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Question Paper -2025

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GS- 2025 ADMISSION TEST

Instructions for all candidates appearing for the Physics test for Ph.D. or Integrated Ph.D.

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

- You may not keep with you any books, papers, mobile phones, or any electronic devices which can be used to get/store information. Use of **scientific, non-programmable calculators is permitted.** Calculators which plot graphs are not allowed. Multiple use devices, such as smart phone, etc. CANNOT be used as calculator.
- This test consists of TWO sections.
  - **SECTION A comprises 25 questions, numbered Q1 - Q25. ----**These are questions on basic topics.
  - **SECTION B comprises 15 questions, numbered Q1 - Q15. ----**These may require somewhat more thought/knowledge.
  - **SECTION C comprises 15 questions, numbered Q1 - Q15. ----**These may require somewhat more thought/knowledge.
- ALL questions are Multiple-Choice Type. In each case, ONLY ONE option is correct. Answer them by clicking the radio button next to the relevant option.
- If your calculated answer does not match any of the given options exactly, you may mark the closest one if it is reasonably close.
- The **grading scheme** will be as follows:
  - Section A : **+3** marks if correct; **–1** mark if incorrect; **0** marks if not attempted
  - Section B : **+5** marks if correct; **0** marks if incorrect or not attempted, i.e. NO negative marks.
  - Section C : **+5** marks if correct; **0** marks if incorrect or not attempted, i.e. NO negative marks.

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Section-A

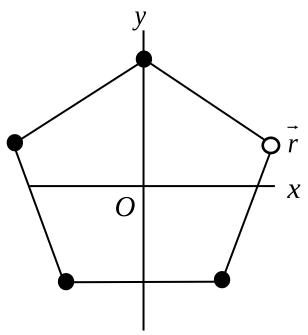
(For both Integrated M.Sc.--Ph.D. and Ph.D. candidates)

Q1. Consider the triangle subtended on the surface of a sphere of radius 1 by joining the points $\left(\frac{1}{2},\frac{\sqrt{3}}{2},0\right)$, $\left(-\frac{1}{2},\frac{\sqrt{3}}{2},0\right)$, and $(0,0,1)$ with arcs of great circles. The area subtended by this triangle on the surface of the sphere is given by: (Hint: Drawing a figure might help.)

(a) $\sqrt{3}\pi$ (b) $\sqrt{3}\pi/2$ (c) $\pi/3$ (d) $2\pi/3$

Ans.: (c)

Q2. The Figure on the right shows a regular pentagon. The black solid circles on its vertices represent point charges with charge $-q$. There is no charge at the position of the white circle at $\vec{r}$ (measured from the origin, O, placed at the center of the pentagon). The electric field at O is given by:



(a) $\vec{E}=\frac{-q\vec{r}}{4\pi\epsilon_0 r^3}$ (b) $\vec{E}=\frac{-4q\vec{r}}{4\pi\epsilon_0 r^3}$

(c) $\vec{E}=\frac{q\vec{r}}{4\pi\epsilon_0 r^3}$ (d) $\vec{E}=\frac{-4q\left(\sin\frac{\pi}{10}\hat{x}+\cos\frac{\pi}{10}\hat{y}\right)}{4\pi\epsilon_0 r^2}$

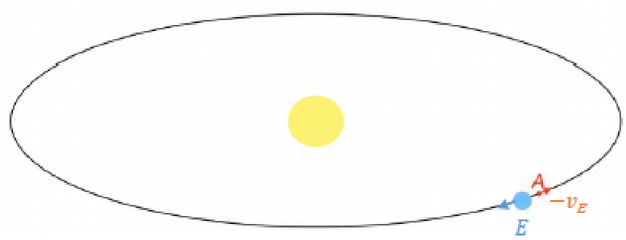
Ans.: (a)

Q3. Consider a two-dimensional insulating solid crystal. At low temperature, how does the specific heat at constant area $c_a=\frac{d\varepsilon}{dT}$, where ε is the energy per unit area, depend on T?

(a) $c_a \sim T^3$ (b) $c_a \sim T^2$ (c) $c_a \sim T$ (d) c_a is independent of T

Ans.: (b)

Q4. The Figure below shows a rocket (red arrow) launched from the earth which is now at a point A where the Earth's gravitational field is negligible. The rocket thrusters have stopped. In the rest frame of the Sun, the velocity of the rocket at A is same in magnitude but opposite in direction to that of the earth was, when it was at the same point. Which of the following statements is correct?



(a) The rocket will move exactly on the earth's elliptical orbit shown in the figure and eventually collide with the earth

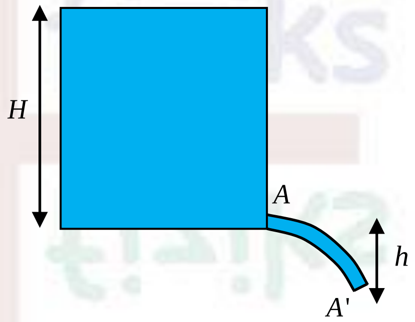
(b) The rocket will eventually escape the Sun's gravitational field

(c) The rocket will eventually reverse its direction and follow the earth

(d) The rocket will turn towards the sun and eventually collide with it

Ans.: (a)

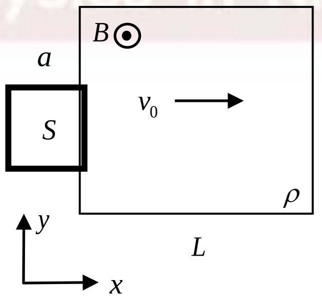
Q5. Water is flowing out of a small horizontal opening of area, A, at the bottom of a tank of height H. The flow is a laminar flow under the influence of gravity. What is the area, A', of the stream transverse to the fluid velocity, at a height h below the opening? (Neglect atmospheric pressure and dissipation effects. The thickness of the stream is negligible compared to H and h.)



(a) $A\sqrt{1+\frac{h}{H}}$ (b) $A\frac{H}{h+H}$ (c) $A\left(1+\frac{h}{H}\right)$ (d) $A\sqrt{\frac{H}{h+H}}$

Ans.: (d)

Q6. A small metallic wire with mass m and electric resistance R is bent into a closed square shape S with sides a. It passes through a region ρ of length $L>a$ with magnetic field $B\hat{z}$. The initial velocity $v_0\hat{x}$ of the square is large enough that it emerges out of ρ from the right. What is the final velocity of S after it completely emerges from ρ?



