
Q1. How many times would the digit 6 be used in numbering a book having 999 pages?

(a) 250 (b) 300 (c) 325 (d) 350

Q2. John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange (in rupees) is

(a) 2 (b) 3 (c) 4 (d) 5

Q3. Find the area of the figure shown below if



$AB \parallel ED$ and $AE \parallel BC$

(a) $250\,cm^2$ (b) $270\,cm^2$ (c) $280\,cm^2$ (d) $300\,cm^2$

Q4. The HCF and LCM of two numbers are 33 and 264 respectively. When one of the numbers is divided by 2, the quotient is 33. The other number is

(a) 126 (b) 130 (c) 132 (d) 136

Q5. A grocer has a sale of Rs. 6435, Rs. 6927, Rs. 7230, Rs. 6855 and Rs. 6562 for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs. 6500 per month.

(a) 4788 (b) 3275 (c) 3896 (d) 4991

Q6. Out of 40 consecutive integers, two are chosen at random. Find the probability that their sum is odd.

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{20}{39}$ (d) $\frac{21}{40}$

Q7. The ratio of first and second class fares between two stations is $6:4$ and the number of passengers traveling by first and second class is in the ratio $1:30$. If Rs. 2100 is collected as total fare, what is the amount collected from first class passengers?

(a) 100 (b) 150 (c) 200 (d) 150

Q8. In an exam 43% passed in Math, 52% passed in physics and 52% passed in chemistry. Only 8% passed in all the three subjects. 14% passed in Math and physics, 21% passed in Math and chemistry and 20% passed in physics and chemistry. The number of students who took the exam is 200. The number of students passing in math only is

(a) 28 (b) 30 (c) 32 (d) 36

Q9. How many distinct eight-lettered words can be formed out of the word PROWLING such that each word starts with R and ends with W?

(a) 80 (b) 100 (c) 110 (d) 120

Q10. A VCR is sold at a profit of 20%. If the cost price and selling price each are reduced by Rs.1000 there is an increase in profit by $\frac{5}{3}\%$. The cost price of VCR (in rupees) is

(a) 10000 (b) 11000 (c) 12000 (d) 13000

Q11. Ravi takes 6 hours to type 32 pages on a computer. While kumar takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?

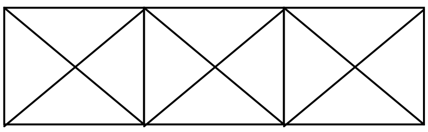
(a) 7 hour (b) 8 hour

(c) 8 hour 15 minutes (d) 7 hour 15 minutes

Q12. In covering a distance of $30\,km$, Abhay takes 2 hours more than Sameer. If Abhay doubles his speed, then he would take 1 hour less than Sameer. Abhay's speed is

(a) $3\,km/hr$ (b) $5\,km/hr$ (c) $6\,km/hr$ (d) $8\,km/hr$

Q13. How many distinct triangles are there in figure shown below?

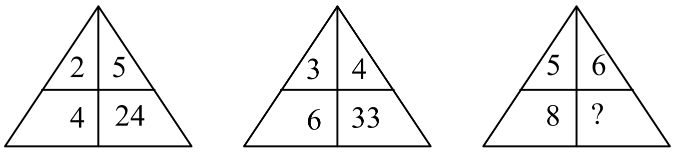


(a) 26 (b) 28 (c) 30 (d) 32

Q14. In the xy plane the straight line joining $(1,2)$ and $(2,-2)$ is perpendicular to the line joining $(8,2)$ and $(4,p)$. The value of p is

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 2

Q15. Which number will come at the missing place $(?)$.



(a) 30 (b) 66 (c) 73 (d) 87

Q16. Given below are for orientations of a dice. How many dots are there on the dice face opposite the one with three dots?

(a) 2 (b) 1 (c) 5 (d) 6

Q17. Given below is a statement and two conclusions based on it.

Statement: Some ants are parrots. All the parrots are apples.

Conclusions:

1. All apples are parrots
2. Some ants are apples

Which of the conclusions definitely follows from the given statement.

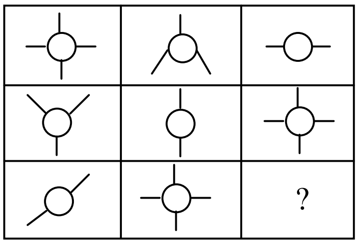
(a) Only 1 (b) Only 2

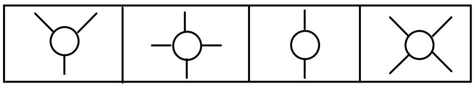
(c) Both 1 and 2 (d) Neither 1 nor 2

Q18. A 10 meter long flag is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flag are 60^0 and 45^0 respectively. Find the height of the tower.

