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GATE Physics-2024

Solution-Mathematical Physics

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Q.11 – Q.35 Carry ONE mark Each

Q30. Consider a volume integral $I = \int_V \nabla^2 \left(\frac{1}{r}\right) dV$

over a volume V, where $r = \sqrt{x^2 + y^2 + z^2}$. Which of the following statement is/are correct?

(A) $I = -4\pi$, if $r = 0$ is inside the volume V

(B) Integrand vanishes for $r \neq 0$

(C) $I = 0$, if $r = 0$ is not inside the volume V

(D) Integrand diverges as $r \to \infty$

Ans: (A), (B), (C)

Solution:

$$\nabla^2\left(\frac{1}{r}\right) = \vec{\nabla}\cdot\left(\vec{\nabla}\frac{1}{r}\right) = \vec{\nabla}\cdot\left(\frac{-\hat{r}}{r^2}\right) = -4\pi\delta^3(\vec{r})$$

$$I = \int_V \nabla^2\left(\frac{1}{r}\right) dV = -\int_V 4\pi\delta^3(\vec{r}) dV \, ; \qquad \delta^3(\vec{r}) = \begin{cases} \infty, \vec{r} = 0 \\ 0, \vec{r} \neq 0 \end{cases}$$

$I = -4\pi$ if $r = 0$ is inside volume V.

$I = 0$ if $r = 0$ is not inside V.

$I = 0$ if $r \neq 0$

Q31. The complex function $e^{-\left(\frac{2}{z-1}\right)}$ has ________________

(A) a simple pole at $z = 1$

(B) an essential singularity at $z = 1$

(C) a residue equal to -2 at $z = 1$

(D) a branch point at $z = 1$

Ans: (B), (C)

Solution:

$$f(z) = e^{-\left(\frac{2}{z-1}\right)} = 1 - \frac{2}{z-1} + \frac{4}{(z-1)^2}\cdot\frac{1}{2!} - \frac{8}{(z-1)^3}\cdot\frac{1}{3!} + \ldots$$

Since there is infinite negative powers of $(z-1)$ so it has essential singularity at $z = 1$

Residue at $z = 1$ is $= (-2)$

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Q.36 – Q.65 Carry TWO marks Each

Q43. Consider two matrices: $P=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$ and $Q=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Which of the following statement is/are true?

(A) P and Q have same set of eigenvalues

(B) P and Q commute with each other

(C) P and Q have different sets of linearly independent eigenvectors

(D) P is diagonalizable

Ans: (A), (B), (C)

Solution:

Characteristic equation: $|P-\lambda I|=0\Rightarrow\begin{vmatrix}1-\lambda & 2\\0 & 1-\lambda\end{vmatrix}=0\Rightarrow(1-\lambda)^2=0\Rightarrow\lambda=1,1$

and $|Q-\lambda I|=0\Rightarrow\begin{vmatrix}1-\lambda & 0\\0 & 1\lambda\end{vmatrix}=0\Rightarrow(1-\lambda)^2\Rightarrow\lambda=1,1$

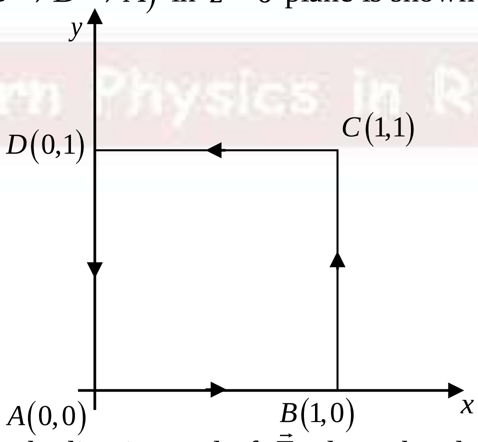
$\because\ Q=I$ so $PQ=P=QP$

$\because\ PX=1X\Rightarrow\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}x_1\\x_2\end{bmatrix}\Rightarrow x_1+2x_2=x_1\Rightarrow x_2=0$

$\Rightarrow X_1=\begin{bmatrix}x_1\\0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ and $X_1^TX_2=0\Rightarrow\begin{bmatrix}x_1 & 0\end{bmatrix}\begin{bmatrix}x_1\\0\end{bmatrix}=x_1^2=0\Rightarrow x_1=0,\ \ X_2=\begin{bmatrix}0\\0\end{bmatrix}$

$\because\ Q=I$ so any non-zero vector is an eigenvector of Q having eigenvalue.

Q45. Consider a vector field $\vec{F}=\left(2xz+3y^2\right)\hat{y}+4yz^2\hat{z}$. The closed path $(\Gamma:A\to B\to C\to D\to A)$ in $z=0$ plane is shown in figure.



$\oint_\Gamma \vec{F}\cdot\vec{dl}$ denotes the line integral of $\vec{F}$ along the closed path Γ. Which of the following option is/are true?

(A) $\oint_\Gamma \vec{F}\cdot d\vec{l}=0$ (B) $\vec{F}$ is non-conservative

(C) $\vec{\nabla}\cdot\vec{F}=0$ (D) $\vec{F}$ can be written as the gradient of a scalar field

Ans: (A), (B)

Solution:

$\vec{F}=\left(2xz+3y^2\right)\hat{y}+4yz^2\hat{z}$

$$\vec{\nabla}\cdot\vec{F}=\frac{\partial}{\partial x}(0)+\frac{\partial}{\partial y}\left(2xz+3y^2\right)+\frac{\partial}{\partial z}\left(4yz^2\right)=6y+8yz=6y\neq 0 \quad \because z=0$$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 2xz+3y^2 & 4yz^2\end{vmatrix}=\hat{x}\left[4z^2-2x\right]-\hat{y}\left[0-0\right]+\hat{z}\left[2z-0\right]=\left(4z^2-2x\right)\hat{x}+2z\hat{z}$$

$\Rightarrow \vec{\nabla}\times\vec{F}=-2x\hat{x}\neq 0 \quad \because z=0$ [$\vec{F}$ is non-conservative]

Now $\oint_\Gamma \vec{F}\cdot d\vec{l}=\int_S\left(\vec{\nabla}\times\vec{F}\right)\cdot d\vec{a}=\iint\left(-2x\hat{x}\right)\cdot\left(dxdy\hat{z}\right)=0$

Q51. A^α and $B_\beta\left(\alpha,\beta=1,2,3,...,n\right)$ are contravariant and covariant vectors, respectively. By convention, any repeated indices are summed over. Which of the following expression is/are tensors?

(A) $A^\alpha B_\beta$ (B) $\frac{A^\alpha B_\beta}{A^\alpha B_\alpha}$ (C) $\frac{A^\alpha}{B_\beta}$ (D) $A^\alpha+B_\beta$

Ans: (A), (B)

Q56. The Fourier transform and its inverse transform are respectively defined as $\tilde{f}(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x)e^{i\omega x}dx$ and $f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{f}(\omega)e^{-i\omega x}d\omega$. Consider two functions f and g. Another function $f*g$ is defined as

$$(f*g)(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(y)g(x-y)dy$$

Which of the following relation is/are true?

Note: Tilde (~) denote the Fourier transform.

(A) $f*g=g*f$ (B) $\widetilde{f*g}=\widetilde{g*f}$

(C) $\widetilde{f*g}=\widetilde{fg}$ (D) $\widetilde{f*g}=\tilde{f}\tilde{g}$

Ans: (A), (B), (D)

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Solution-Classical Mechanics

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Q.11 – Q.35 Carry ONE mark Each

Q11. If $F_1(Q,q)=Qq$ is the generating function of a canonical transformation from (p,q) to (P,Q), then which one of the following relations is correct?

(A) $\frac{p}{P}=\frac{Q}{q}$ (B) $\frac{P}{p}=\frac{Q}{q}$

(C) $\frac{p}{P}=-\frac{Q}{q}$ (D) $\frac{P}{p}=-\frac{Q}{q}$

Ans: (C)

Solution: $\because F_1=Qq$, so $p=\frac{\partial F_1}{\partial q}=Q$ ...(1) and $P=-\frac{\partial F_1}{\partial Q}=-q$ ...(2)

From equation (1) and (2) $\frac{p}{P}=-\frac{Q}{q}$

Q23. Let $\rho(\vec{p},\vec{q},t)$ be the phase space density of an ensemble of a system. The Hamiltonian of the system is $H(\vec{p},\vec{q})$. If $\{A,B\}$ denotes the Poisson bracket of A and B, then $\frac{d\rho}{dt}=0$ implies

(A) $\frac{\partial\rho}{\partial t}=0$ (B) $\frac{\partial\rho}{\partial t}\propto\{\rho,H\}$

(C) $\frac{\partial\rho}{\partial t}\propto\left\{\rho,\frac{\vec{p}\cdot\vec{q}}{2}\right\}$ (D) $\frac{\partial\rho}{\partial t}\propto\left\{\rho,\frac{\vec{q}\cdot\vec{q}}{2}\right\}$

Ans: (B)

Solution: The time evolution of a function $\rho(\vec{p},\vec{q},t)$: $\frac{d\rho}{dt}=\{\rho,H\}+\frac{\partial\rho}{\partial t}$

If $\frac{d\rho}{dt}=0$ then $\frac{\partial\rho}{\partial t}=-\{\rho,H\}$

Q25. An inertial observer sees two spacecrafts S and T flying away from each other along x-axis with individual speed $0.5c$, where c is the speed of light. The speed of T with respect to S is

(A) $\frac{4}{5}c$ (B) $\frac{4}{3}c$ (C) c (D) $\frac{2}{3}c$

Ans: (A)

Solution: $v_{TS}=\frac{(0.5c)-(-0.5c)}{1-\frac{(0.5c)(-0.5c)}{c^2}}=\frac{c}{1.25}=\frac{4}{5}c$

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Q.36 – Q.65 Carry TWO marks Each

Q38. Consider the Lagrangian $L = m\dot{x}\dot{y} - m\omega_0^2 xy$. If p_x and p_y denote the generalized momenta conjugate to x and y, respectively, then the canonical equations of motion are

(A) $\dot{x} = \frac{p_x}{m},\ \dot{p}_x = -m\omega_0^2 x,\ \dot{y} = \frac{p_y}{m},\ \dot{p}_y = -m\omega_0^2 y$

(B) $\dot{x} = \frac{p_x}{m},\ \dot{p}_x = m\omega_0^2 x,\ \dot{y} = \frac{p_y}{m},\ \dot{p}_y = m\omega_0^2 y$

(C) $\dot{x} = \frac{p_y}{m},\ \dot{p}_x = -m\omega_0^2 y,\ \dot{y} = \frac{p_x}{m},\ \dot{p}_y = -m\omega_0^2 x$

(D) $\dot{x} = \frac{p_y}{m},\ \dot{p}_x = m\omega_0^2 y,\ \dot{y} = \frac{p_x}{m},\ \dot{p}_y = m\omega_0^2 x$

Ans: (C)

Solution:

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{y} \Rightarrow \dot{y} = \frac{p_x}{m},\quad p_y = \frac{\partial L}{\partial \dot{y}} = m\dot{x} \Rightarrow \dot{x} = \frac{p_y}{m}$$

$$\dot{p}_x = \frac{\partial L}{\partial x} = -m\omega_0^2 y,\quad \dot{p}_y = \frac{\partial L}{\partial y} = -m\omega_0^2 x$$

Q41. The equation of motion for the forced simple harmonic oscillator is

$$\ddot{x}(t) + \omega^2 x(t) = F\cos(\omega t)$$

where $x(t=0)=0$ and $\dot{x}(t=0)=0$.Which one of the following options is correct?

(A) $x(t) \propto t\sin(\omega t)$

(B) $x(t) \propto t\cos(\omega t)$

(C) $x(t) = \infty$

(D) $x(t) \propto e^{\omega t}$

Ans: (A)

Solution:

(i) $x(t) = c_1 t\sin(\omega t)$:

At $t = 0 \to x(t=0) = 0$

(ii) $\dot{x}(t) = c_1\sin(\omega t) + c_1\omega t\cos(\omega t)$:

At $t = 0 \to \dot{x}(t=0) = 0$

Q60. Lagrangian of a particle of mass m is $L=\frac{1}{2}m\dot{x}^2-\lambda x^4$, where λ is a positive constant. If the particle oscillates with total energy E, then the time period of oscillations is $a\int_0^{\left(\frac{E}{\lambda}\right)^{\frac{1}{4}}}\frac{dx}{\sqrt{\left(\frac{2}{m}\right)\left(E-\lambda x^4\right)}}$. The value of a is ____________ (in integer).