(a) $v_0\left(1-\frac{a^3B^2}{mRv_0}\right)\hat{x}$ (b) $v_0e^{-\frac{a^2B^2L}{mRv_0}}\hat{x}$ (c) $v_0\left(1-\frac{a^3B^2}{mRv_0}\right)^2\hat{x}$ (d) $v_0\hat{x}$

Ans.: (c)

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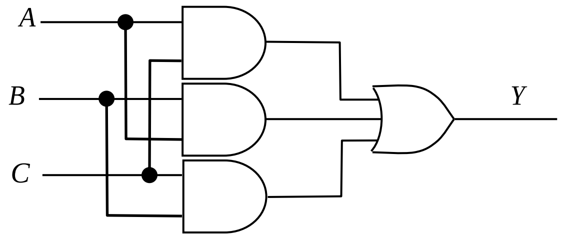
Q7. The output pulse train Y of the circuit shown on the right, with three synchronized input trains, $A = 00001111$; $B = 00110011$; $C = 01010101$ will be



(a) 00010111 (b) 00100111 (c) 01010101 (d) 00010001

Ans.: (a)

Q8. Consider a stationary electron in a uniform, time-independent magnetic field of strength $B_0/4$ oriented in the $\hat{z}$-direction. The Hamiltonian for this system is expressed as

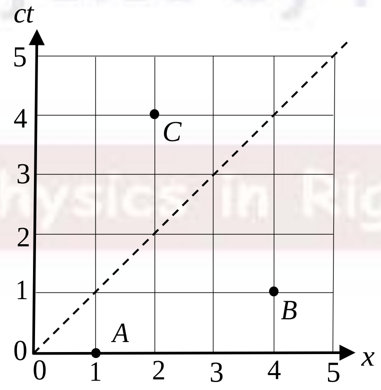
$$H = -\frac{e}{m}\vec{S}\cdot\vec{B}$$

where $\vec{S}$ is the spin-1/2 operator for electrons. The initial electron spin is oriented in the $\hat{x}$-direction. The spin precession frequency of the electrons is:

(a) $\frac{|e|B_0}{4m}$ (b) $\frac{|e|B_0}{8m}$ (c) $\frac{|e|B_0}{2m}$ (d) 0

Ans.: ***(NB: Due to a typo in the options all test takers will be awarded the full score.)***

Q9. Consider the following space-time diagram which indicates three events A, B and C for an inertial observer. Which of the following statements is true?



(a) It is always possible to find an inertial observer for whom events A and B are simultaneous. However, no inertial observer can be found for whom events A and C are simultaneous.

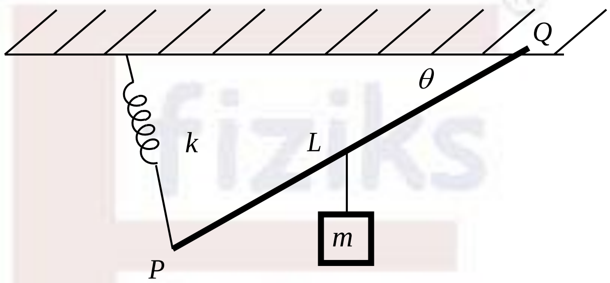
(b) It is always possible to find an inertial observer for whom events A and C are simultaneous. However, no inertial observer can be found for whom events A and B are simultaneous.

(c) It is always possible to find an inertial observer for whom events A and B are simultaneous. Similarly, an inertial observer can also be found for whom events A and C are simultaneous.

(d) It is impossible to find an inertial observer for whom events A and B are simultaneous. Similarly, no inertial observer can be found for whom events A and C are simultaneous.

Ans.: (a)

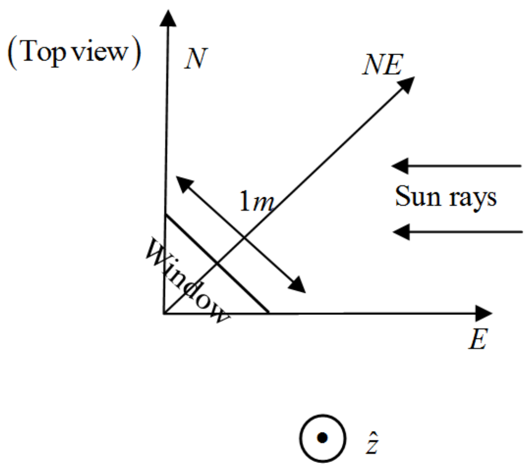
Q10. A massless rigid rod of length L is suspended with an ideal spring of spring constant k at one end P, and by a hinge on the other end, Q. The rest length of the spring is zero. A mass m is suspended from the mid-point of the rod. This results in titling of the rod by angle θ. What is the angle θ?



(a) $\sin^{-1}\left(\frac{mg}{2kL}\right)$ (b) $\tan^{-1}\left(\frac{mg}{2kL}\right)$ (c) $\cos^{-1}\left(\frac{mg}{kL}\right)$ (d) $\sec^{-1}\left(\frac{mg}{kL}\right)$

Ans.: (b)

Q11. There is an open window of dimension $1m\times 1m$ on the north east (NE) facing wall of a house. At 9 AM, the sun shines through the window and illuminates a certain part of the floor of the house. What is the area A illuminated by the sun? (Assume that the sun rises in the east (E) at 6 AM and is directly overhead $(\hat{z})$ at 12 noon.)



(a) $\sqrt{2}m^2$ (b) $1m^2$ (c) $\frac{1}{2}m^2$ (d) $\frac{1}{\sqrt{2}}m^2$

Ans.: (d)

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Q12. For a one-dimensional quantum harmonic oscillator, at time $t=0$, the particle is in the ground state. What is the expectation value of the position and momentum operator at time t?

(a) $\langle x(t)\rangle = \langle p(t)\rangle = 0$

(b) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}}\sin\omega t, \langle p(t)\rangle = 0$

(c) $\langle x(t)\rangle = \sqrt{\frac{\hbar}{m\omega}}\sin\omega t, \langle p(t)\rangle = \sqrt{\hbar m\omega}\cos\omega t$

(d) $\langle x(t)\rangle = 0, \langle p(t)\rangle = \sqrt{\hbar m\omega}\cos\omega t$

Ans.: (a)

Q13. Consider two ideal gases A and B with atomic masses m_A and m_B respectively such that $m_A > m_B$. The two gases with same number of moles are kept at the same temperature and confined in containers with the same volume. Which of the gases will exert more pressure and molecules of which gas will have a higher RMS momentum?

(a) Both will exert the same pressure but molecules of Gas A will have more RMS momentum

(b) Gas A will exert more pressure and molecules of Gas B will have more RMS momentum

(c) Gas B will exert more pressure but molecules of Gas A will have more RMS momentum

(d) Both will exert the same pressure and molecules of both gases have the same RMS momentum

Ans.: (a)

Q14. For a given measurement of particles in a counter, a 10-minute data collection resulted in a statistical uncertainty of 2.5%. How much additional time must be allocated to reduce the statistical uncertainty to 0.5%?