(a) $5\left(\sqrt{3}+1\right)m$ (b) $5\left(\sqrt{3}+3\right)m$ (c) $10\left(\sqrt{3}-1\right)m$ (d) $10\left(\sqrt{3}+1\right)m$

Q19. Select a suitable figure from the four alternatives that would complete the figure matrix.





(a) (b) (c) (d)

Q20. A watch which gains 5 seconds in 3 minutes was set right at 7 *AM*. In the afternoon of the same day, when the watch indicated 4.15 *PM*, the true time is

(a) 3.15 *PM* (b) 3.30 *PM* (c) 3.45 *PM* (d) 4.00 *PM*

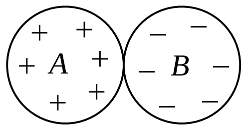
PART B

ANSWER ANY 20 QUESTONS

Q21. The wave function of a particle at a certain time is $\psi(x) = Ae^{-\mu|x|}$. Normalized $\psi(x)$ in momentum representation is given as (using k = p/ħ)

(a) $\frac{1}{\sqrt{2\pi\hbar}}\frac{2\mu^{3/2}}{\mu^2+k^2}$ (b) $\frac{1}{\sqrt{2\pi\hbar}}\frac{2\mu}{\mu^2+k^2}$ (c) $\frac{1}{\sqrt{2\pi\hbar}}\frac{2\mu^{1/2}}{\mu^2+k^2}$ (d) $\frac{1}{\sqrt{2\pi\hbar}}\frac{2}{\mu^2+k^2}$

Q22. Two uniformly charged insulating solid spheres A and B, both of radius a, carry total charges $+Q$ and $-Q$, respectively. The spheres are placed touching each other as shown in the figure.



If the potential at the centre of the sphere A is V_A and that at the centre of B is V_B then the difference $V_B - V_A$ is

(a) $\frac{Q}{4\pi\varepsilon_0 a}$ (b) $\frac{-Q}{2\pi\varepsilon_0 a}$ (c) $\frac{Q}{2\pi\varepsilon_0 a}$ (d) $\frac{-Q}{4\pi\varepsilon_0 a}$

Q23. If the Hamiltonian of a dynamical system in two dimensions is $H = \frac{1}{m}p_x p_y - \frac{1}{2m}p_y^2$, then its Lagrangian is

(a) $L = -\frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$ (b) $L = \frac{1}{2}m\dot{x}^2 + m\dot{x}\dot{y}$

(c) $L = -\frac{1}{2}m\dot{y}^2 + m\dot{x}\dot{y}$ (d) $L = \frac{1}{2}m\dot{y}^2 + m\dot{x}\dot{y}$

Q24. A particle of mass m is in potential

$$V(x) = \begin{cases} \infty & x < 0 \\ \frac{-32\hbar^2}{ma^2} & 0 \le x \le a \\ 0 & x > 0 \end{cases}$$

How many bound state are possible?

(a) 2 (b) 3 (c) 4 (d) 5

Q25. A solid cylinder of radius R has total charge Q distributed uniformly over its volume and uniform mass density has total mass M. It is rotating about its axis with angular speed ω. If its angular momentum is L and magnetic moment is μ, then the ratio $\frac{\mu}{L}$ is

(a) $\frac{Q}{M}$ (b) $\frac{Q}{2M}$ (c) $\frac{Q}{3M}$ (d) $\frac{Q}{8M}$

Q26. For central force potential $V = kr^4$, if T is kinetic energy then average value of total energy E is given by

(a) $\langle E\rangle = 2\langle V\rangle$ (b) $\langle E\rangle = 3\langle V\rangle$ (c) $\langle E\rangle = 4\langle V\rangle$ (d) $\langle E\rangle = 5\langle V\rangle$

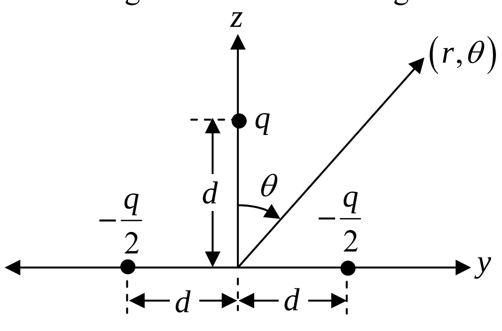
Q27. The potential is given by $\begin{cases} V(x) = \infty, x < 0 \\ = \frac{1}{2}m\omega^2 x^2, x > 0 \end{cases}$, If a particle of mass m interacts with this potential then what is the expectation value of energy on state

$$\psi = \sqrt{\frac{2}{3}}\phi_g + \frac{1}{\sqrt{3}}\phi_{1st}$$

where φ_g is ground state and φ_{ist} is the first excited state?