Ans: 4

Solution: $\because \int_0^T dt = 4\int_0^{(E/\lambda)^{1/4}}\frac{dx}{v} \Rightarrow T = 4\int_0^{(E/\lambda)^{1/4}}\frac{dx}{\sqrt{\frac{2}{m}\left(E-\lambda x^4\right)}} \Rightarrow \alpha = 4 \qquad \because E=\frac{1}{2}mv^2+\lambda x^4$

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Solution-Electromagnetic Theory

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Q.11 – Q.35 Carry ONE mark Each

Q12. An unpolarized plane electromagnetic wave in a dielectric medium 1 is incident on a plane interface that separates medium 1 from another dielectric medium 2. Medium 1 and medium 2 have refractive indices n_1 and n_2, respectively, with $n_2 > n_1$. If the angle of incidence is $\tan^{-1}\left(\frac{n_2}{n_1}\right)$, which one of the following statements is true?

(A) The reflected wave is unpolarized

(B) The reflected wave is polarized parallel to the plane of incidence

(C) The reflected wave is polarized perpendicular to the plane of incidence

(D) There is no transmitted wave

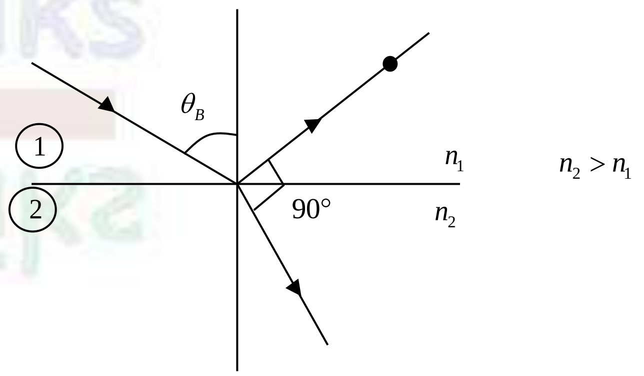
Ans: (C)

Solution:

At Brewster Angle

$$\theta = \theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

The reflected wave is polarized perpendicular to the plane of incidence.



Q20. An infinitely long cylinder of radius R carries a frozen-in magnetization $\vec{M} = ke^{-s}\hat{z}$, where k is a constant and s is the distance from the axis of cylinder. The magnetic permeability of free space is μ_0. There is no free current present anywhere. The magnetic flux density $\left(\vec{B}\right)$ inside the cylinder is

(A) 0

(B) $\mu_0 ke^{-R}\hat{z}$

(C) $\mu_0 ke^{-s}\hat{z}$

(D) $\mu_0 ke^{-s}\left(\frac{R}{s}\right)\hat{z}$

Ans: (C)

Solution:

$\because\ I_{\text{free}} = 0$ and there is cylindrical symmetry so $\oint \vec{H}\cdot d\vec{\ell} = I_{\text{free}}$

$$\vec{H} = 0 \Rightarrow \frac{\vec{B}}{\mu_0} - \vec{M} = 0 \Rightarrow \vec{B} = \mu_0\vec{M} \Rightarrow \vec{B} = \mu_0 ke^{-sz}\hat{z}$$

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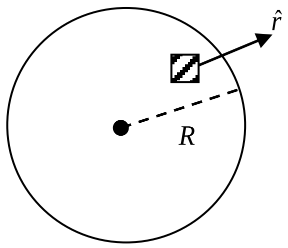
Q34. The electric field in a region depends only on x and y coordinates as

$$\vec{E} = k\frac{(x\hat{x}+y\hat{y})}{x^2+y^2}$$

where k is a constant. The flux of $\vec{E}$ through the surface of a sphere of radius R with its center at the origin is $n\pi kR$, where the value of n is ___________ (in integer).

Ans: 4

Solution:



$x = R\sin\theta\cos\phi,\ y = R\sin\theta\sin\phi,\ d\vec{a} = R^2\sin\theta d\theta d\phi\hat{r}$

$$\phi_E = \oint_S \vec{E}\cdot d\vec{a}$$

$$\vec{E}\cdot d\vec{a} = \frac{k(x\hat{x}+y\hat{y})}{x^2+y^2}\cdot\left(R^2\sin\theta d\theta d\phi\hat{r}\right) = k\left[\frac{x(\hat{x}\cdot\hat{r})+y(\hat{y}\cdot\hat{r})}{x^2+y^2}\right]R^2\sin\theta d\theta d\phi$$

$$= k\left[\frac{R\sin\theta\cos\phi(\sin\theta\cos\phi)+R\sin\theta\sin\phi(\sin\theta\sin\phi)}{R^2\sin^2\theta\left(\cos^2\phi+\sin^2\phi\right)}\right]R^2\sin\theta d\theta d\phi$$

$$= k\left[\frac{R\sin^2\theta\left(\cos^2\phi\right)+\sin^2\phi}{R^2\sin^2\theta}\right]R^2\sin\theta d\theta d\phi \qquad \Rightarrow \vec{E}\cdot d\vec{a} = kR\sin\theta d\theta d\phi$$

$$\Rightarrow \phi_E = \oint_S \vec{E}\cdot d\vec{a} = kR\int_0^{\pi}\int_0^{2\pi}\sin\theta d\theta d\phi = kR\times 2\times 2\pi = 4\pi kR \quad \Rightarrow n = 4$$

Q.36 – Q.65 Carry TWO marks Each

Q40. In a parallel plate capacitor, the plate at $x=0$ is grounded and the plate at $x=d$ is maintained at a potential V_0. The space between the two plates is filled with a linear dielectric of permittivity $\in=\in_0\left(1+\frac{x}{d}\right)$, where $\in_0$ is the permittivity of free space. Neglecting the edge effects, the electric field $\left(\vec{E}\right)$ inside the capacitor is

(A) $-\frac{V_0}{(d+x)\ln 2}\hat{x}$ (B) $-\frac{V_0}{d}\hat{x}$

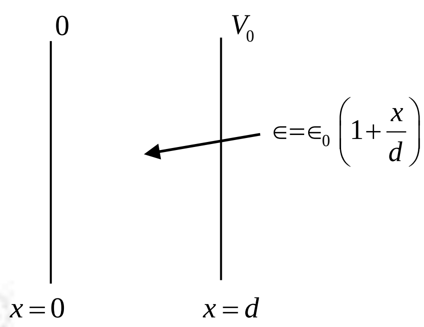
(C) $-\frac{V_0}{(d+x)}\hat{x}$ (D) $-\frac{V_0 d}{(d+x)x}\hat{x}$

Ans: (A)

Solution:

Let charge on capacitor plate is Q then

$$\vec{E} = \frac{Q}{\in_0 \in_r A}\hat{x} \Rightarrow \vec{E}\cdot d\vec{l} = \frac{\theta}{\in_0\left(1+\frac{x}{d}\right)A}dx$$



$$V_0 = -\int_0^d \vec{E}\cdot d\vec{l} = -\frac{Q}{\in_0 A}\int_0^d \frac{1}{1+x/d}dx = -\frac{Q}{\in_0 A}\left[d\ln\left(1+\frac{x}{d}\right)\right]_0^d$$

$$\Rightarrow V_0 = -\frac{Qd}{\in_0 A}ln2 \Rightarrow \frac{Q}{A} = \frac{-\in_0 V_0}{dln2}$$

Thus $\vec{E} = \frac{1}{\in_0\in_r}\cdot\frac{Q}{A}\hat{x} = -\frac{1}{\in_0\left(1+\frac{x}{d}\right)}\times\frac{\in_0 V_0}{dln2}\hat{x} = -\frac{V_0}{(x+d)ln2}\hat{x}$

Q46. Two point charges of charge $+q$ each are placed a distance $2d$ apart. A grounded solid conducting sphere of radius a is placed midway between them. Assume $a^2 << d^2$. Which of the following statement is/are true?

(A) If $a > \frac{d}{8}$, the net force acting on the charges is directed towards each other

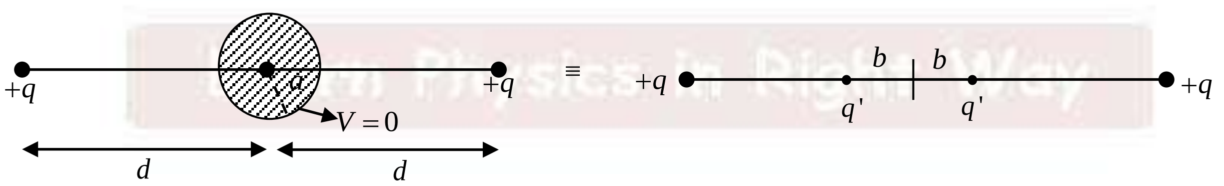
(B) The potential at the surface of the sphere is zero

(C) Total induced charge on the sphere is $\left(-\frac{2aq}{d}\right)$

(D) The potential at the center of the sphere is non-zero

Ans: (A), (B), (C)

Solution: $a^2 \ll d^2$

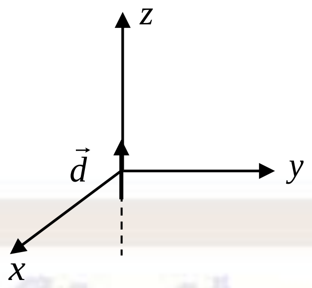


$q' = -\frac{a}{d}q$ and $b = \frac{a^2}{d}$

Total induced charge $q_T' = 2q' = -\frac{2a}{d}q$

Potential at the center = 0

Q55. An oscillating electric dipole of moment $\vec{d}(t) = d_0 \cos(\omega t)\hat{z}$ is placed at origin as shown in figure. Consider a point $P(r,\theta,\phi)$ at a very large distance from the dipole. Here r,θ and ϕ are spherical polar coordinates. Which of the following statement is/are true for intensity of radiation?



(A) Intensity is zero if P is on the z axis

(B) Intensity is zero at $P\left(r = R, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{4}\right)$

(C) Intensity at $P\left(r = R, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{4}\right)$ is greater than that at $P\left(r = R, \theta = \frac{\pi}{4}, \phi = \frac{\pi}{4}\right)$

(D) Intensity at $P\left(r = R, \theta = \frac{\pi}{2}, \phi = \frac{\pi}{4}\right)$ is equal to that at $P\left(r = R, \theta = \frac{\pi}{4}, \phi = \frac{\pi}{4}\right)$

Ans: (A), (C)

Solution:

$$\vec{p}(t) = q\vec{d} = qd_0 \cos\omega t\hat{z} = p_0 \cos\omega t\hat{z} \quad \Rightarrow I = \langle \vec{S} \rangle = k\frac{\sin^2\theta}{r^2}$$

(A) On z-axis, $\theta = 0° \Rightarrow I = 0$

(B) $P\left(R, \frac{\pi}{2}, \frac{\pi}{4}\right)$: $\quad I_1 = k\frac{\sin^2 \pi/2}{R^2} = \frac{k}{R^2} \neq 0$

(C) $P\left(R, \frac{\pi}{4}, \frac{\pi}{4}\right)$: $\quad I_2 = \frac{k\sin^2 \pi/4}{R^2} = \frac{k}{2R^2} = \frac{I_1}{2} \Rightarrow I_1 > I_2$

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Solution-Quantum Mechanics

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Q.11 – Q.35 Carry ONE mark Each

Q13. The wavefunction of a particle in an infinite one-dimensional potential well at time t is

$$\psi(x,t)=\sqrt{\frac{2}{3}}e^{-iE_1t/\hbar}\psi_1(x)+\frac{1}{\sqrt{6}}e^{i\pi/6}e^{-iE_2t/\hbar}\psi_2(x)+\frac{1}{\sqrt{6}}e^{i\pi/4}e^{-iE_3t/\hbar}\psi_3(x)$$

where ψ_1,ψ_2 and ψ_3 are the normalized ground state, the normalized first excited state and the normalized second excited state, respectively. E_1,E_2 and E_3 are the eigen-energies corresponding to ψ_1,ψ_2 and ψ_3, respectively. The expectation value of energy of the particle in state $\psi(x,t)$ is

(A) $\frac{17}{6}E_1$ (B) $\frac{2}{3}E_1$

(C) $\frac{3}{2}E_1$ (D) $14E_1$

Ans: (A)

Solution: $\psi(x,t)=c_1\psi_1(x)+c_2\psi_2(x)+c_3\psi_3(x)$

where $c_1=\sqrt{\frac{2}{3}}e^{-iE_1t/\hbar}$, $c_2=\frac{1}{\sqrt{6}}e^{i\pi/6}e^{-iE_2t/\hbar}$ and $c_3=\frac{1}{\sqrt{6}}e^{i\pi/4}e^{-iE_3t/\hbar}$

The expectation value of energy is

$$\langle E\rangle=\frac{\sum E_iP(E_i)}{\sum P(E_i)}=\frac{E_1|c_1|^2+E_2|c_2|^2+E_3|c_3|^2}{|c_1|^2+|c_2|^2+|c_3|^2}=\frac{E_1\times\frac{2}{3}+E_2\times\frac{1}{6}+E_3\times\frac{1}{6}}{\frac{2}{3}+\frac{1}{6}+\frac{1}{6}}=\frac{1}{6}(4E_1+E_2+E_3)$$

where $E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$; $E_1=\frac{\pi^2\hbar^2}{2mL^2}$, $E_2=4E_1$ and $E_3=9E_1$

$\therefore\langle E\rangle=\frac{1}{6}(4E_1+4E_1+9E_1)=\frac{17}{6}E_1$. Thus correct answer is option (A)

Q22. A particle is subjected to a potential

$$V(x)=\begin{cases}\infty, & x\le 0\\ V_0, & a\le x\le b\\ 0, & \text{elsewhere}\end{cases}$$

Here, $a>0$ and $b>a$. If the energy of the particle $E<V_0$, which one of the following schematics is a valid quantum mechanical wavefunction (ψ) for the system?