(a) 40 minutes (b) 240 minutes (c) 250 minutes (d) 50 minutes

Ans.: (b)

Q15. Laser light is incident normally on a thin film of material with a refractive index (n_S) larger than that of air $(n_a \approx 1)$. As the wavelength of the laser light is varied, the intensity of the transmitted light through the film shows a peak at $633\,nm$. If the thickness of the film is $118\,nm$, the minimum n_S is closest to:

(a) 3.68 (b) 5.36 (c) 1.34 (d) 2.68

Ans.: (d)

Q16. The asymptotic expansion of the following function for $x \to \infty$

$$x \tanh^{-1}\frac{1}{x}$$

is given by:

(a) $1-\frac{1}{3x^2}+\frac{1}{5x^4}-\frac{1}{7x^6}+\ldots$

(b) $1+\frac{1}{3x^2}+\frac{1}{5x^4}+\frac{1}{7x^6}+\ldots$

(c) $x+\frac{1}{2x}+\frac{1}{4x^3}+\frac{1}{6x^5}+\ldots$

(d) $1+\frac{1}{2x^2}+\frac{1}{4x^4}+\frac{1}{6x^6}+\ldots$

Ans.: (b)

Q17. The $n \times n(n>4)$ matrix M, with all entries equal to 1 has:

(a) Precisely $n-1$ degenerate eigenvalues and one other non-degenerate eigenvalue

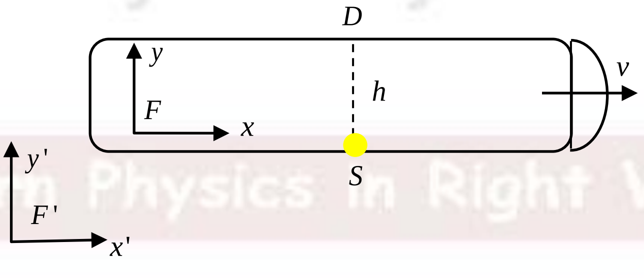
(b) Precisely $n-2$ degenerate eigenvalues and two other non-degenerate eigenvalues

(c) Precisely 2 degenerate eigenvalues and $n-2$ other non-degenerate eigenvalues

(d) No degenerate eigenvalues

Ans.: (a)

Q18. A spaceship is moving with a constant relativistic velocity $v\hat{x}'$ with respect to an inertial frame F'. In the frame F moving with spaceship, light is emitted from the source S and is detected at the detector D with displacement $h\hat{y}$ from S. In the frame F', what is the time t' taken for the light to reach from S to D?



(a) $\left(\frac{h}{c}\right)$

(b) $\left(\frac{h}{c}\right)\sqrt{1-v^2/c^2}$

(c) $\left(\frac{h}{c}\right)\sqrt{\frac{1-v/c}{1+v/c}}$

(d) $\frac{\left(\frac{h}{c}\right)}{\sqrt{1-v^2/c^2}}$

Ans.: (d)

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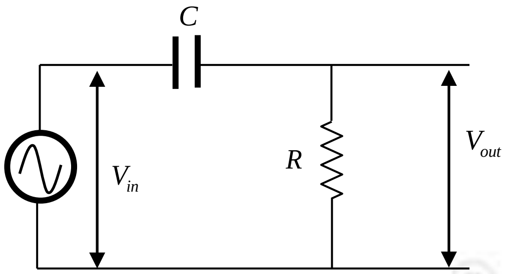
Q19. The sinusoidal signal $V_{in} = V_i \sin(2\pi ft)$ is given to a high-pass filter (see Figure).

The output signal is given by $V_{out} = V_i |A| \sin(2\pi ft + \phi)$.

What is the value of $|A|$?



(a) $\dfrac{1}{\left|1+\left(\dfrac{1}{2\pi RCf}\right)^2\right|}$

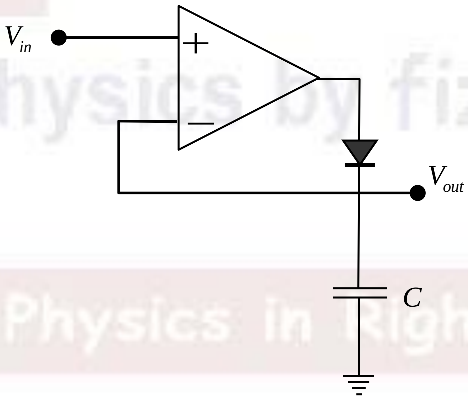
(b) $\dfrac{1}{\left|1+\left(\dfrac{1}{2\pi RCf}\right)\right|}$

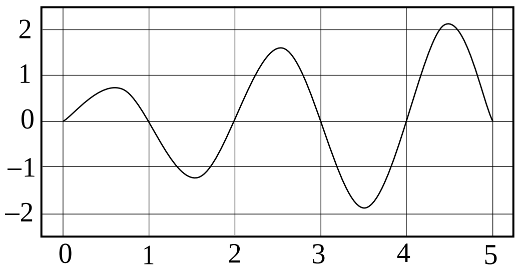
(c) $\dfrac{1}{\left|1+\left(\dfrac{1}{2\pi RCf}\right)^2\right|^{1/2}}$

(d) $\dfrac{1}{\left|1+\left(\dfrac{1}{2\pi RCf}\right)\right|^{1/2}}$

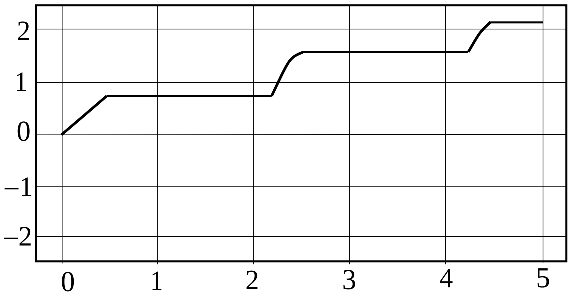
Ans.: (c)

Q20. For the circuit on the right, which graph represents V_{out} correctly for the V_{in} shown below?

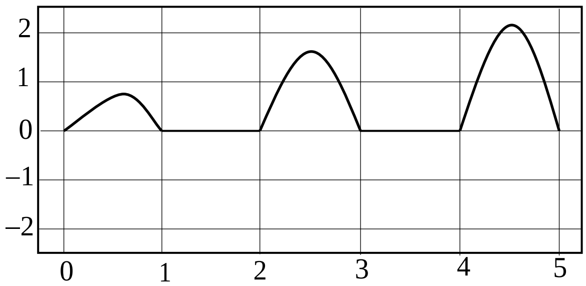




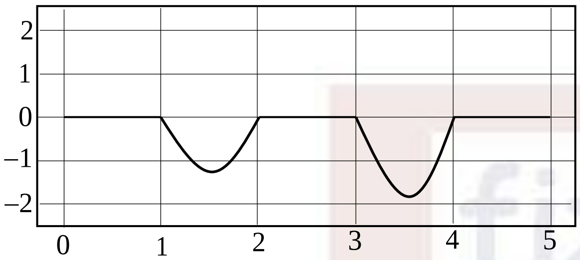
(a)



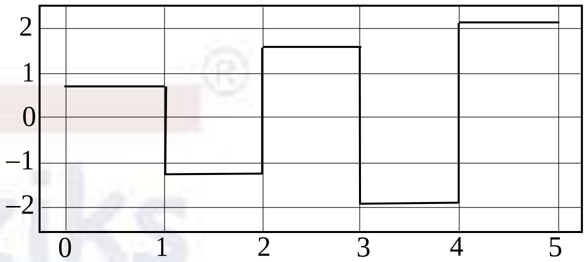
(b)