(a) $\frac{4\hbar\omega}{3}$ (b) $\frac{11\hbar\omega}{6}$ (c) $\frac{5\hbar\omega}{3}$ (d) $\frac{13\hbar\omega}{6}$

Q28. Consider a system of three charges as shown in the figure below:



For $r = 10\ m; \theta = 60$ degrees; $q = 10^{-6}$ Coulomb, and $d = 10^{-3}\ m$, the electric dipole potential in volts (rounded off to three decimal places) at a point (r,θ) is

(a) 15 mV (b) 30 mV (c) 45 mV (d) 60 mV

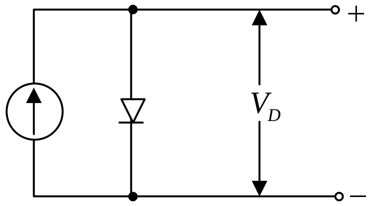
Q29. In the figure, silicon diode is carrying a constant current of $1\,mA$. When the temperature of the diode is $20^0 C, V_D$ is found to be $700\,mV$. If the temperature rises to $40^0 C, V_D$ becomes approximately equal to

(a) $740\,mV$

(b) $660\,mV$

(c) $680\,mV$

(d) $700\,mV$



Q30. The value of $\int_{-\pi/2}^{\pi/2} \cos x\,\delta(\sin x)\,dx + \int_{-2\pi}^{2\pi} e^x \sin x\,\delta'(x-\pi)\,dx$ is.

(a) $1+e^{-\pi}$ (b) $1-e^{\pi}$ (c) $1+e^{\pi}$ (d) $1-e^{-\pi}$

Q31. A particle of mass m moves under a potential energy $V(x) = \dfrac{cx}{(x^2+a^2)}$ where c and a are positive constants then which of following is correct?

(a) Point $x = a$ and $x = -a$ both are stable equilibrium points.

(b) Point $x = a$ and $x = -a$ both are unstable equilibrium points.

(c) Point $x = a$ is stable equilibrium point and $x = -a$ is unstable equilibrium point.

(d) Point $x = a$ is unstable equilibrium point and $x = -a$ is stable equilibrium point

Q32. Which of the following represents generating function $g(x,t)$ of Bessel's differential equation?

(a) $e^{-(x/2)(t-1/t)}$ (b) $e^{-(x/2)(t-1/t)^2}$ (c) $e^{(x/2)^2(t-1/t)^2}$ (d) $e^{(x/2)(t-1/t)}$

Q33. For a BJT, the common-base current gain $\alpha = 0.98$ and the collector base junction reverse bias saturation current $I_{CO} = 0.6\,\mu A$. This BJT is connected in the common emitter mode and operated in the active region with a base drive current $I_B = 20\,\mu A$. The collector current I_C for this mode of operation is

(a) $0.98\,mA$ (b) $0.99\,mA$ (c) $1.0\,mA$ (d) $1.01\,mA$

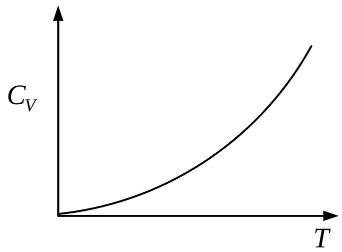
Q34. A particle moves in the one-dimensional potential $V(x)=\alpha x^6$, where $a>0$ is a constant. If the total energy of the particle is E, its frequency in a periodic motion is proportional to

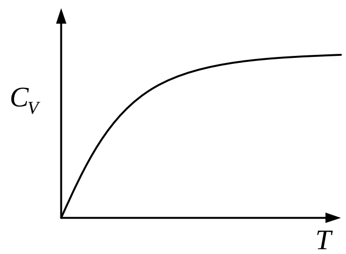
(a) $E^{-1/3}$ (b) $E^{-1/2}$ (c) $E^{1/3}$ (d) $E^{1/2}$

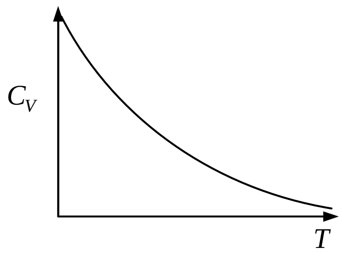
Q35. A circular loop made of a thin wire has radius $2\,cm$ and resistance $2\,\Omega$. It is placed perpendicular to a uniform magnetic field of magnitude $\left|\vec{B}_0\right|=0.01$ Tesla. At time $t=0$ the field starts decaying as $\vec{B}=\vec{B}_0 e^{-t/t_0}$, where $t_0=1s$. The total charge that passes through a cross section of the wire during the decay is Q. The value of Q in μC is:

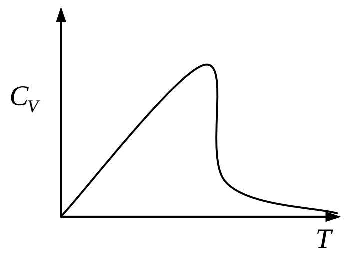
(a) 6.3 (b) 4.3 (c) 2.3 (d) 1.3

Q36. Partition function for a gas of photons is given as, $\ln Z=\dfrac{\pi^2 V\left(k_B T\right)^3}{45\hbar^3 C^3}$. The specific heat of the photon gas varies with temperature as

(a)


(b)


(c)


(d)


Q37. Let $w(t)$ be the wronskian of two linearly independent solution of the differential equation $\dfrac{d^2y}{dt^2}+\sin t\dfrac{dy}{dt}+y=0$. If $w(0)=e$ then the value of $w(2\pi)$ is

(a) $\dfrac{1}{e}$ (b) e (c) e^2 (d) $\dfrac{1}{e^2}$

Q38. The general expression for the molar entropy of an ideal monatomic gas?

(a) $S = nR\ln\left[\dfrac{RTe^{5/2}}{\Lambda^2 N_A P}\right]$ (b) $S = nR\ln\left[\dfrac{RTe^{5/2}}{\Lambda^1 N_A P}\right]$

(c) $S = nR\ln\left[\dfrac{RTe^{5/2}}{\Lambda^{-1} N_A P}\right]$ (d) $S = nR\ln\left[\dfrac{RTe^{5/2}}{\Lambda^3 N_A P}\right]$

Where $\Lambda^3 = \left(\dfrac{h^2}{2\pi mkT}\right)^{3/2}$

Q39. A monochromatic and linearly polarized light is used in a Young's double slit experiment. A linear polarizer, whose pass axis is at an angle 45^0 to the polarization of the incident wave, is placed in front of one of the slits. If $I_{\max}$ and $I_{\min}$, respectively, denote the maximum and minimum intensities of the interference pattern on the screen, the ratio $\dfrac{I_{\max}}{I_{\min}}$, is

(a) $\dfrac{\sqrt{2}}{3}$ (b) $\dfrac{2}{3}$ (c) 5 (d) $\dfrac{1}{5}$

Q40. Consider three inertial frames of reference A, B and C. Frame B moves with a velocity $\dfrac{c}{2}$ with respect to A, and C moves with a velocity $\dfrac{c}{10}$ with respect to B in the opposite direction. The velocity of C as measured in A is

(a) $\dfrac{3c}{7}$ (b) $\dfrac{4c}{7}$ (c) $\dfrac{c}{7}$ (d) $\dfrac{8}{19}c$

Q41. Imagine tossing a coin 50 times. What are the probabilities of observing heads 25 times (i.e., 25 successful experiments) and just 10 times?

(a) $\left(\dfrac{50!}{(10!)(40!)}\right)\left(\dfrac{1}{2}\right)^{10}\left(\dfrac{1}{2}\right)^{40}$ (b) $\left(\dfrac{50!}{(10!)(40!)}\right)\left(\dfrac{1}{2}\right)^{40}\left(\dfrac{1}{2}\right)^{10}$

(c) $\left(\dfrac{1}{2}\right)^{10}\left(\dfrac{1}{2}\right)^{40}$ (d) $50!$

Q42. The inverse Laplace transform of the function $H(s)=\ln\frac{s-a}{s-b}+\int_s^\infty \frac{1}{(s-a)}ds$ is (The Laplace transform of a function $f(t)$ is $F(s)=\int_0^\infty e^{-as}f(t)dt$).

(a) $\frac{e^{at}}{t}$ (b) $\frac{e^{bt}}{t}$ (c) $\frac{e^{at}+e^{bt}}{t}$ (d) $\frac{e^{at}-e^{bt}}{t}$

Q43. Determine the heat capacity for an ensemble consisting of units that have only two energy levels separated by an arbitrary amount of energy, hv.

(a) $\frac{Nk\beta^2(hv)^2 e\beta hv}{\left(e^{\beta hv}-1\right)^2}$ (b) $\frac{Nk\beta^2(hv)^2 e\beta hv}{\left(e^{2\beta hv}+1\right)^2}$

(c) $\frac{Nk\beta^2(hv)^2 e\beta hv}{\left(e^{\beta hv}+1\right)^2}$ (d) $\frac{Nk\beta^2(hv)^2 e\beta hv}{\left(e^{\beta hv}+1\right)}$

Q44. Given that matrix $A=\begin{bmatrix}0 & 1 & 1\\ -1 & 0 & 1\\ -1 & -1 & 0\end{bmatrix}$, pick out the **INCORRECT** statement.