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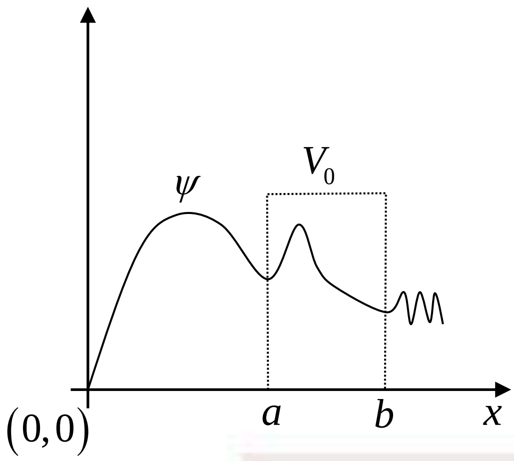
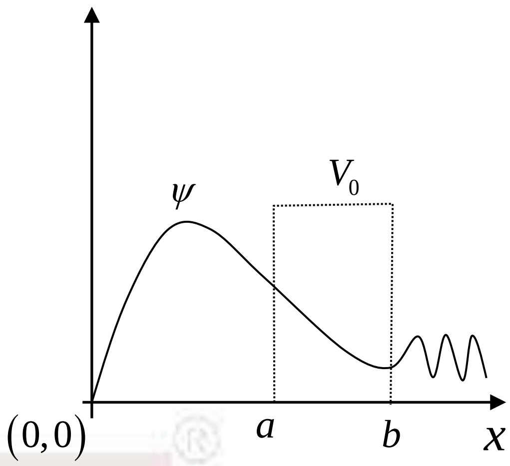
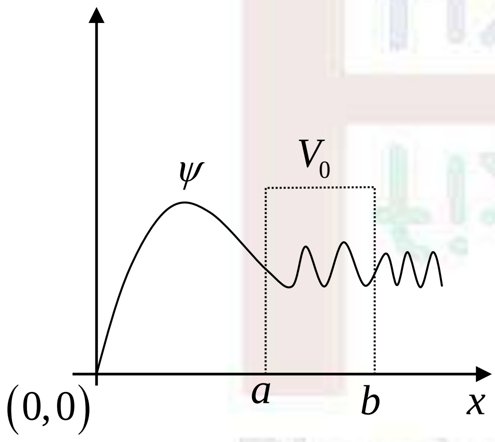
(A)



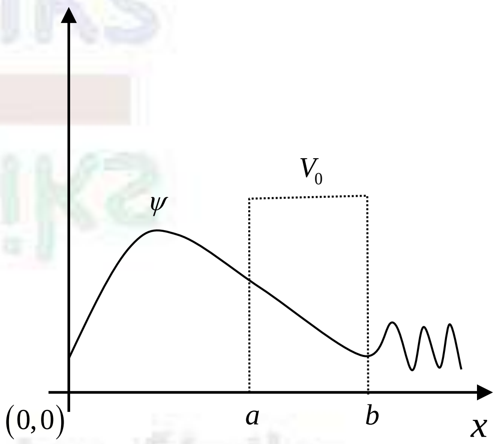
(B)



(C)



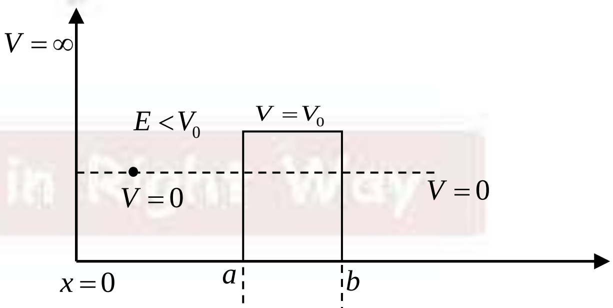
(D)

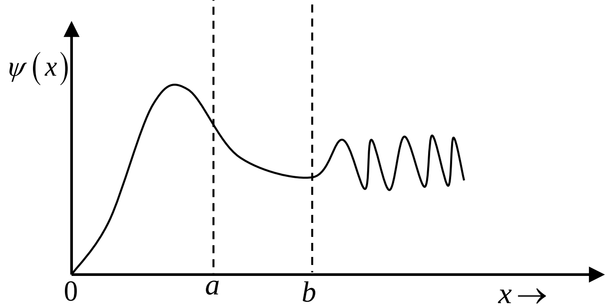


Ans: (B)

Solution:

$$V(x) = \begin{cases} \infty; & x \le 0 \\ V_0; & a \le x \le b \\ 0; & \text{elsewhere} \end{cases}$$





The $\psi(x) = 0$ at $x = 0$ as potential is infinite.

The $\psi(x)$ must decay exponentially in $a \le x \le b$ as $E < V_0$ and oscillatory in a region where $V = 0$. Thus option (B) is correct.

Q28. Following trial wavefunctions $\phi_1 = e^{-Z'(r_1+r_2)}$

and $\phi_2 = e^{-Z'(r_1+r_2)}\left(1+g\left|\vec{r}_1-\vec{r}_2\right|\right)$

are used to get a variational estimate of the ground state energy of the helium atom. Z' and g are the variational parameters, $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the electrons. Let E_0 be the exact ground state energy of the helium atom. E_1 and E_2 are the variational estimates of the ground state energy of the helium atom corresponding to ϕ_1 and ϕ_2, respectively. Which one of the following options is true?

(A) $E_1 \le E_0, E_2 \le E_0, E_1 \ge E_2$ (B) $E_1 \ge E_0, E_2 \le E_0, E_1 \ge E_2$

(C) $E_1 \le E_0, E_2 \ge E_0, E_1 \le E_2$ (D) $E_1 \ge E_0, E_2 \ge E_0, E_1 \ge E_2$

Ans: (D)

Solution: Variational principle always gives an upper bound to ground state energy (E_0)

$\therefore\ E_1 \ge E_0$ and $E_2 \ge E_0$

only option (D) satisfying the condition. Therefore, option (D) is correct answer.

Q29. The wavefunction for particle is given by the form $e^{-(i\alpha x+\beta)}$, where α and β real constants. In which one of the following potentials $V(x)$, the particle is moving?

(A) $V(x) \propto \alpha^2 x^2$ (B) $V(x) \propto e^{-\alpha x}$

(C) $V(x) = 0$ (D) $V(x) \propto \sin(\alpha x)$

Ans: (C)

Solution: Method-I: The given wave function $\psi(x) = e^{-i\alpha x}e^{-\beta} = Ae^{-i\alpha x}$ is a plane wave function. This wave function is valid for a system in which either potential is zero $(V(x)=0)$ or constant $(V(x) =$ constant). Thus only option (C) is correct.

Method-II: The Schrodinger wave equation is $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$

Since, $\psi(x) = Ae^{-i\alpha x} \Rightarrow \frac{d\psi}{dx} = (-i\alpha)\psi \Rightarrow \frac{d^2\psi}{dx^2} = -\alpha^2\psi$

$$\therefore\ -\frac{\hbar^2}{2m}\left[-\alpha^2\psi\right] + V(x)\psi = E\psi$$

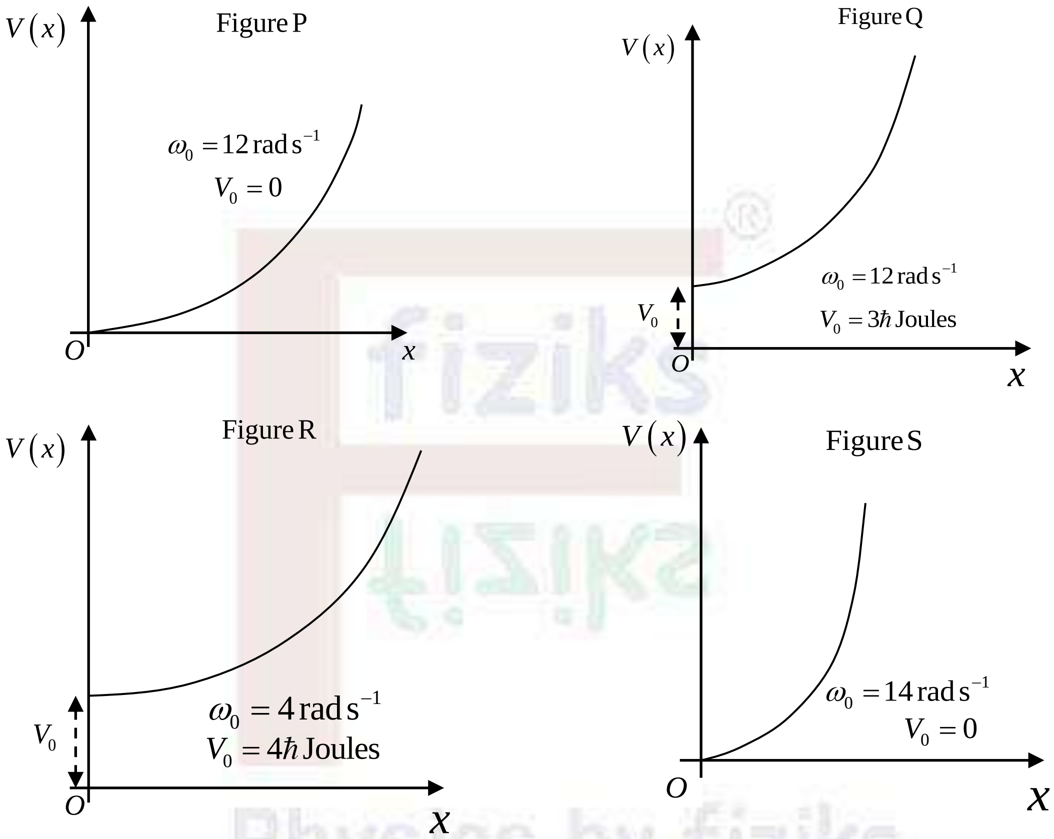
Compare x-dependent terms on L.H.S. and R.H.S. $\therefore\ V(x) = 0$

The correct option is (C)

Q.36 – Q.65 Carry TWO marks Each

Q47. A particle of mass m is moving in the potential $V(x)=\begin{cases} V_0+\frac{1}{2}m\omega_0^2x^2, & x>0 \\ \infty, & x\le 0 \end{cases}$

Figures, P,Q,R and S show different combinations of the values of ω_0 and V_0.



$E_j^{(P)},E_j^{(Q)},E_j^{(R)}$ and $E_j^{(S)}$ with $j=0,1,2,\ldots$, are the eigen-energies of the j-th level for the potentials shown in figures P,Q,R and S, respectively. Which of the statement is/are true?