(c)

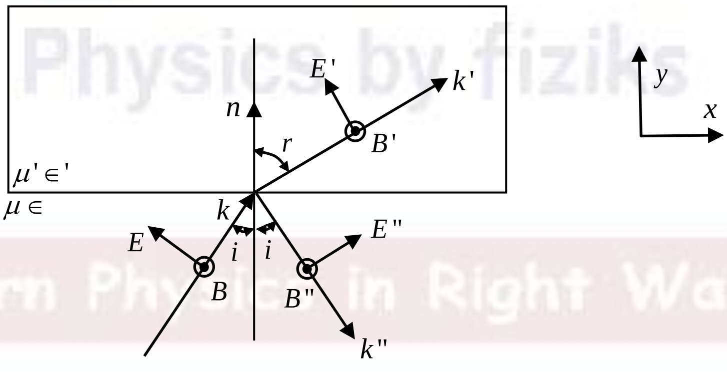


(d)



Ans.: (a)

Q21. Light in medium with electric permittivity $\in$ and magnetic permeability μ is incident on a medium with electric permittivity $\in'$ and magnetic permeability μ'. The angle of incidence is i. The E field is linearly polarized in the plane as shown, and the B field is in the $\hat{z}$ direction. Which of the following is a correct boundary condition on the fields?



(a) $\in(E\cos i - E''\cos i) = \in' E'\cos r$

(b) $\in(E\sin i + E''\sin i) = \in' E'\sin r$

(c) $E\sin i + E''\sin i = E'\sin r$

(d) $\mu(B + B'') = \mu' B'$

Ans.: (b)

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Q22. An experimental set-up needs to be kept in an environment with zero magnetic field by minimizing the Earth's magnetic field. This can be achieved by:

(a) Keeping the setup in a place completely covered with a sheet of metal of very high magnetic permeability

(b) Keeping the setup in a place completely covered with a sheet of metal of very high permittivity

(c) Keeping the setup at the center of the interior of a long solenoid

(d) Keeping the setup in a place completely covered with a sheet of an insulating material

Ans.: (a)

Q23. Two types of particles A and B have the same mass, but are distinguished by an internal degree of freedom. A classical ideal gas in a volume V at temperature T contains (X) $2N$ particles of A-type and (Y) N particles of A-type and N particles of B-type. Which of the following is true?

(a) Pressure of (X) and (Y) are same; (Y) has more entropy than (X)

(b) Pressure of (X) and (Y) are same; (X) has more entropy than (Y)

(c) Pressure of (X) is greater than pressure of (Y); (X) has more entropy than (Y)

(d) Pressure of (X) is greater than pressure of (Y); (Y) has more entropy than (X)

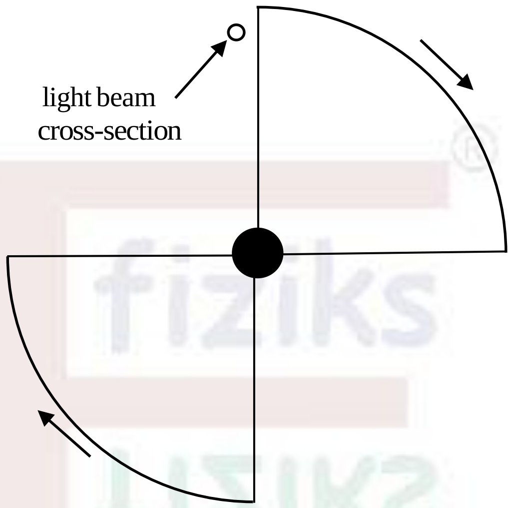
Ans.: (a)

Q24. A negative electric charge $-q$ moves in a (classical) circular orbit at a non-relativistic speed v around a positive charge. The magnetic field at the center of the circular orbit due to the negative charge is found to be B_1. Now, consider another situation where a negative charge $-2q$ moves in a circular orbit at the same speed v around the same positive charge. The magnetic field at the center of the circular orbit in this case is B_2. What is the ratio B_2/B_1?

(a) 1/2 (b) 1 (c) 2 (d) 4

Ans.: ***(NB: Due to an ambiguity in the question all test takers will be awarded the full score.)***

Q25. Consider a fan with blades rotating with frequency f, as shown in the Figure. It is used to periodically block a light beam of intensity I_0. The beam has a very small cross-sectional area and hits the blade near its outer edge, as shown. The transmitted beam is detected by a photo-detection unit which gives out a voltage signal V proportional to the transmitted intensity I. If this voltage signal pattern is displayed on an oscilloscope, what would best describe the signal pattern?



(a) $V_0 \sum_n \left[\cos^2(2n\pi ft) - \sin^2(2n\pi ft)\right],\ n = 2,6,10,14...$

(b) $V_0 \left[\frac{1}{2} + \sum_n \frac{4}{\pi n} \sin(2n\pi ft)\right],\ n = 2,6,10,14...$

(c) $V_0 \left[\frac{1}{2} + \frac{1}{2} \sin(4\pi ft)\right]$

(d) $V_0 \left[\cos^2(4\pi ft)\right]$

Ans.: (b)

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Section B

(only for Integrated M.Sc.-Ph.D. candidates)

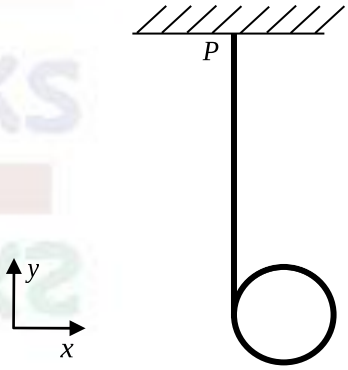
Q1. Let $|nlm\rangle$ denote the energy eigenstates of non-relativistic hydrogen atoms without spin, and a_0 is the Bohr radius. The matrix element

$$\langle n=2, l=1, m_z=0|\hat{x}|n=2, l=0, m_z=0\rangle \text{ is:}$$

(a) $\sqrt{2}a_0$ (b) 0 (c) a_0 (d) $\sqrt{3}a_0$

Ans.: (b)

Q2. The given Figure shows some charges and their coordinates in the $x-y$ plane. The electric potential at a point, r, far from the origin, is given by:



(a) $\phi = \dfrac{qd\left(\vec{r}\cdot\hat{x}+2\vec{r}\cdot\hat{y}\right)}{4\pi\in_0 r^3}$

(b) $\phi = \dfrac{qd}{4\pi\in_0 r^2}$

(c) $\phi = \dfrac{qd\vec{r}\cdot\hat{z}}{4\pi\in_0 r^3}$

(d) $\phi = \dfrac{qd\vec{r}\cdot\hat{x}}{4\pi\in_0 r^3}$

Ans.: (a)

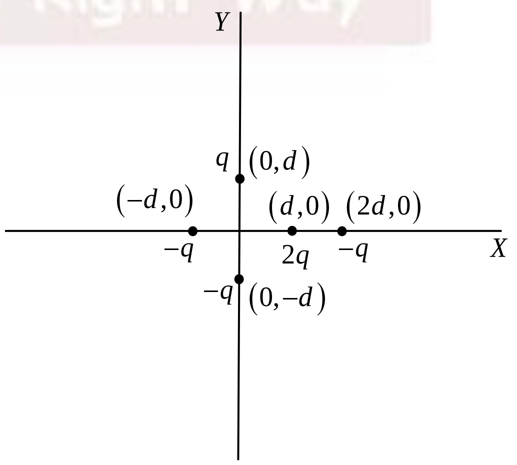
Q3. Consider a massless string with one end fixed on the ceiling (P). The total length of the string is L and it is initially completely wrapped around a solid uniform disc of radius R (assume $R \ll L$) and mass m. The disc is released from rest from the ceiling at time $t=0$ and falls under gravity. The thread unwinds from the disc, always remaining tight. What is its velocity at a time t before the thread unwinds fully?