(a) The determinant of the matrix A is 0.

(b) The matrix A^{18} is symmetric.

(c) The eigenvalues of the matrix are $0,\sqrt{2}\,i$ and $-\sqrt{2}i$ where $i^2=-1$.

(d) $\left(A^T\right)^2-A^2$ is a null matrix, where A^T denotes the transpose of matrix A.

Q45. The present output Q_n of an edge trigged JK flip-flop is logic 0, if $J=1$, then Q_{n+1}

(a) can not be determined (b) will be logic 0

(c) will be logic 1 (d) will race around

PART C

ANSWER ANY 20 QUESTONS

Q46. Consider a particle of mass m moving in three dimensional potential

$$V(x,y,z)=\begin{cases}2m\omega^2 z^2, & 0<x<a,\ 0<y<a,\quad 0<z<\infty \\ \infty, & \text{otherwise}\end{cases}$$

The ground state energy is given by

(a) $\frac{\pi^2\hbar^2}{2ma^2}+\frac{3}{2}\hbar\omega$ (b) $\frac{\pi^2\hbar^2}{2ma^2}+3\hbar\omega$ (c) $\frac{\pi^2\hbar^2}{ma^2}+\frac{3}{2}\hbar\omega$ (d) $\frac{\pi^2\hbar^2}{ma^2}+3\hbar\omega$

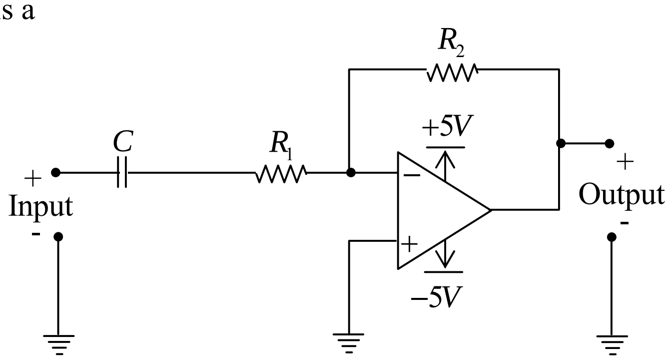
Q47. An electromagnetic wave is travelling in free space (of permittivity ε_0) with electric field

$$\vec{E}=\cos k\left(x-ct\right)\hat{z}\ \ V/m$$

The average power (per unit area) crossing planes parallel to $4x+3y=0$ will be

(a) 1.1 mV/m (b) 2.1 mV/m (c) 3.1 mV/m (d) 4.1 mV/m

Q48. The circuit shown is a



(a) Low pass filter with $f_{3dB}=\frac{1}{\left(R_1+R_2\right)C}rad/s$

(b) High pass with $f_{3dB}=\frac{1}{R_1C}rad/s$

(c) Low with pass filter $f_{3dB}=\frac{1}{R_2C}rad/s$

(d) High pass filter with $f_{3dB}=\frac{1}{\left(R_1+R_2\right)C}rad/s$

Q49. The dominant interactions underlying the following process

A. $\tau^- + \tau^+ \to K^- + K^+$

B. $\overline{K^0} + p \to \pi^+ + \Lambda^0$

C. $K^+ \to \pi^+ + \pi^0$

(a) A: Weak; B: Strong; C: Electromagnetic

(b) A: Weak; B: Strong; C: Weak

(c) A: Electromagnetic; B: Weak; C: Weak

(d) A: Electromagnetic; B: Strong; C: Weak

Q50. A particle of mass m moves in one dimension in the periodic potential

$$V(x) = V_0 \cos\left(\frac{2\pi x}{\alpha}\right)$$

We know that the energy eigenstates can be divided into classes characterized by an angle θ with wave functions $\phi(x)$ that obey $\phi(x+a) = e^{i\theta}\phi(x)$ for all x. For the class $\theta = \pi$, this becomes $\phi(x+a) = -\phi(x)$ (antiperiodic over length a). Even when $V_0 = 0$, we can still classify eigenstates by θ. When V_0 is small (i.e. $V_0 << \hbar^2 / ma^2$), calculate the energy eigenvalues for $\theta = \pi$ by first order perturbation theory for ground state .