(A) $E_0^{(P)}=E_0^{(Q)}$ (B) $E_0^{(Q)}=E_0^{(S)}$

(C) $E_0^{(P)}=E_1^{(R)}$ (D) $E_0^{(R)}\neq E_0^{(Q)}$

Ans: (B), (C), (D)

Solution: The energy eigenvalue is $E_j=V_0+\left(2j+\frac{3}{2}\right)\hbar\omega_0$; $j=0,1,2,3,\ldots$

$E_0^{(P)}=0+\left(0+\frac{3}{2}\right)\hbar\times 12=18\hbar$; $E_0^{(Q)}=3\hbar+\left(0+\frac{3}{2}\right)\hbar\times 12=21\hbar$

$E_1^{(R)}=4\hbar+\left(2+\frac{3}{2}\right)\hbar\times 4=4\hbar+14\hbar=18\hbar$; $E_0^{(S)}=0+\left(0+\frac{3}{2}\right)\hbar\times 14=21\hbar$

Thus, $E_0^{(P)} \neq E_0^{(Q)}$; option (A) is wrong; $E_0^{(Q)} = E_0^{(S)} = 21\hbar$; option (B) is correct

$E_0^{(P)} = E_1^{(R)} = 18\hbar$; option (C) is correct; $E_0^{(R)} \neq E_0^{(Q)}$; option (D) is correct

Thus, correct options are (B), (C) and (D)

Q48. The non-relativistic Hamiltonian for a single electron atom is $H_0 = \frac{p^2}{2m} - V(r)$ where $V(r)$ is the Coulomb potential and m is the mass of the electron. Considering the spin-orbit interaction term $H' = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\vec{L}\cdot\vec{S}$ added to H_0, which of the following statement is/are true?

(A) H' commutes with L^2

(B) H' commutes with L_z and S_z

(C) For a given value of principal quantum number n and orbital angular momentum quantum number l, there are $2(2l+1)$ degenerate eigenstates of H_0

(D) H_0, L^2, S^2, L_z and S_z have a set of simultaneous eigenstates

Ans: (A), (C), (D)

Solution: (i) $\left[H', L^2\right] = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\left[\vec{L}\cdot\vec{S}, L^2\right]$

Since $\left[L^2, L_i\right] = 0$ and $\left[L^2, S_i\right] = 0$. Therefore $\left[\vec{L}\cdot\vec{S}, L^2\right] = 0$. Thus $\left[H', L^2\right] = 0$

The statement (A) is correct.

(ii) $\left[H', L_z\right] = \frac{1}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\left[\vec{L}\cdot\vec{S}, L_z\right]$. Now $\left[\vec{L}\cdot\vec{S}, L_z\right] = \left[L_xS_x + L_yS_y + L_zS_z, L_z\right]$

Since $\left[L_i, L_j\right] = i\hbar \in_{ijk} L_k$. Therefore $\left[\vec{L}\cdot\vec{S}, L_z\right] \neq 0$, thus statement (B) is not correct.

(iii) For given value of n and ℓ, the degeneracy is $g = 2\ell(\ell+1)$

Thus statement (C) is correct.

(iv) The unperturbed Hamiltonian H_0 is spherically symmetric and depends only on r.

Thus $\left[H, L^2\right] = 0$ and $\left[H, L_z\right] = 0$

H also commutes with S^2 and S_z, because H does not depend on the spin degree of freedom.

$\left[H, S^2\right] = 0$ and $\left[H, S_z\right] = 0$

Thus statement (D) is correct.

The correct statements are (A), (C) and (D).

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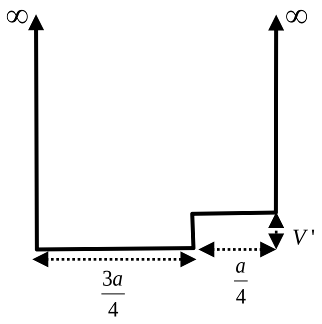
Q61. A particle of mass m in an infinite potential well of width a is subjected to a perturbation, $V' = \frac{h^2}{40ma^2}$ as shown in figure, where h is Planck's constant.



The first order energy shift of the fourth energy eigenstate due to this perturbation is $\left(\frac{h^2}{Nma^2}\right)$

The value of N is____________ (*in integer*).

Ans: 160

Solution:

$$E_n^{(1)} = \frac{\text{Strength of perturbation} \times \text{length of the perturbation}}{\text{length of the well}} = \frac{\frac{\hbar^2}{40ma^2} \times \frac{a}{4}}{a} = \frac{\hbar^2}{160ma^2}$$

Thus $N = 160$

Q64. An electron in the Coulomb field of a proton is in the following state of coherent superposition of orthonormal states ψ_{nlm}

$$\psi = \frac{1}{3}\psi_{100} + \frac{1}{\sqrt{3}}\psi_{210} - \frac{\sqrt{5}}{3}\psi_{320}$$

Let E_1, E_2, and E_3 represent the first three energy levels of the system. A sequence of measurements is done on the same system at different times. Energy is measured first at time t_1 and the outcome is E_2. Then total angular momentum is measured at time $t_2 > t_1$ and finally energy is measured again at $t_3 > t_2$. The probability of finding the system in a state with energy E_2 after the final measurement is $P/9$. The value of P is _____ (*in integer*).

Ans: 9

Solution: $$|\psi\rangle = \frac{1}{3}|100\rangle + \frac{1}{\sqrt{3}}|210\rangle - \frac{\sqrt{5}}{3}|320\rangle$$

First energy measurement at t_1: Since energy measurement collapses the wavefunction to the eigenstate corresponding to the measured energy, the system collapses to the state $|210\rangle$ after the first measurement

$$\hat{H}|\psi\rangle = E_2|210\rangle$$

Total angular momentum measurement at $t_2 > t_1$.

$$L^2|210\rangle = 2\hbar^2|210\rangle$$

Second energy measurement at $t_3 > t_2$: Since the system is still is the state $|210\rangle$, which corresponds to the energy E_2.

$$\therefore \ \hat{H}|210\rangle = E_2|210\rangle$$

The probability of getting E_2 is $P(E_2) = 1 = \frac{9}{9} = \frac{P}{9}$. Thus $P = 9$.

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Solution-Thermodynamics and Statistical Mechanics

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Q.11 – Q.35 Carry ONE mark Each

Q14. If a thermodynamical system is adiabatically isolated and experiences a change in volume under an externally applied constant pressure, then the thermodynamical potential minimized at equilibrium is the

(A) enthalpy (B) Helmholtz free energy

(C) Gibbs free energy (D) grand potential

Ans: (A)

Solution: $dH = TdS + VdP$; $\because S = \text{Constant}$ and $P = \text{Constant}$ then $dH = 0$.

Thus enthalpy is minimized at equilibrium.

Q33. The vapor pressure (P) of solid ammonia is given by $\ln(P) = 23.03 - \frac{3754}{T}$, while that of liquid ammonia is given by $\ln(P) = 19.49 - \frac{3063}{T}$, where T is the temperature in K.

The temperature of the triple point of ammonia is ____________ K (*rounded off to two decimal places*).

Ans: 195.10 to 195.30

Solution: For Triple point $\ln(P) = 23.03 - \frac{3754}{T} = 19.49 - \frac{3063}{T} \Rightarrow \frac{3063}{T} - \frac{3754}{T} = 19.49 - 23.03$

$$\Rightarrow \frac{3754}{T} - \frac{3063}{T} = 23.03 - 19.49 \Rightarrow \frac{691}{T} = 3.54 \Rightarrow T = \frac{691}{3.54} = 195.20\,K$$

Q35. The Hamiltonian of a system of N particles in volume V at temperature T is

$$H = \sum_{i=1}^{2N} a_i q_i^2 + \sum_{i=1}^{2N} b_i p_i^2$$

where a_i and b_i are positive constants. The ensemble average of the Hamiltonian is $\alpha N k_B T$, where k_B is the Boltzmann constant. The value of α is ________(in integer).

Ans: 2

Solution: Given, Hamiltonian $H = \sum_{i=1}^{2N} a_i q_i^2 + \sum_{i=1}^{2N} b_i p_i^2$ ...(1)

For $N = 1$, we have $H_1 = \sum_{i=1}^{2} a_i q_i^2 + \sum_{i=1}^{2} b_i p_i^2$. This means motion is confined to 2-D plane.

$\therefore \langle H_1 \rangle = 2 \times \frac{kT}{2} + 2 \times \frac{kT}{2} = 2kT$. For $N = N$, $\langle H \rangle = 2NkT$, $\alpha = 2$

Note: If question is asked in 3D: $H = \sum_{i=1}^{3N} a_i q_i^2 + \sum_{i=1}^{3N} b_i p_i^2$ then $\langle H \rangle = 3NkT$

Q.36 – Q.65 Carry TWO marks Each

Q59. The canonical partition function of an ideal gas is $Q(T,V,N)=\frac{1}{N!}\left[\frac{V}{(\lambda(T))^3}\right]^N$

where T,V,N and $\lambda(T)$ denote temperature, volume, number of particles, and thermal de Broglie wavelength, respectively. Let k_B be the Boltzmann constant and μ be the chemical potential. Take $\ln(N!)=N\ln(N)-N$.

If the number density $\left(\frac{N}{V}\right)$ is $2.5\times10^{25}\,\text{m}^{-3}$ at a temperature T, then $\frac{e^{\mu/(k_BT)}}{(\lambda(T))^3}\times10^{-25}$ is ________ m^{-3} (*rounded off to one decimal place*).

Ans: 2.5

Solution: Given $Q(T,V,N)=\frac{1}{N!}\left[\frac{V}{(\lambda(T))^3}\right]^N$ ...(1)

Helmholtz free energy is $A=-kT\ln Q=-kT\ln\left[\frac{1}{N!}\left\{\frac{V}{\lambda^3}\right\}^N\right]=-kT\left[-\ln N!+N\ln\left(\frac{V}{\lambda^3}\right)\right]$

$$A=-kT\left[-N\ln N+N+N\ln\left(\frac{V}{\lambda^3}\right)\right]=kT\left[N\ln N-N-N\ln\left(\frac{V}{\lambda^3}\right)\right] \quad ...(2)$$

$$\mu=\left(\frac{\partial A}{\partial N}\right)=kT\left[N\times\frac{1}{N}+\ln N-1-\ln\frac{V}{\lambda^3}\right]\Rightarrow\frac{\mu}{kT}=\ln N-\ln\frac{V}{\lambda^3}=\ln\left(\frac{N}{V}\lambda_T^3\right)$$

$$\Rightarrow e^{\frac{\mu}{kT}}=\frac{N}{V}\lambda_T^3\Rightarrow\frac{e^{\frac{\mu}{kT}}}{\lambda_T^3}=\frac{N}{V}=2.5\times10^{25}m^{-3}\Rightarrow\frac{e^{\frac{\mu}{kT}}}{\lambda_T^3}\times10^{-25}=2.5m^{-3}$$

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Solution-Electronics

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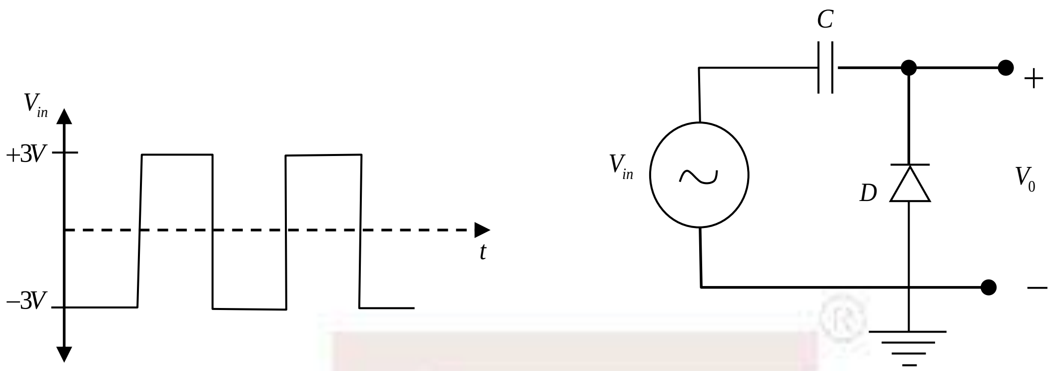
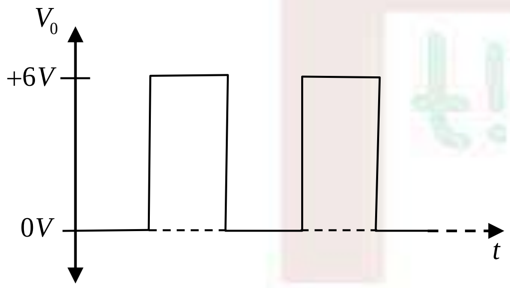
Q.11 – Q.35 Carry ONE mark Each

Q18. The symbols C, D, V_{in} and V_0 shown in the figure denote capacitor, ideal diode, input voltage and output voltage, respectively.