(a) $-\dfrac{gt}{2}\hat{y}$

(b) $-gt\,\hat{y}$

(c) $-\dfrac{3gt}{4}\hat{y}$

(d) $-\dfrac{2gt}{3}\hat{y}$



Ans.: (d)

Q4. A quantum particle is in the ground state of an infinite potential well of length L with

$$V(x)=\begin{cases} 0 & \text{for } x\in[0,L] \\ +\infty & \text{otherwise} \end{cases}$$

What is the expectation value of the operator, $\hat{O}=\hat{x}\hat{p}+\hat{p}\hat{x}$ in this state?

(a) $i\hbar$ (b) 0 (c) $\hbar/2$ (d) $-i\hbar$

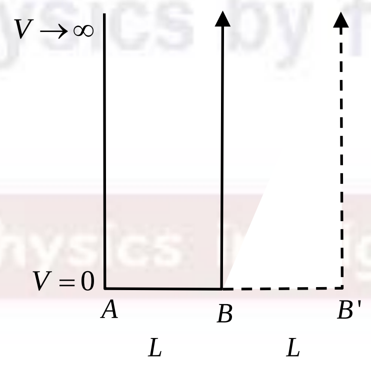
Ans.: (b)

Q5. Consider a random walker on a 2D plane which starts at the origin. At every step it either moves one unit along the positive x-axis with probability 1/2 or along the positive y-axis with probability 1/2. The distance from the origin after n steps is denoted by r_n. What is the mean square displacement $\langle r_n^2 \rangle$?

(a) $n(n-1)$ (b) $n(n-1)/2$ (c) n^2 (d) $n(n+1)/2$

Ans.: (d)

Q6. Consider a quantum particle of mass m in an infinite one-dimensional potential well of length L between points A and B. The particle is in the ground state with an energy E_g. The wall at B is suddenly shifted to B' where AB' has length $2L$. We measure the energy again, and obtain the value E_1. What is the probability that $E_1 \neq E_g$?



(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) 0

Ans.: (a)

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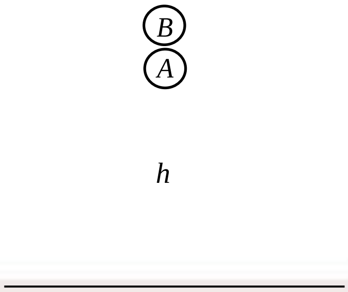
Q7. A ball A is dropped on the floor from height h. It bounces up to height $h/4$ on the first bounce. Now identical balls A and B are dropped together from height h as shown. How high does the ball B bounce on the first bounce? Assume that the coefficient of restitution between balls A and B is 1. (Ignore the size and the small initial separation of the balls.)



(a) $h/2$ (b) h (c) $h/4$ (d) $h/8$

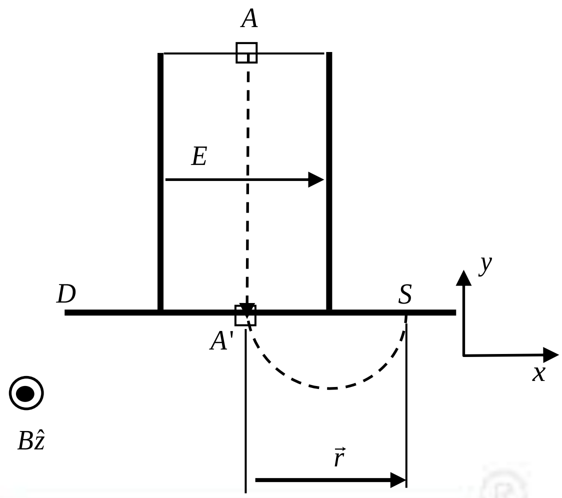
Ans.: (c)

Q8. Consider a spherical planet with a radius $R = 6400\,km$. The density $\rho(r)$ for r varies as $\rho(r) \propto r$, where r is the distance from the center of the planet. A tunnel is dug through the center, and the escape speed is measured at various distances r. At the planet's surface, the escape speed is found to be 11.2 km.sec, and at a distance of $3200\,km$ from the center, it is $12.7\,km/\text{sec}$. What is the escape speed at the center of the planet?

(a) 12.9 km/sec (b) 14.2 km/sec (c) 13.2 km/sec (d) 0 km/sec

Ans.: (a)

Q9. There is a uniform electric field $E\hat{x}$ between two parallel plates of a capacitor (parallel to the xz plane). The plates are placed in a uniform magnetic field $B\hat{z}$ which fills the entire region (inside and outside the capacitor). Charged particles enter the capacitor through a small aperture A. They exit from a small aperture A' at the other end, if they do not deviate from a straight line path. There is a detector plate D in the xz plane passing through A'. D detects where the particles impinge. What is the displacement vector $\vec{r}$ between the impact point S and the aperture A' for a particle with mass m and charge q? (This device is a simple version of a mass spectrometer.)



(a) $\frac{-mE}{2qB^2}\hat{x}$ (b) $\frac{-mE}{qB^2}\hat{x}$ (c) $\frac{-2mE}{qB^2}\hat{x}$ (d) $\frac{mE}{qB^2}\hat{x}$

Ans.: (c)

Q10. A stream of electrons, each having an energy of $0.5eV$, impinges on a pair of extremely thin slits separated by $10\,\mu m$. The distance between adjacent minima on a screen $20m$ behind the slits would be closest to:

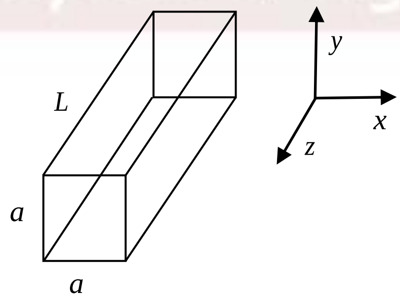
(a) $3.48mm$ (b) $1.74mm$ (c) $6.96cm$ (d) $5m$

Ans.: (a)

Q11. A very long square pipe with length L and cross-sectional area $a^2\left(L \gg a\right)$ has ideal conducting walls. A travelling mode with

$$E_z\left(\vec{r},t\right) = A\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}e^{ikz-i\omega t}$$

is excited in the pipe. What is the relation between k and ω? (Assume $\in\approx\in_0, \mu\approx\mu_0$.)