(a) $E_1 = \frac{\hbar^2\pi^2}{2ma^2}$, $E_2 = \frac{\hbar^2\pi^2}{2ma^2} + \frac{V_0}{2}$

(b) $E_1 = \frac{\hbar^2\pi^2}{2ma^2}$, $E_2 = \frac{\hbar^2\pi^2}{2ma^2} + V_0$

(c) $E_1 = \frac{\hbar^2\pi^2}{2ma^2} - \frac{V_0}{2}$, $E_2 = \frac{\hbar^2\pi^2}{2ma^2} + \frac{V_0}{2}$

(d) $E_1 = \frac{\hbar^2\pi^2}{2ma^2} - V_0$, $E_2 = \frac{\hbar^2\pi^2}{2ma^2} + V_0$

Q51. The viscosity η of a liquid is given by poiseuille's formula $\eta = \frac{\pi P a^4}{8lV}$. Assume that l and V can be measured very accurately, but the pressure P has an rms error of 2% and the radius a has an independent rms error of 4%. The rms error of the viscosity is closet to

(a) 10% (b) 12% (c) 14% (d) 16%

Q52. The frame S' is moving with respect to frame S with speed v in positive x direction. If a'_x is x component of acceleration of particle A with respect to S' frame then the acceleration of A with respect to frame S is given by

(a) $a_x = \dfrac{a'_x\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{1+\dfrac{u'_x v}{c^2}}$

(b) $a_x = \dfrac{a'_x\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{\left(1+\dfrac{u'_x v}{c^2}\right)^2}$

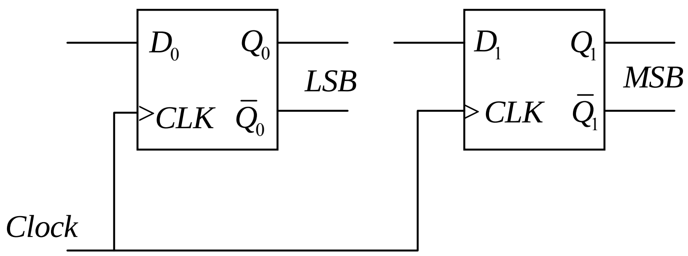
(c) $a_x = \dfrac{a'_x\left(1-\dfrac{v^2}{c^2}\right)^{3/2}}{\left(1+\dfrac{u'_x v}{c^2}\right)^2}$

(d) $a_x = \dfrac{a'_x\left(1-\dfrac{v^2}{c^2}\right)^{3/2}}{\left(1+\dfrac{u'_x v}{c^2}\right)^3}$

Q53. The spectral line $\lambda = 4878\,\mathring{A}$ of Ca is produced in a transition between 4^1F_3 upper state and a 3^1D_2 lower state. In the presence of external magnetic field, each time levels split into Zeeman levels. If $B = 1\text{T}$ the Zeeman separation is

(a) $0\cdot 11\,\mathring{A}$ (b) $0\cdot 01\,\mathring{A}$ (c) $0\cdot 01\,\mathring{A}$ (d) $0\cdot 001\,\mathring{A}$

Q54. Two D-flip-flop, as shown below are to be connected as a synchronous counter that goes through the following Q_1Q_0 sequence $00 \to 01 \to 11 \to 10 \to 00 \to$ ...... the input D_0 and D_1 respectively should be connected as



(a) $\bar{Q}_1$ and Q_0 (b) $\bar{Q}_0$ and Q_1

(c) $\bar{Q}_1 Q_0$ and $\bar{Q}_1 Q_0$ (d) $\bar{Q}_1\bar{Q}_0$ and Q_1Q_0

Q55. The ground state energy of the attractive delta function potential

$$V(x) = -b\delta\left(x - \frac{a}{2}\right),$$

where $b > 0$, calculated with the variational trial function

$$\psi(x) = \begin{Bmatrix} A\sin\frac{\pi x}{a}, & \text{for} & 0 < x < a, \\ 0, & & \text{otherwise}, \end{Bmatrix} \text{ is}$$

(a) $-\frac{mb^2}{\pi^2\hbar^2}$ (b) $-\frac{2mb^2}{\pi^2\hbar^2}$ (c) $-\frac{mb^2}{2\pi^2\hbar^2}$ (d) $-\frac{mb^2}{4\pi^2\hbar^2}$

Q56. In the region far from a source, the time dependent electric field at a point (r,θ,ϕ) is

$$\vec{E}(r,\theta,\phi) = \hat{\phi}\frac{E_0}{\sqrt{2}}\omega^2\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t - \frac{r}{c}\right)\right]$$

where ω is angular frequency of the source. The total power radiated (averaged over a cycle) is

(a) $\frac{2\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (b) $\frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (c) $\frac{4}{3\pi}\frac{E_0^2\omega^4}{\mu_0 c}$ (d) $\frac{2}{3}\frac{E_0^2\omega^4}{\mu_0 c}$