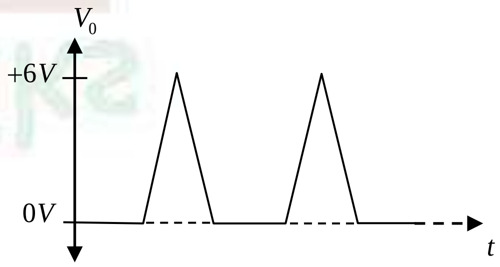


Which one of the following output waveforms (V_0) is correct for the given input waveform (V_{in})?

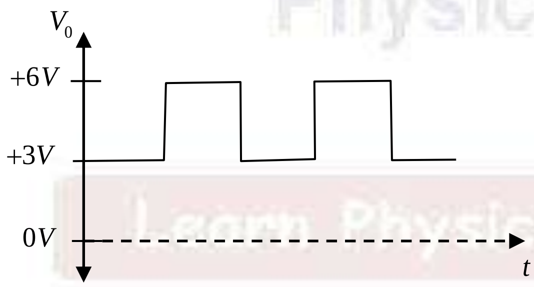
(A)



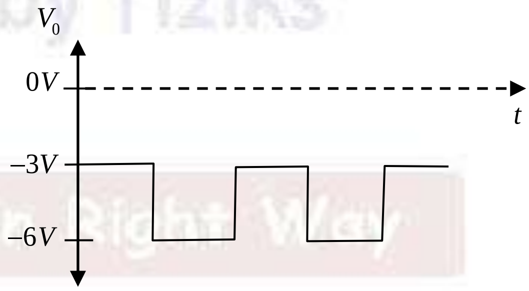
(B)



(C)



(D)



Ans: (A)

Solution:

The circuit shown in the figure is a clamper circuit. In this voltage level will shift in the direction of diode current i.e. in upward direction and peak to peak voltage remains the same.

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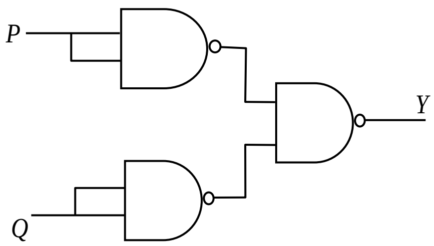
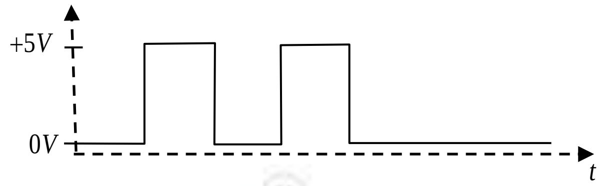
Q24. Consider the following circuit:



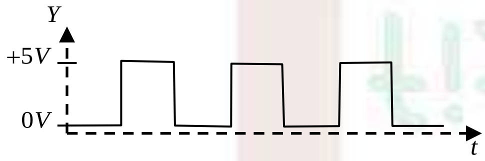
Suppose the input signal P is



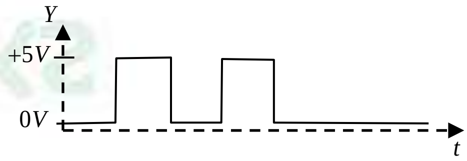
and the input signal Q is

Which one of the following output signals is correct?

(A)



(B)



(C)

(D)

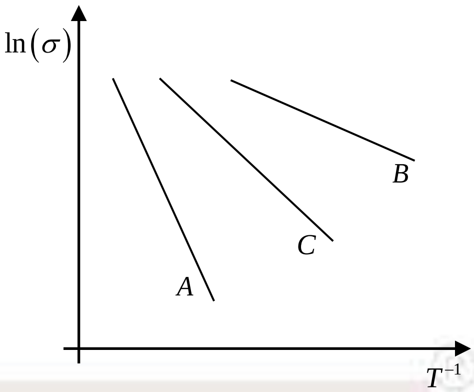
Ans: (A)

Solution:

$$Y = \overline{\overline{P}\,\overline{Q}} = \overline{\overline{P}} + \overline{\overline{Q}} = P + Q$$

So output Y is P OR Q.

Q27. The temperature dependence of the electrical conductivity (σ) of three intrinsic semiconductors A, B and C is shown in figure.



Let E_A, E_B and E_C be the bandgaps of A, B and C, respectively. Which one of the following relations is correct?

(A) $E_C > E_A > E_B$ (B) $E_B > E_C > E_A$

(C) $E_A > E_B > E_C$ (D) $E_A > E_C > E_B$

Ans: (D)

Solution: $\sigma = n_i e(\mu_e + \mu_n)$ where $n_i = \sqrt{N_c N_v} e^{-E_g/2k_BT}$

$$ln\sigma = lnn_i + ln\left[e(\mu_e + \mu_n)\right] = ln\sqrt{N_c N_v} - \frac{E_g}{2k_BT} + ln\left[e(\mu_e + \mu_n)\right]$$

$ln\sigma = c - \frac{E}{2k_BT} \Rightarrow y = mx + c$ where Slope $m = -\frac{E_g}{2k_B}$ and c is some constant.

We have assumed that N_c, N_v, μ_e and μ_n are independent of temperature.

$\because m_A > m_C > m_B \Rightarrow E_A > E_C > E_B$

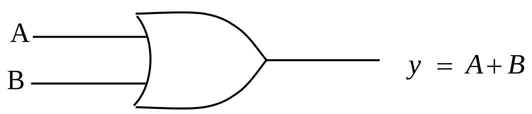
Q32. The minimum number of basic logic gates required to realize the Boolean expression $B\cdot(A+B) + A\cdot(\bar{B}+A)$ is ______________ (in integer).

Ans: 1

Solution:

$$Y = B(A+B) + A(\bar{B}+A) = AB + B + A\bar{B} + AB \Rightarrow Y = AB + B + A\bar{B} = B(1+A) + A\bar{B} = B + \bar{B}A$$

$\Rightarrow Y = B + A$



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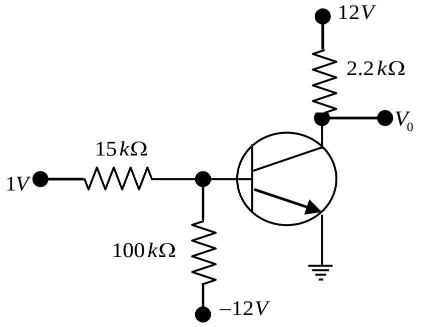
Q.36 – Q.65 Carry TWO marks Each

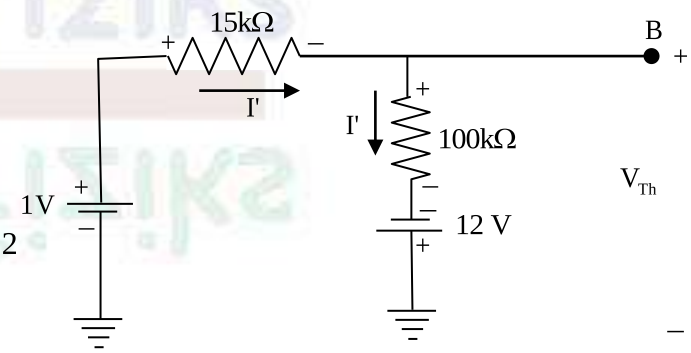
Q58. A typical biasing of a silicon transistor is shown in figure.



The value of common-emitter current gain β for the transistor is 100. Ignore reverse saturation current. The output voltage V_0 (in V) is ____ (*in integer*).

Ans: 12

Solution: Applying K.V.L.



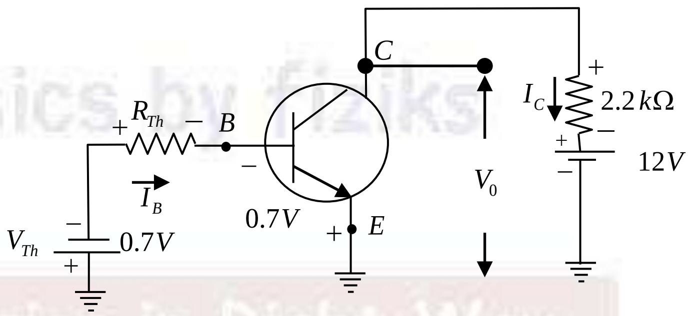
$-1V + 15I' + 100I' - 12V = 0$

$\Rightarrow I' = \frac{13}{115} mA$

$\Rightarrow V_{Th} = 100I' - 12 = \frac{1300}{115} - 12 = 11.3 - 12$

$\Rightarrow V_{Th} = -0.7V$

Applying K.V.L. in input section



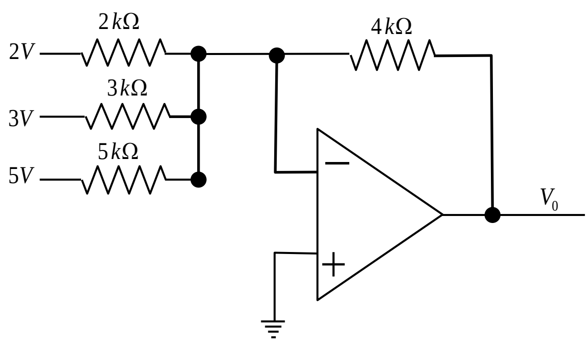
$+0.7V + I_B R_{Th} - 0.7V = 0$

$\Rightarrow I_B = 0 \Rightarrow I_C = 0$

$\Rightarrow V_{CE} = V_{CC} = 12V$

$\Rightarrow V_0 = V_{CE} = 12V$

Q63. Consider the operational amplifier circuit shown in figure.



The output voltage V_0 is ______________V (*integer*).

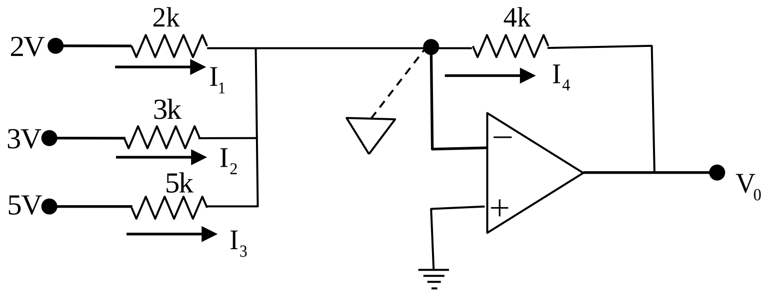
Ans: -12

Solution:

$\because I_1 + I_2 + I_3 = I_F$

$$\frac{2V-0}{2k} + \frac{3V-0}{3k} + \frac{5V-0}{5k} = \frac{0-V_0}{4k}$$

$$\Rightarrow V_0 = -4(1+1+1) = -12V$$



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GATE Physics-2024

Solution-Atomic and Molecular Physics

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Q.11 – Q.35 Carry ONE mark Each

Q15. The mean distance between the two atoms of HD molecule is r, where H and D denote hydrogen and deuterium, respectively. The mass of the hydrogen atom is m_H. The energy difference between two lowest lying rotational states of HD in multiples of $\hbar^2/\left(m_H r^2\right)$ is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) 6 (D) $\frac{4}{3}$

Ans: (A)

Solution:

Rotational energy levels is $E_J = BJ\left(J+1\right)$ where $B = \frac{\hbar^2}{2\mu r^2}$

The energy spacing between two consecutive levels is $\Delta E = E_{J+1} - E_J = 2B\left(J+1\right)$

The energy difference between two lowest levels is

$$\Delta E = E_1 - E_0 = 2B = \frac{\hbar^2}{\mu r^2} \qquad \text{where } \mu = \frac{m_H \times m_D}{m_H + m_D} = \frac{2m_H^2}{3m_H} = \frac{2}{3}m_H$$

$$\therefore \ \Delta E = \frac{\hbar^2}{\frac{2}{3}m_H r^2} = \frac{3}{2}\left(\frac{\hbar^2}{m_H r^2}\right)$$

Thus correct answer is option (A).

Q21. Atomic numbers of V, Cr, Fe and Zn are 23, 24, 26 and 30, respectively. Which one of the following materials does NOT show an electron spin resonance (ESR) spectra?