(a) $k=\sqrt{\frac{\omega^2}{c^2}+\frac{2\pi^2}{a^2}}$ (b) $k=\sqrt{\frac{\omega^2}{c^2}-\frac{2\pi^2}{a^2}}$ (c) $k=\frac{\omega}{c}$ (d) $k=\sqrt{\frac{\omega^2}{c^2}+\frac{\pi^2}{a^2}}$

Ans.: (b)

Q12. The general solution of the equation

$$\frac{d^3y}{dx^3}+k^3y=0\left(k>0\right)$$

is given by:

(a) $C_1e^{-kx}+C_2e^{\frac{kx}{2}}\cos\left(\sqrt{3}kx/2\right)+C_3e^{\frac{kx}{2}}\sin\left(\sqrt{3}kx/2\right)$

(b) $C_1e^{-kx}+C_2e^{\frac{-kx}{2}}\cos\left(\sqrt{3}kx/2\right)+C_3e^{\frac{-kx}{2}}\sin\left(\sqrt{3}kx/2\right)$

(c) $C_1e^{-kx}+C_2e^{\frac{kx}{2}}\cos\left(\sqrt{3}kx/2\right)+C_3e^{\frac{kx}{2}}\sin\left(kx/2\right)$

(d) $C_1e^{-kx}+C_2e^{\frac{kx}{2}}\cos\left(kx/2\right)+C_3e^{\frac{kx}{2}}\sin\left(\sqrt{3}kx/2\right)$

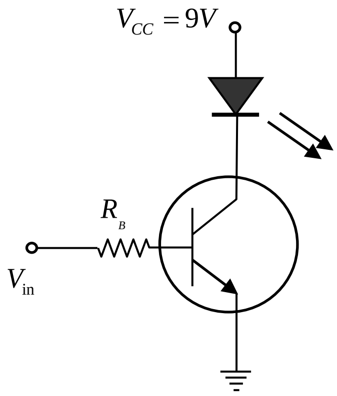
Ans.: (a)

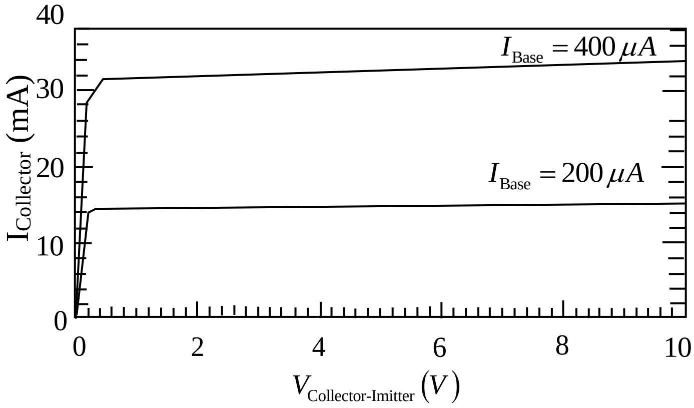
Q13. Radiation from the Big-Bang observed today has a black-body spectrum with $T=2.7K$ and its energy density as a function of the wavelength peaks at $\lambda=1.1mm$. This radiation is red-shifted to longer wave lengths compared to the black-body spectrum when the universe was hotter, say at 270 K. What was the photon energy corresponding to the wavelength at which the energy density peaked, when the universe was at 270 K?

(a) 0.12 eV (b) 1.2 meV (c) 23 meV (d) 0.23 eV

Ans.: (a)

Q14. Consider the Collector-Emitter characteristics of a silicon NPN transistor in the Figure below. The circuit on the right is for lighting and LED with an input voltage $V_{in}=1V$. The LED needs $20mA$ current, that will be provided by the transistor. A forward biased silicon PN junction has a $0.7V$ drop across it. What is the closest value of resistor R_B needed for this purpose?





(a) $15k\Omega$ (b) 560Ω (c) $4.2k\Omega$ (d) $1.3k\Omega$

Ans.: (d)

Q15. A classical ideal gas at temperature T is placed in a spherically symmetric potential

$$V(r) = cr^3$$

What is $\langle V(r) \rangle$ per particle?

(a) kT (b) $3kT/2$ (c) $kT/2$ (d) $kT/3$

Ans.: (a)

Section C

(only for Ph.D. candidates)

Q1. Consider two random variables x and y described by the joint distribution

$$P(x,y)=\frac{1}{2\pi\sqrt{1-a^2}}e^{\frac{2axy-x^2-y^2}{2(1-a^2)}}$$

with $0<a<1$. If the above distribution is written in terms of orthogonal coordinates $z=x-y$ and $u=x+y$, the probability distribution in z is given by:

(a) A Gaussian with mean 0 and standard deviation $\sqrt{2(1-a)}$

(b) A Gaussian with mean $\sqrt{a}$ and standard deviation $\sqrt{2(1-a)}$

(c) A Gaussian with mean 0 and standard deviation $\sqrt{2(1-a^2)}$

(d) Not a Gaussian distribution

Ans.: (a)

Q2. Consider a particle with mass m in a quantum harmonic oscillator potential with a frequency ω, such that its Hamiltonian is

$$\hat{H}=\frac{\hat{p}^2}{2m}+\frac{m\omega^2\hat{x}^2}{2}$$

The Hamiltonian is perturbed by adding a term to the potential

$$\Delta\hat{H}=\lambda\sin\hat{x}$$

where λ is small compared to $\hbar\omega$. The relative change in the ground state energy, to the leading order in $\lambda/(\hbar\omega)$ is given by:

(a) $O\left(\frac{\lambda}{(\hbar\omega)}\right)$

(b) $O\left(\frac{\lambda^2}{(\hbar\omega)^2}\right)$

(c) $O(1)$

(d) The ground state energy does not change

Ans.: (b)

Q3. A relativistic particle moving under the central force of gravity experiences the following effective potential:

$$V_{eff}(r)=-\frac{GMm}{r}+\frac{l^2}{2mr^2}-\frac{GMl^2}{mc^2r^3}$$

where the last term is the relativistic correction to the Newtonian formula. The smallest radius at which a stable circular orbit can exist for some value of the angular momentum l is given by:

(a) $\frac{6GM}{c^2}$ (b) $\frac{3GM}{c^2}$

(c) $\frac{2GM}{c^2}$ (d) There are no stable circular orbits

Ans.: (a)