Q57. The ground state of $^{125}_{451}Sb$ nucleus has spin-parity $\left(\frac{7}{2}\right)^+$ while the first excited state has spin – parity $\left(\frac{9}{2}\right)^+$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited to ground states is

(a) $E1$ (b) $M1$ (c) $M2$ (d) $E3$

Q58. The value of the integral $\int_0^\infty \frac{\ln x^{-2}}{\left(x^2+1\right)^2}dx$ is

(a) 0 (b) $\frac{-\pi}{4}$ (c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

Q59. The differential scattering cross-section $D(\theta)=\frac{d\sigma}{d\Omega}$ for the central potential $V(r)=\frac{\beta}{r}e^{-\mu r}$, where β and μ are positive constants, is calculated in the first Born approximation. If θ is scattering angle, then differential scattering cross section is given by

(a) $D(\theta)=\left(\frac{2m\beta}{\left(\mu^2\hbar^2+2mE\sin^2\theta\right)}\right)^2$

(b) $D(\theta)=\left(\frac{2m\beta}{\hbar^2\left(\mu^2+2mE\sin^2\theta\right)}\right)^2$

(c) $D(\theta)=\left(\frac{2m\beta}{\hbar^2\left(\mu^2+2mE\sin^2\frac{\theta}{2}\right)}\right)^2$

(d) $D(\theta)=\left(\frac{2m\beta}{\left(\mu^2\hbar^2+2mE\sin^2\frac{\theta}{2}\right)}\right)^2$

Q60. A box of volume, $V=L^3$, contains an ideal gas of N identical atoms, each of which has spin, $s=1/2$ and magnetic moment, μ. A magnetic field, B, is applied to they system, the magnetization of the system is given by?

(a) $N\mu\tanh\left[(\beta\mu B)/2\right]$

(b) $(1/2)\mu\tanh\left[(\beta\mu B)/2\right]$

(c) $\mu\tanh\left[(\beta\mu B)/2\right]$

(d) $(1/2)N\mu\tanh\left[(\beta\mu B)/2\right]$

Q61. The cut-off wavelength of TE_{10} and TE_{20} mode in the hollow rectangular waveguide of dimensions a = 3 cm, b = 1.5 cm are respectively

(a) $3\,cm$ and $6\,cm$

(b) $6\,cm$ and $3\,cm$

(c) $4\,cm$ and $3\,cm$

(d) $2\,cm$ and $3\,cm$

Q62. According to the shell model the spin and parity of two nuclei $^{49}_{24}Cr$ and $^{49}_{25}Mn$ are respectively

(a) $\left(\frac{7}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$

(b) $\left(\frac{7}{2}\right)^-$ and $\left(\frac{5}{2}\right)^+$

(c) $\left(\frac{7}{2}\right)^-$ and $\left(\frac{7}{2}\right)^-$

(d) $\left(\frac{5}{2}\right)^-$ and $\left(\frac{7}{2}\right)^+$

Q63. If the Hamiltonian of a system is $H = \frac{p^2 \exp(-2\gamma t)}{2m} + \frac{1}{2} m\omega^2 q^2 \exp(2\gamma t)$ and F_2 type generating function is given by $F_2 = \exp(\gamma t) qP$ then the solution of equation of motion with boundary condition $Q(t=0)=0$ is given by

(a) $Q = A\sin\left(\omega^2 - \gamma^2\right)t$ (b) $Q = A\sin\sqrt{\left(\omega^2 - \gamma^2\right)}t$

(c) $Q = A\sin\left(\omega^2 + \gamma^2\right)t$ (d) $Q = A\sin\sqrt{\left(\omega^2 + \gamma^2\right)}t$

Q64. Suppose the frequency of phonons in a two – dimensional chain of atoms is proportional to the wave vector. If n is the number density of atoms and υ_s is the speed of the phonons, the Debye frequency is

(a) $\upsilon_s (2\pi n)^{1/2}$ (b) $\upsilon_s \left(\frac{n}{2\pi}\right)^{1/2}$ (c) $\upsilon_s \left(\frac{n}{\pi}\right)^{1/2}$ (d) $\upsilon_s (\pi n)^{1/2}$

Q65. For the case of Boundary at infinity

$\left(\frac{d^2}{dx^2} + k^2\right)\psi(x) = g(x)$. The green's function for this differential equation is

(a) $G(x,x') = \frac{-i}{2k}\exp\left(i|x-x'|\right)$ (b) $G(x,x') = \frac{i}{2k}\exp\left(i|x-x'|\right)$