(A) V (B) Cr (C) Fe (D) Zn

Ans: (D)

Solution: ESR is exhibited by atoms which possesses unpaired of electrons in their electronic configurations

$V \to 3d^3$	↑	↑	↑		
$Cr \to 3d^4$	↑	↑	↑	↑	
$Fe \to 3d^6$	↑↓	↑	↑	↑	↑
$Zn \to 3d^{10}$	↑↓	↑↓	↑↓	↑↓	↑↓

Q.36 – Q.65 Carry TWO marks Each

Q37. The spin-orbit interaction in a hydrogen-like atom is given by the Hamiltonian

$$H' = -k\vec{L}\cdot\vec{S}$$

where k is a real constant. The splitting between levels ${}^2p_{3/2}$ and ${}^2p_{1/2}$ due to this interaction is

(A) $\frac{1}{2}k\hbar^2$ (B) $\frac{3}{2}k\hbar^2$ (C) $\frac{3}{4}k\hbar^2$ (D) $2k\hbar^2$

Ans: (B)

Solution:

$$\Delta E_{l,s} = k\left(l+\frac{1}{2}\right)\hbar^2 \Rightarrow \Delta E_{l,s} = k\left(1+\frac{1}{2}\right)\hbar^2 = \frac{3}{2}k\hbar^2$$

Q42. An atom is subjected to a weak uniform magnetic field $\vec{B}$. The number of lines in its Zeeman spectrum for transition from $n=2, l=1$ to $n=1, l=0$ is

(A) 8 (B) 10

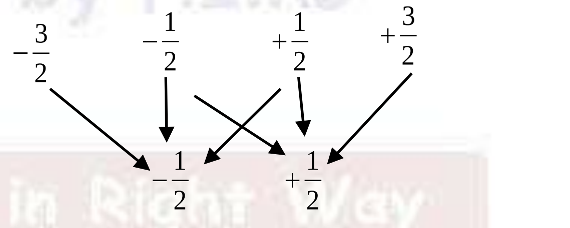
(C) 12 (D) 5

Ans: (B)

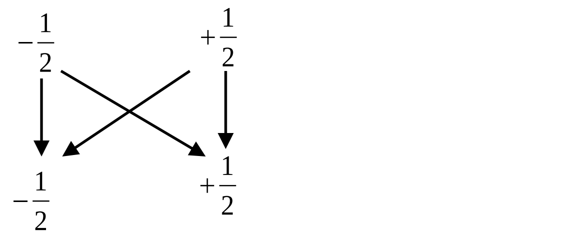
Solution:

$$n=2, l=1, s=\frac{1}{2} \rightarrow 2^2p_{3/2}, 2^2p_{1/2}; \quad n=1, l=0, s=\frac{1}{2} \rightarrow 1^2s_{1/2}$$

(i) Transition $2^2p_{3/2} \rightarrow 1^2s_{1/2}$

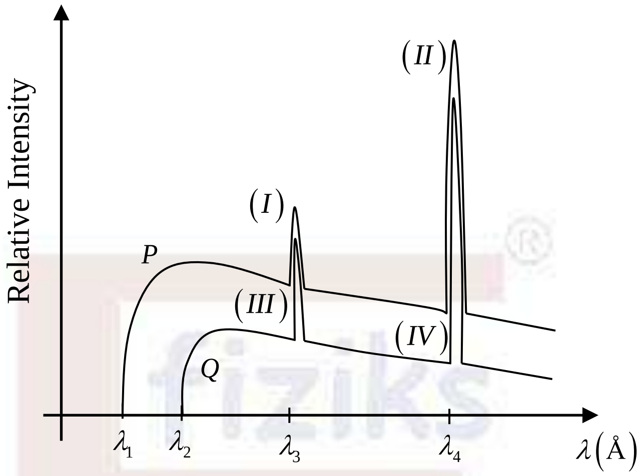
$2^2p_{3/2}$	m_J	
$2^2s_{1/2}$	m_J	

(ii) Transition $2^2p_{1/2} \rightarrow 1^2s_{1/2}$

$2^2p_{1/2}$	m_J	
$2^2s_{1/2}$	m_J	

So, total number of lines is 10.

Q53. The curves P and Q schematically show the variation of X-ray intensity with wavelength at two different accelerating voltages for a given target material. In the figure $\lambda_1 = 0.25\text{Å}, \lambda_2 = 0.5\text{Å}, \lambda_3 = 1.0\text{Å}$ and $\lambda_4 = 2.25\text{Å}$. Take Planck's constant as $6.6\times10^{-34}\ Js$, speed of light as $3\times10^{8}\ \text{ms}^{-1}$ and elementary charge as 1.6×10^{-19} C.



Which of the following statement is/are true?

(A) The accelerating potential corresponding to curve P is greater than that of curve Q

(B) The accelerating potential applied to obtain curve Q is 24750 V

(C) Peaks (II) and (IV) correspond to radiative transitions from L to K shells

(D) Peaks (I) and (III) correspond to radiative transitions from N to K shells

Ans: (A), (B), (C)

Solution:

(a) $\lambda_{\min} = \dfrac{12375}{V}\text{Å};\quad \lambda^{P}_{\min} < \lambda^{Q}_{\min},\quad V_P > V_Q$

(b) $V_Q = \dfrac{12375}{\lambda^{Q}_{\min}} = \dfrac{12375}{0.5} = 24750\text{Å}$

(c) and (d)

$\lambda_{L\to K} > \lambda_{M-K} > \lambda_{L-K}$

$$\lambda = \frac{hc}{E_K - E_L} \text{ for } K_\alpha;\ \lambda = \frac{hc}{E_K - E_M} \text{ for } K_\beta;\ \lambda = \frac{hc}{E_K - E_N} \text{ for } K_\gamma$$

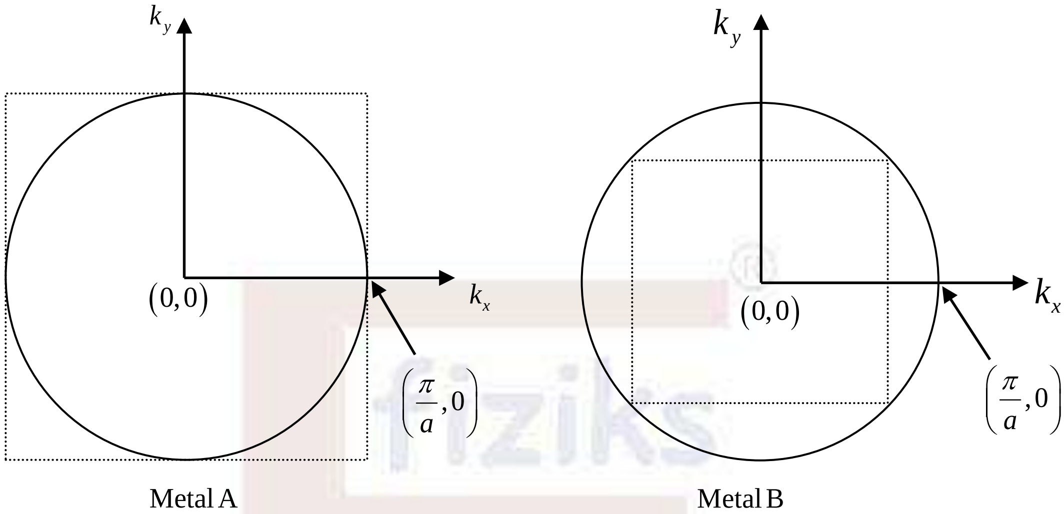
GATE Physics-2023

Solution-Solid State Physics

Be Part of Disciplined Learning

Q.11 – Q.35 Carry ONE mark Each

Q16. Crystal structures of two metals A and B are two-dimensional square lattices with same lattice constant a. Electrons in metals behave as free electrons. The Fermi surfaces corresponding to A and B are shown by solid circles in figures.



The electron concentrations in A and B are n_A and n_B, respectively. The value of $\left(\frac{n_B}{n_A}\right)$ is

(A) 3 (B) 2 (C) $3\sqrt{3}$ (D) $\sqrt{2}$

Ans: (D)

Solution:

The Fermi radius in two-dimension is $k_F = 2\pi n$ where n is electron concentrated

The Fermi radius for metal A is $K_F = \frac{\pi}{a}$

The Fermi radius for metal B is $K_F = \sqrt{2}\frac{\pi}{a}$

$\therefore$ For metal A: $\frac{\pi}{a} = 2\pi n_A$ ...(1) and For metal B: $\frac{\sqrt{2}\pi}{a} = 2\pi n_B$ ...(2)

Thus $\frac{2\pi n_B}{2\pi n_A} = \frac{\sqrt{2}\pi/a}{\pi/a} \Rightarrow \frac{n_B}{n_A} = \sqrt{2}$

Therefore, the correct option is (D)

Q.36 – Q.65 Carry TWO marks Each

Q39. The X-ray diffraction pattern of a monatomic cubic crystal with rigid spherical atoms of radius 1.56Å shows several Bragg reflections of which the reflection appearing at the lowest 2θ value in from (111) plane. If the wavelength of X-ray used is 0.78Å, Bragg angle (in 2θ, *rounded off to one decimal place*) corresponding to this reflection and the crystal structure, respectively, are

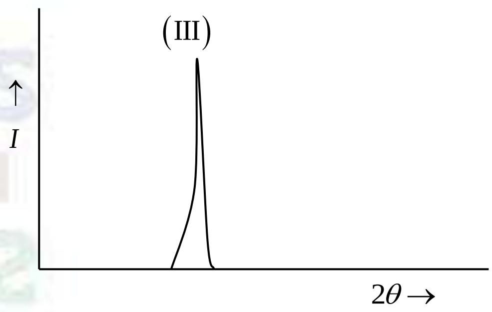
(A) 21.6° and body centered cubic (B) 17.6° and face centered cubic

(C) 10.8° and body centered cubic (D) 8.8° and face centered cubic

Ans: (B)

Solution:

According to the question, the first peak appears for the plane (III). Thus the lattice is face centered cubic.



For FCC; $\sqrt{2}a = 4r$

$$a = \frac{4r}{\sqrt{2}} = \frac{4\times 1.56}{\sqrt{2}} A^\circ = 4.41 A^\circ$$

Bragg's law is $2d\sin\theta = \lambda \Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{2a}\sqrt{h^2+k^2+l^2}\right) = \sin^{-1}\left(\frac{0.78A^\circ}{2\times 4.41A^\circ}\sqrt{3}\right) = \sin^{-1}(0.15) = 8.8^\circ \Rightarrow 2\theta = 17.6^\circ$$

Thus correct option is (B)

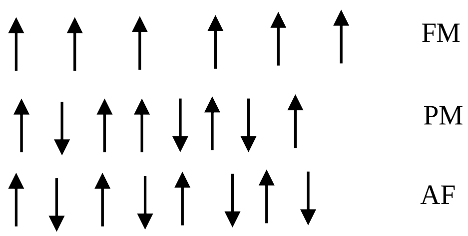
Q44. An infinite one dimensional lattice extends along x-axis. At each lattice site there exits an ion with spin $\frac{1}{2}$. The spin can point either in $+z$ or $-z$ direction only. Let S_P, S_F and S_A denote the entropies of paramagnetic, ferromagnetic and antiferromagnetic configurations, respectively. Which of the following relation is/are true?

(A) $S_P > S_F$ (B) $S_A > S_F$ (C) $S_A = 4S_F$ (D) $S_P > S_A$

Ans: (A), (D)

Solution:

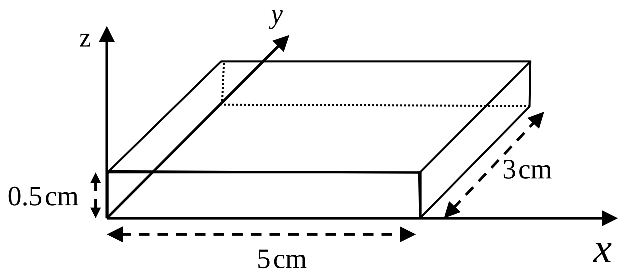
$S_P > S_F$



$S_P > S_A$, also $S_A > S_F$ must be true.

(A), (B) and (D) must be true.