Q4. The integral

$$\int_{-\infty}^{+\infty} dk \frac{e^{-ikx}}{k^2+1}$$

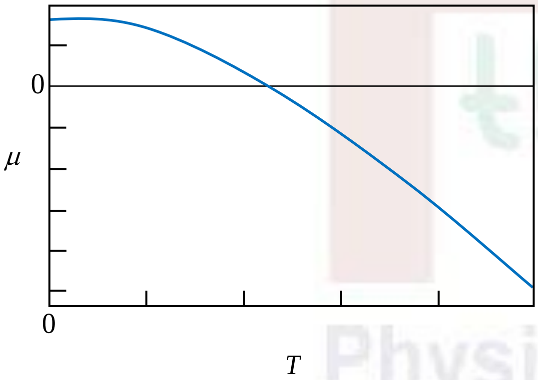
is given by:

(a) πe^{-x} (b) πe^{x} (c) $-\pi e^{-x}$ (d) $-\pi e^{x}$

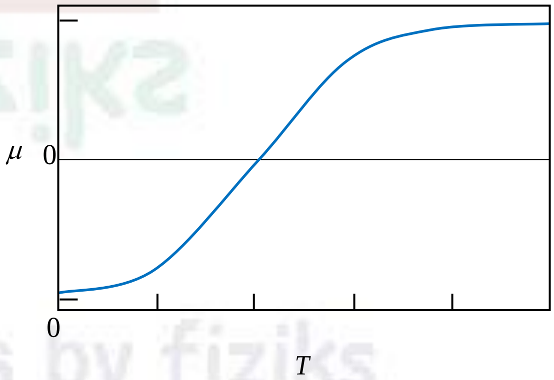
Ans.: ***(NB: Due to an ambiguity in the question all test tankers will be awarded the full score.)***

Q5. Consider a (non-relativistic) gas of fermions in a container with a fixed density n. Which plot best describes how the chemical potential μ changes with T?

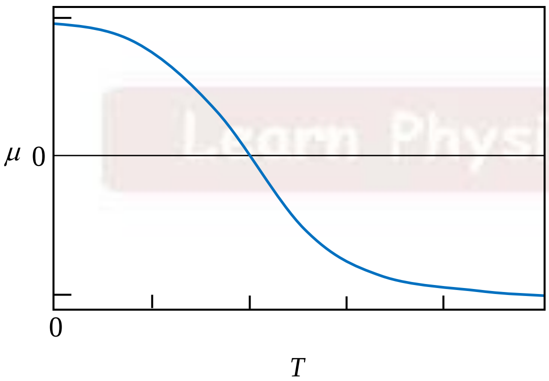
(a)



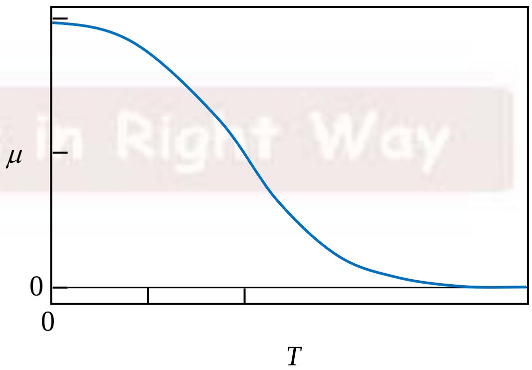
(b)



(c)



(d)



Ans.: (a)

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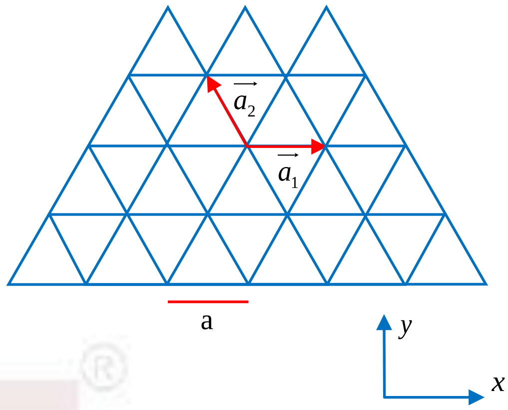
Q6. A triangular lattice with lattice constant a has primitive vectors $\vec{a_1}$ and $\vec{a_2}$, as shown in the Figure. The primitive wavevectors for the reciprocal lattice are given by:



(a) $\vec{b_1} = \frac{2\pi}{a}\hat{x} + \frac{2\pi}{a\sqrt{3}}\hat{y}, \quad \vec{b_2} = \frac{4\pi}{a\sqrt{3}}\hat{y}$

(b) $\vec{b_1} = \frac{2\pi}{a\sqrt{3}}\hat{x} + \frac{2\pi}{a}\hat{y}, \quad \vec{b_2} = \frac{4\pi}{a\sqrt{3}}\hat{y}$

(c) $\vec{b_1} = \frac{2\pi}{a}\hat{x} - \frac{2\pi}{a\sqrt{3}}\hat{y}, \quad \vec{b_2} = \frac{4\pi}{a\sqrt{3}}\hat{x}$

(d) $\vec{b_1} = \frac{2\pi}{a\sqrt{3}}\hat{x} - \frac{2\pi}{a}\hat{y}, \quad \vec{b_2} = \frac{4\pi}{a\sqrt{3}}\hat{x}$

Ans.: (a)

Q7. Consider a free-particle in 3 spatial dimensions described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m}$$

It is initially in a state described by a normalized wavefunction

$$\psi(r, t=0) = \left(\frac{\gamma}{\pi}\right)^{3/4} e^{-\gamma r^2/2}$$

What is the probability density of finding the particle with energy E at time t?

(Hint: Express the wavefunction in momentum space.)

(The following integral might be useful: $\int_{-\infty}^{+\infty} dx \frac{1}{\sqrt{2\pi}} e^{-ikx} e^{-\gamma x^2/2} = \frac{1}{\sqrt{\gamma}} e^{-k^2/(2\gamma)}$)

(a) $\frac{2\pi m}{\hbar^2}(\gamma\pi)^{-1} e^{-2mE/(\gamma\hbar\text{^}2)}$

(b) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{\frac{2m\hbar}{t}} e^{-2mE/(\gamma\hbar\text{^}2)}$

(c) $\frac{2m}{\hbar}\frac{1}{\sqrt{2mE}}(\gamma\pi)^{-1/2} e^{-2mE/(\gamma\hbar\text{^}2)}$

(d) $\frac{4\pi m}{\hbar^3}(\gamma\pi)^{-3/2}\sqrt{2mE} e^{-2mE/(\gamma\hbar\text{^}2)}$

Ans.: (d)

Q8. Consider two relativistic particles, each with mass m and momentum of magnitude p, colliding head-on. As a result of the collision, two heavier particles are produced, each with mass αm, where $\alpha > 1$. The minimum value of p required for this collision to occur is:

(a) $(\alpha - 1)mc$ (b) $\sqrt{\alpha^2 - 1}mc$ (c) $2\alpha mc$ (d) $(\sqrt{\alpha} - 1)^2 mc$

Ans.: (b)

Q9. Consider a particle P moving on a one-dimensional discrete lattice with lattice constant a. P can hop from one side to a neighboring site. The probabilities of moving to the right and left are p and $q = 1 - p$, respectively. Starting from the origin $x = 0$ at time $t = 0$, what is the mean square displacement $\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle$ after N steps, where $\langle x \rangle$ is the average position at time t?