(c) $G(x,x') = \frac{-i}{2k}\exp\left(i|x+x'|\right)$ (d) $G(x,x') = \frac{-i}{2}\exp\left(i|x-x'|\right)$

Q66. If the co-efficient of simulated emissions for transitions is $6\cdot 5\times 10^{19} m^2 W^{-1} s^{-1}$ and the emitted photon is at wavelength $4000\,\overset{0}{\text{A}}$, then the lifetime of the excited state is approximately

(a) $59\,ns$ (b) $45\,ns$ (c) $30\,ns$ (d) $10\,ns$

Q67. The points where the series solution of the LDE

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0 \text{ will diverge are located at}$$

(a) 0 and 1 (b) 0 and -1 (c) -1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

Q68. If the leading anharmonic correction to the energy of n^{th} vibrational level of diatomic molecule is $x_e\left(n+\frac{1}{2}\right)^2\hbar\omega$ with $x_e = 0\cdot 004$, the total number of every levels positive is approximately.

(a) 50 (b) 95 (c) 125 (d) 176

Q69. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, (as measured by the moving observer) The current per uint length is given by

(a) 0 (b) $\frac{vQ}{L}$ (c) $\frac{vQ}{L}\sqrt{1-\frac{v^2}{c^2}}$ (d) $\frac{vQ}{L\sqrt{1-\frac{v^2}{c^2}}}$

Q70. One mole of an ideal gas at $300\,K$ is reversibly and isothermally compressed from a volume of $25.0\,L$ to a volume of $10.0\,L$. Because the water bath thermal reservoir in the surroundings is very large, T remains essentially constant at $300\,K$ during the process. The ΔS_{total} is given by

(a) $-2.285\times10^3\,J$ (b) 0 (c) -7.62 JK^{-1} (d) 7.62 JK^{-1}

Q71. The value of $I = \int_0^1 \frac{1}{1+x}dx$ using Simpson 1/3 rule with $h = 0.5$ is

(a) 0.69 (b) 0.59 (c) 0.49 (d) 0.39

Q72. The dispersion relation for the electrons is given by

$$E = E_0 - 2A\alpha\left(1-\frac{k^2a^2}{2}\right)e^{-\alpha a}$$

where E_0, A, α are constant and a is lattice constant. If ω_c is the cyclotron resonance frequency of the electrons in a magnetic field B, the value of A is

(a) $\frac{\hbar^2\omega_c}{2eBa^2e^{-\alpha a}}$ (b) $\frac{\hbar^2\omega_c}{2eBAa^2e^{-\alpha a}}$ (c) $\frac{\hbar^2\omega_c}{eBe^{-\alpha a}}$ (d) $\frac{2\hbar^2\omega_c}{eBAa^2e^{-\alpha a}}$

Q73. The electrons in graphene can be thought of as a two-dimensional gas with a linear energy-momentum relation $E=|\vec{p}|v$, where $\vec{p}=(p_x,p_y)$ and v is a constant. If ρ is the number of electrons per unit area, the energy per unit area is proportional to $\frac{E}{L^2}$

(a) $\frac{E}{L^2}=\frac{NE_{av}}{L^2}=\frac{2}{3}\sqrt{2\pi}\hbar v\rho^{3/2}$

(b) $\frac{E}{L^2}=\frac{NE_{av}}{L^2}=\frac{2}{3}\sqrt{2\pi}\hbar v\rho^{1/2}$

(c) $\frac{E}{L^2}=\frac{NE_{av}}{L^2}=\frac{2}{3}\sqrt{2\pi}\hbar v\rho^{2}$

(d) $\frac{E}{L^2}=\frac{NE_{av}}{L^2}=\frac{2}{3}\sqrt{2\pi}\hbar v\rho^{3}$

Q74. Potassium chloride (KCl) crystal is a face – centered cubic lattice with a basis consisting of K^+ and Cl^- ions separated by half the body diagonal of a unit cube. Which of the planes corresponding to the Miller indices given below will not give rise to Bragg's reflection of X -rays?

(a) (111) (b) (200) (c) (220) (d) (222)

Q75. A comet moves toward the sun with initial velocity v_0. The mass of the sun is M and radius is R. Find the total cross section σ for striking the sun. (Take Sun to be at rest and ignore all other bodies. Assume that Sun and comet interacts via kepler's potential with gravitational constant G.)

(a) $\pi R^2\left(1+\frac{2GM}{v_0^2R}\right)$

(b) $\pi R^2\left(1-\frac{2GM}{v_0^2R}\right)$

(c) $4\pi R^2\left(1+\frac{2GM}{v_0^2R}\right)$

(d) $4\pi R^2\left(1-\frac{2GM}{v_0^2R}\right)$