Q50. An extrinsic semiconductor shown in figure carries a current of $2\,mA$ along its length parallel to $+x$ axis. When the majority charge carrier concentration is $12.5\times10^{13}\ \text{cm}^{-3}$ and the sample is exposed to a constant magnetic field applied along the $+z$ direction, a Hall voltage of 20 mV is measured with the negative polarity at $y=0$



plane. Take the electric charge as 1.6×10^{-19} C. The concentration of minority charge carrier is negligible. Which of the following statement is/are true?

(A) The majority charge carrier is electron

(B) The magnitude of the applied magnetic field is 1 Tesla

(C) The electric field corresponding to the Hall voltage is in the $+y$ direction

(D) The magnitude of Hall coefficient is 50,000 m^3C^{-1}

Ans: (A), (B)

Solution: According to the question $\vec{V}=-V\hat{i}$, $\vec{B}=B\hat{k}$. $\vec{F}_L=q\left(\vec{V}\times\vec{B}\right)=-q\left(V\hat{i}\times B\hat{k}\right)=+qVB\hat{j}$

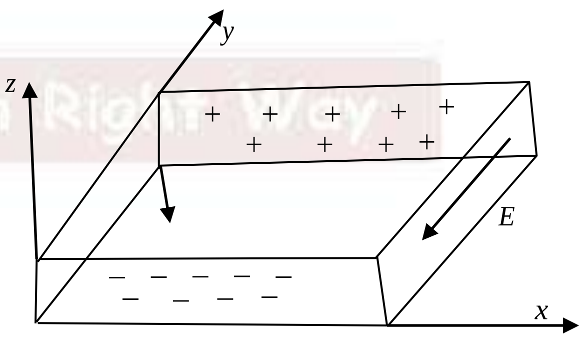
If $q=-e$, $\vec{F}_L=-eVB\hat{j}$ as a result electron will accumulate at $y=0$ plane. Thus, majority career are electrons. Therefore, option (A) is correct.

(B) The Hall voltage is $V_H=\dfrac{IB}{new}$

$$B=\frac{V_H new}{I}=\frac{20\times10^{-3}\times12.5\times10^{13}\times10^{6}m^{-3}\times1.6\times10^{-19}\times0.5\times10^{-2}m}{2\times10^{-3}}=1\,T$$

Thus option (B) is correct.

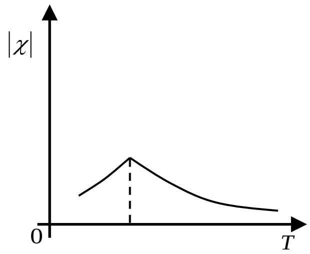
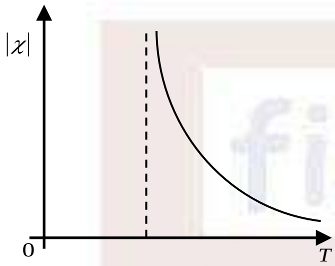
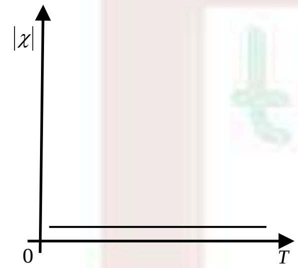
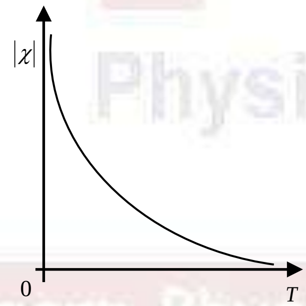
(C) The electric field corresponding to the Hall voltage is in the $-y$ direction. Thus option (c) is not correct.



(D) $R_H=\dfrac{1}{ne}=\dfrac{1}{12.5\times10^{13}\times10^{6}\times1.6\times10^{-19}}=0.05m^3C^{-1}$

Thus option (D) is not correct.

Q52. The temperature T dependence of magnetic susceptibility χ (Column I) of certain magnetic materials (Column II) are given below. Which of the following option is/are correct?

Column I	Column II
(1) 	(P) Diamagnetic
(2) 	(Q) Paramagnetic
(3) 	(R) Ferromagnetic
(4) 	(S) Antiferromagnetic

(A) 2 − P, 4 − Q, 3 − S (B) 4 − P, 1 − Q, 2 − R

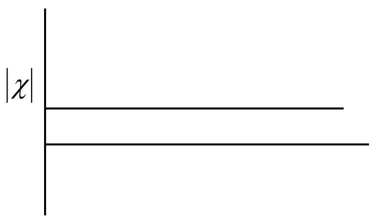
(C) 4 − Q, 2 − R, 1 − S (D) 3 − P, 4 − Q, 2 − R

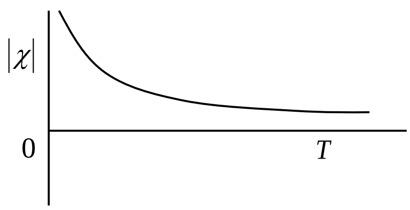
Ans: (C), (D)

Solution:

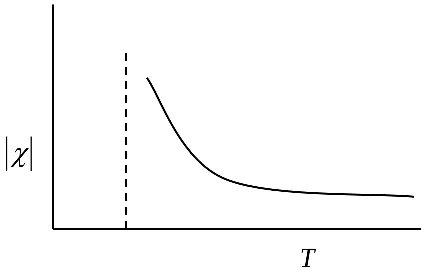
$\chi_{\text{diamagnet}} = -\text{ constant}$

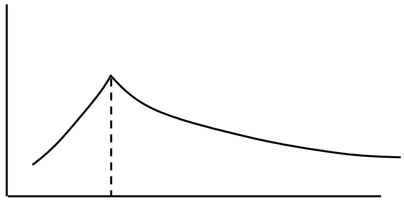
$\chi_{\text{Paramagnet}} = \dfrac{C}{T}$





$\chi_{\text{Ferromagnet}} = \dfrac{C}{T - T_C}$ $\qquad\qquad$ $\chi_{\text{Anti-Ferro}} = \dfrac{C}{T + T_N}$





Thus options (C) and (D) are correct.

Q54. Apart from the acoustic modes, 9 optical modes are identified from the measurements of photon dispersions of a solid with chemical formula $A_n B_m$, where A and B denote the atomic species, and n and m are integers. Which of the following combination of n and m is/are possible?

(A) $n = 1, m = 1$ $\qquad$ (B) $n = 2, m = 2$

(C) $n = 3, m = 1$ $\qquad$ (D) $n = 4, m = 4$

Ans: (B), (C)

Solution:

No. of optical modes = 9, $\quad$ No. of Acoustical modes = 3

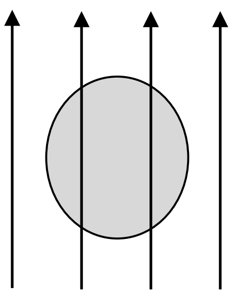
Total no. of modes $= 3 \times p = 12$

Thus no. of atoms per primitive cell $= p = 4$.

$p = n + m = 4$. Thus $n = 2, m = 2$ and $n = 3,\ m = 1$

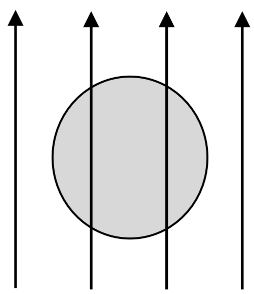
Q57. A material behaves as a superconductor below a critical temperature T_c and as a normal conductor above T_c. A magnetic field $\vec{B} = B\hat{z}$ is applied when $T > T_c$. The material is then cooled below T_c in the presence of $\vec{B}$. Which of the following figure represent the correct configuration of magnetic field lines?

(A)



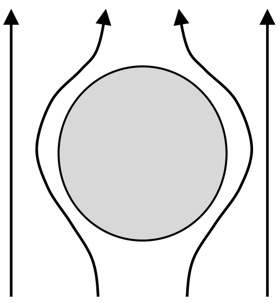
$T > T_C$

(B)



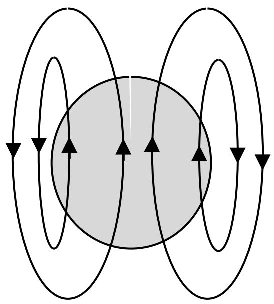
$T < T_C$

(C)



$T < T_C$

(D)



$T < T_C$

Ans: (A), (C)

Solution:

$B = \mu_0 (H + M)$ at $T > T_C$

$B = 0$ at $T < T_C$

Thus options (A) and (C) are correct.

Q62. Consider a three-dimensional system of non-interacting bosons with zero chemical potential. The energy of the system $\in \propto k^2$, where k is the wavevector. The low temperature specific heat of the system at constant volume depends on the temperature as $C_V \propto T^{\frac{n}{2}}$. The value of n is _____ (in integer).

Ans: 3

Solution:

$C_V \propto T^{\frac{n}{2}}$

For three-dimensions $n = 3$

Physics by fiziks

Learn Physics in Right Way

GATE Physics-2024

Solution-Nuclear and Particle Physics

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Q.11 – Q.35 Carry ONE mark Each

Q17. Consider the induced nuclear fission reaction ${}^{235}_{92}U + n \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2n$ where neutron momenta in both initial and final states are negligible. The ratio of the kinetic energies (KE) of the daughter nuclei, $\dfrac{KE\left({}^{93}_{37}Rb\right)}{KE\left({}^{141}_{55}Cs\right)}$ is ______________

(A) $\dfrac{93}{141}$ (B) $\dfrac{141}{93}$ (C) 1 (D) 0

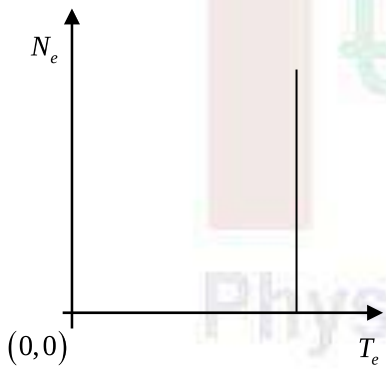
Ans: (B)

Solution:

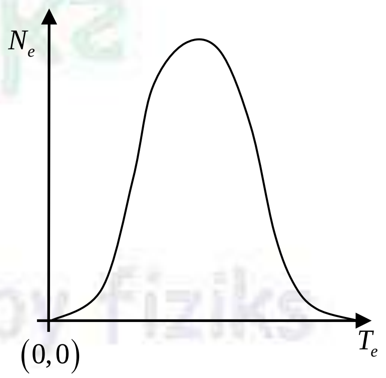
$$|\vec{p}_{Rb}| = |\vec{p}_{Cs}| = p; \quad \frac{KE_{Rb}}{KE_{Cs}} = \frac{p^2/2M_{Rb}}{p^2/2M_{Cs}} = \frac{M_{Cs}}{M_{Rb}} = \frac{141}{93}$$

Q19. Let N_e and T_e, respectively, denote number and kinetic energy of electrons produced in a nuclear beta decay. Which one of the following distributions is correct?

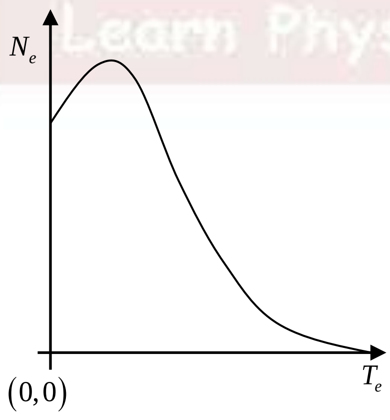
(A)



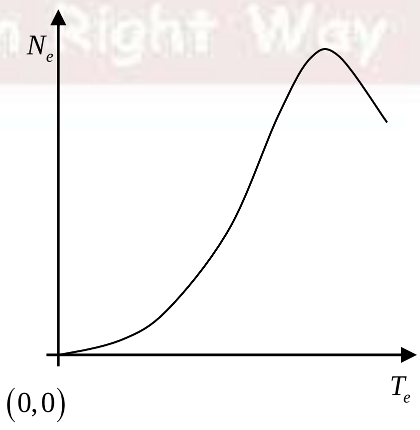
(B)



(C)



(D)



Ans: (C)

Solution: It is a standard energy spectrum for β^- decay.

Q26. Let P, Q and R be three different nuclei. Which one of the following nuclear processes is possible?

(A) $v_e + {}_z^AP \to {}_{z+1}^{A}Q + e^{-1}$ (B) $v_e + {}_z^AP \to {}_{z-1}^{A}R + e^{+}$

(C) $v_e + {}_z^AP \to {}_z^AP + e^{+} + e^{-}$ (D) $v_e + {}_z^AP \to {}_z^AP + \gamma$

Ans: (A)

Solution:

(b) Lepton number is not conserved.