(a) $2Na^2pq$ (b) $4Na^2(p-q)$ (c) $4Na^2pq$ (d) $2Na^2(p-q)$

Ans.: (c)

Q10. In the shell model of a nucleus, states of nucleons (protons or neutrons) in a spherically symmetric potential are labelled as nL_j, where n is the principal quantum number, L is the angular momentum quantum number (s, p, d, f corresponds to $L = 0,1,2,3$ respectively), and $\hat{J} = \hat{L} + \hat{S}$. The spin-orbit interaction is given by

$$\hat{H}_{so} = C\hat{L} \cdot \hat{S}$$

If the strength of spin-orbit interaction is $C = -2\,MeV$, the energy difference between two nucleonic states $1d_{5/2}$ and $1d_{3/2}$ is given by:

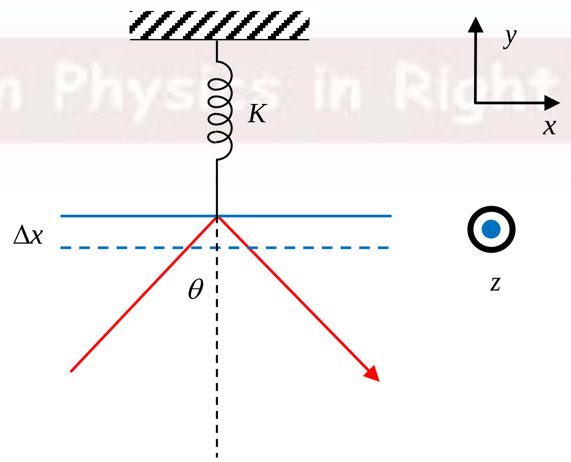
(a) 5 MeV (b) 2 MeV (c) 3 MeV (d) 4 MeV

Ans.: (a)

Q11. A perfect mirror is hanging from the ceiling via a spring of spring constant *K*. A plane wave laser beam with area A and

$$E(\vec{r},t) = E_0 \hat{z} \cos\left(\vec{k} \cdot \vec{r} - \omega t\right)$$

is incident on the mirror at an angle θ, and lifts the mirror by Δx. What is Δx (averaged over a cycle) in terms of $K, E_0, \in_0$, and μ_0?

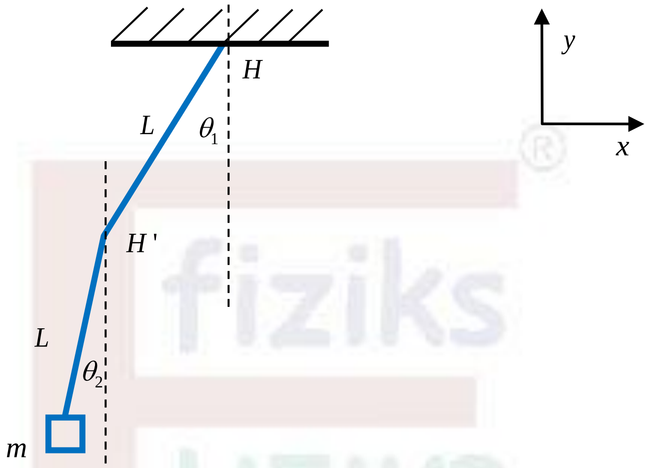


(a) $\dfrac{E_0^2 A\cos\theta}{Kc}\sqrt{\dfrac{\in_0}{\mu_0}}$ (b) $\dfrac{E_0^2 A\cos^2\theta}{Kc}\sqrt{\dfrac{\in_0}{\mu_0}}$

(c) $\dfrac{\epsilon_0 E_0^2 A}{Kc}\sqrt{\dfrac{\epsilon_0}{\mu_0}}$ (d) $\dfrac{2\epsilon_0 E_0^2 A\cos^2\theta}{Kc}\sqrt{\dfrac{\epsilon_0}{\mu_0}}$

Ans.: ***(NB: Due to a typo in the options all test takers will be awarded the full score.)***

Q12. A composite pendulum consists of two massless rods and a weight m. The two rods are connected by a hinge H'. The other end of the first rod is connected to the ceiling by a hinge H. The rods can move freely about H, H' in the xy plane. What is the Lagrangian of the system?



(a) $\dfrac{L^2 m\left(\dot{\theta}_1^2+\dot{\theta}_2^2-2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1-\theta_2\right)\right)}{2}-gLm\left(\cos\theta_1+\cos\theta_2\right)$

(b) $\dfrac{L^2 m\left(\dot{\theta}_1^2+\dot{\theta}_2^2\right)}{2}+gLm\left(\cos\theta_1+\cos\theta_2\right)$

(c) $\dfrac{L^2 m\left(\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\sin\left(\theta_1-\theta_2\right)\right)}{2}-gLm\left(\cos\theta_1+\cos\theta_2\right)$

(d) $\dfrac{L^2 m\left(\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1-\theta_2\right)\right)}{2}+gLm\left(\cos\theta_1+\cos\theta_2\right)$

Ans.: (d)

Q13. Consider the alpha decay of ^{224}U at rest, to ^{220}Th. The atomic masses are given below:

$M_{224U} = 224.0276$ amu; $M_{220Th} = 220.0158$ amu; $M_{4He} = 4.0026$ amu.

What is the estimate of the kinetic energy of the emitted alpha $\left(^4He\right)$ particle? (One amu corresponds to $931.5\,MeV/c^2$)

(a) 8.4163 MeV (b) 8.5698 MeV (c) 8.7261 MeV (d) 8.1066 MeV

Ans.: (a)

Q14. Two students perform a counting experiment independently. Student A measures the counts for 1-minute intervals each and repeats the measurement five times. The obtained counts are given below.

Measurement turn	Counts
1	25
2	35
3	30
4	23
5	27

This student then takes the mean of these counts and reports the count rate (counts/min). The second student (B) makes one measurement for five minutes. She measures 145 counts and reports the count rate (counts/min). If the clock used for all these measurements is accurate up to 0.1 minutes, and there are no other sources of uncertainties, we can conclude that:

(a) The count rate reported by student A will have a larger uncertainty than that reported by student B.

(b) The count rate reported by student B will have a larger uncertainty than that reported by student A.

(c) The reported uncertainty in both results would be identical

(d) Nothing may be concluded about the relative uncertainties between A and B

Ans.: (a)

Q15. O_2 is a linear molecule. (The bond length of the oxygen molecule is 1.2Å, and the mass of an oxygen atom is $2.7\times10^{-26}kg$.) A neutron strikes an O_2 molecule and loses energy by exciting a rotational energy level of O_2. Which of the following is the best estimate of the lowest amount of energy the neutron would have to transfer to the O_2 molecule? (Take the transfer of translational kinetic energy to be negligible.)

(a) $3.6\times10^{-4}eV$ (b) $7.2\times10^{-4}eV$ (c) $1.8\times10^{-4}eV$ (d) $1.4\times10^{-3}eV$

Ans.: (a)