(c) Z number is not conserved.

(d) Lepton number is not conserved.

Q.36 – Q.65 Carry TWO marks Each

Q36. Binding energy and rest mass energy of a two-nucleon bound state are denoted by B and mc^2, respectively, where c is the speed of light. The minimum energy of a photon required to dissociate the bound state is

(A) B (B) $B\left(1+\dfrac{B}{2mc^2}\right)$

(C) $B\left(1-\dfrac{B}{2mc^2}\right)$ (D) $B - mc^2$

Ans: (B)

Solution:

${}_1^2H + \gamma = p + n$; $B = \left(m_P + m_n\right)c^2 - mc^2$

Apply conservation of energy principle,

$$mc^2 + E_\gamma = \sqrt{\left(\frac{E_r}{c}\right)^2 c^2 + \left(m_P + m_n\right)^2 c^4} \Rightarrow mc^2 + E_\gamma = \sqrt{E_\gamma^2 + \left(B + mc^2\right)^2} \Rightarrow E_\gamma = B\left(1+\frac{B}{2mc^2}\right)$$

Q49. Decays of mesons and baryons can be categorized as weak, strong and electromagnetic decays depending upon the interactions involved in the processes. Which of the following option is/are true?

(A) $\pi^0 \to \gamma\gamma$ is a weak decay (B) $\wedge^0 \to \pi^0 + p$ is an electromagnetic decay

(C) $K^0 \to \pi^+ + \pi^-$ is a weak decay (D) $\Delta^{++} \to p + \pi^+$ is a strong decay

Ans: (C), (D)

Solution:

(a) As γ photon is involved, it cannot be a weak interaction.

(b) $\Lambda^0 \to \pi^0 + p$. Here third component of isospin (I_3) is not conserved. $\left[0 \neq 0 + \frac{1}{2}\right]$, so it cannot be electromagnetic interaction.

(c) Charge, B and L are conserved. I, I_3 and S are not conserved. So $k^0 \to \pi^+ + \pi^-$ is a weak interaction.

(d) Charge, B, spin and L are conserved. I, I_3 and S are conserved. So it is a strong interaction.

Q65. According to the nuclear shell model, the absolute value of the difference in magnetic moments of ${}^{15}_{8}O$ and ${}^{15}_{7}N$, in the units of nuclear magneton (μ_N) is $a/3$. The magnitude of a is __________ (*in integer*).

Ans: 2

Solution:

$${}^{15}_{8}O : n_n = 7 : \quad 1s^2_{1/2} 1p^4_{3/2} 1p^1_{1/2} : \ j = \frac{1}{2}$$

$$\langle \mu_z \rangle_0 = \langle \mu_z \rangle_n = 1.91 \frac{j}{j+1} \mu_N = 1.91 \frac{1/2}{1/2+1} \mu_N = 0.63 \mu_N$$

$${}^{15}_{7}N : n_p = 7 : 1s^2_{1/2} 1p^4_{3/2} 1p^1_{1/2} : j = \frac{1}{2}$$

$$\langle \mu_z \rangle_N = \langle \mu_z \rangle_p = \frac{j}{j+1}[j - 1.29]\mu_N = \frac{1/2}{1/2+1}\left[\frac{1}{2} - 1.29\right]\mu_N = -0.26 \mu_N$$

$$\langle \mu_z \rangle_0 - \langle \mu_z \rangle_N = (0.63 + 0.26)\mu_N = 0.89 \mu_N = \frac{2.67}{3} \mu_N \approx \frac{2}{3} \mu_N$$

Physics by fiziks

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GATE Physics-2024

Solution-General Aptitude

Be Part of Disciplined Learning

Section -GA (General Aptitude)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: –1/3).

Q1. If '→' denotes increasing order of intensity, then the meaning of the words [smile → giggle → laugh] is analogous to [disapprove → ________ → chide]. Which one of the given options is appropriate to fill the blank?

(A) reprove (B) praise (C) reprise (D) grieve

Ans: (A)

Solution: [smile → giggle → laugh] is analogous to [disapprove → reprove → chide].

Q2. Find the odd one out in the set: $\{19, 37, 21, 17, 23, 29, 31, 11\}$

(A) 21 (B) 29 (C) 37 (D) 23

Ans: (A)

Solution: All numbers in the set are prime except 21.

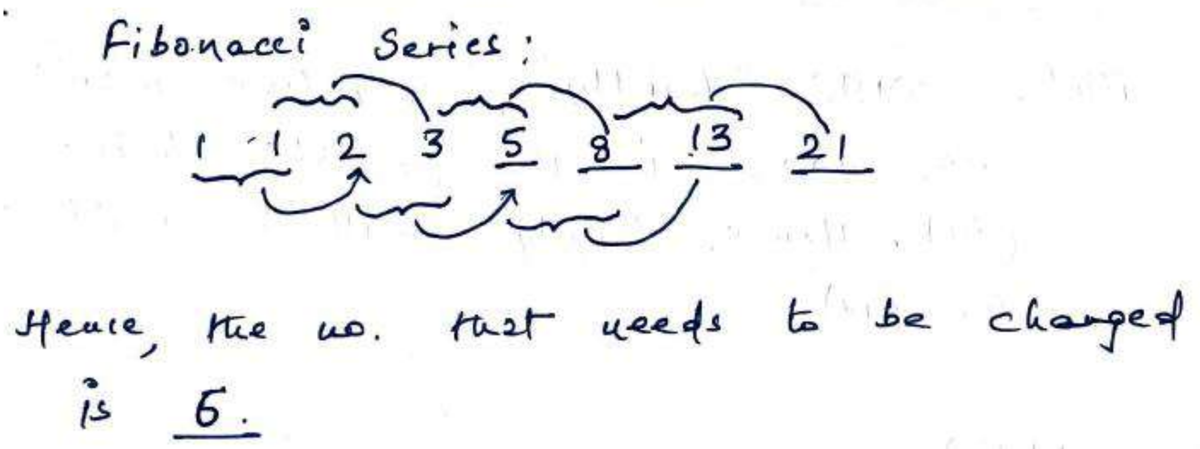
Q3. In the following series, identify the number that needs to be changed to form the Fibonacci series.

1, 1, 2, 3, 6, 8, 13, 21,…

(A) 8 (B) 21 (C) 6 (D) 13

Ans: (C)

Solution:



Q4. The real variables x, y, z and the real constants p, q, r satisfy

$$\frac{x}{pq - r^2} = \frac{y}{qr - p^2} = \frac{z}{rp - q^2}$$

Given that the denominators are non-zero, the value of $px + qy + rz$ is

(A) 0 (B) 1

(C) pqr (D) $p^2 + q^2 + r^2$

Ans: (A)

Solution:

$$\frac{x}{pq-r^2}=\frac{y}{qr-p^2}=\frac{z}{rp-q^2}=\frac{px+qy+rz}{p\left(pq-r^2\right)+q\left(qr-p^2\right)+r\left(rp-q^2\right)}=\frac{px+qy+rz}{0}$$

$\Rightarrow px+qy+rz=0$

Q5. Take two long dice (rectangular parallelepiped), each having four rectangular faces labelled as 2, 3, 5, and 7. If thrown, the long dice cannot land on the square faces and has 1/4 probability of landing on any of the four rectangular faces. The label on the top face of the dice is the score of the throw.

If thrown together, what is the probability of getting the sum of the two long dice scores greater than 11?

If thrown together, what is the probability of getting the sum of the two long dice scores greater than 11?

(A) 3/8 (B) 1/8 (C) 1/16 (D) 3/16

Ans: (D)

Solution:

Table shows all three possibilities of numbers on top faces.

Total probability

$$=\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{3}{16}$$

D_1	D_2
5	7
7	5
7	7

Q. 6 – Q. 10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: –2/3).

Q6. In the given text, the blanks are numbered (i)−(iv). Select the best match for all the blanks.

Prof. P________ (i) merely a man who narrated funny stories.__________ (ii) in his blackest moments he was capable of self-deprecating humor.

Prof. Q__________(iii) a man who hardly narrated funny stories._________(iv) in his blackest moments was he able to find humor.

(A) (i) was (ii) Only (iii) wasn't (iv) Even

(B) (i) wasn't (ii) Even (iii) was (iv) Only

(C) (i) was (ii) Even (iii) wasn't (iv) Only

(D) (i) wasn't (ii) Only (iii) was (iv) Even

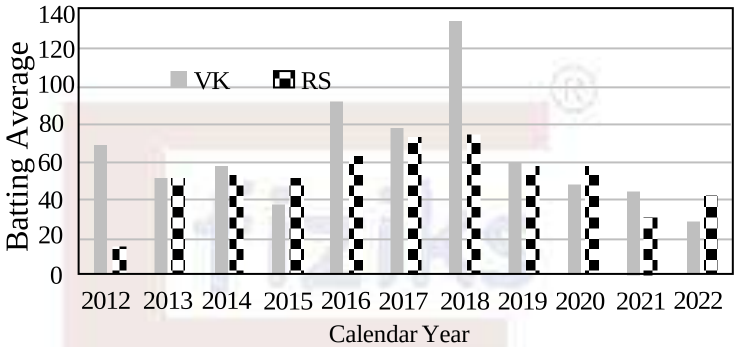
Ans: (B)

Q7. How many combinations of non-null sets A, B, C are possible from the subsets of $\{2,3,5\}$ satisfying the conditions: (i) A is a subset of B, and (ii) B is a subset of C?

(A) 28 (B) 27 (C) 18 (D) 19

Ans: MTA

Q8. The bar chart gives the batting averages of VK and RS for 11 calendar years from 2012 to 2022. Considering that 2015 and 2019 are world cup years, which one of the following options is true?



(A) RS has a higher yearly batting average than that of VK in every world cup year.

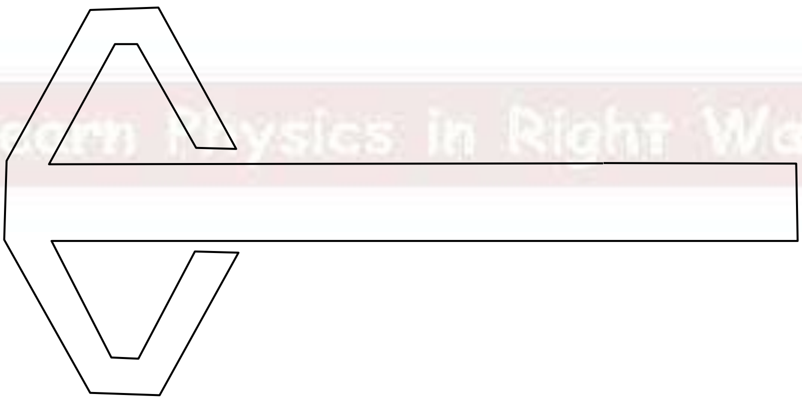
(B) VK has a higher yearly batting average than that of RS in every world cup year.

(C) VK's yearly batting average is consistently higher than that of RS between the two world cup years.

(D) RS's yearly batting average is consistently higher than that of VK in the last three years.

Ans: (C)

Q9. A planar rectangular paper has two V-shaped pieces attached as shown below.



This piece of paper is folded to make the following closed three-dimensional object.

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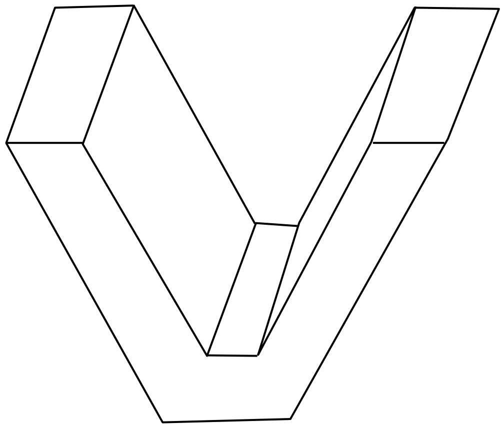
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com



The number of folds required to form the above object is

(A) 9 (B) 7 (C) 11 (D) 8

Ans: (A)

Q10. Four equilateral triangles are used to form a regular closed three-dimensional object by joining along the edges. The angle between any two faces is

(A) 30° (B) 60°

(C) 45° (D) 90°

Ans: MTA