Solution

Full Length Test – 01

Ans. 1: (d)

Solution: Let the four digit number be '$aaab$' or '$baaa$'

Since the number has to be a multiple of 9, therefore $3a+b$ should be either $9, 18, 27$.

Case I: $3a+b$

Possible cases are

$(1116, 6111, 2223, 3222, 3330, 9000)$

Case II: $3a+b=18$

Possible cases are

$(3339, 9333, 4446, 6444, 5553, 3555, 6660)$

Case III: $3a+b=27$

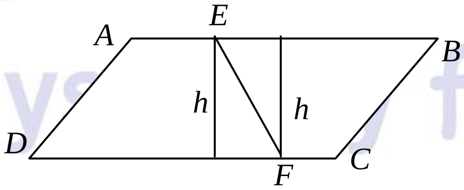
Possible cases are

$(6669, 9666, 8883, 3888, 7776, 6777, 9990)$

Hence the total number of required numbers $=20$

Ans. 2: (b)

Solution: Let A_1 and A_2 be the areas of trapeziums $AEDF$ and $EBCF$ respectively. Let h be the common heights of these trapeziums.



Given, $AB = 20cm, AE = 3cm$

Therefore,

$$EB = 20 - 3 = 17cm$$

$$DC = AB = 20\,cm$$

$$FC = 20 - DF$$

$$A_1 = \frac{1}{2}\times(AE + DF)\times h = \frac{1}{2}\times(3 + DF)\times h$$

$$A_2 = \frac{1}{2}\times(FC + EB)\times h = \frac{1}{2}\times(20 - DF + 17)\times h$$

From the question

$$A_1 = A_2$$

$$\Rightarrow \frac{1}{2}\times(3+DF)\times h=\frac{1}{2}(20-DF+17)\times h$$

$$\Rightarrow 3+DF=37-DF$$

$$\Rightarrow 2DF=34 \Rightarrow DF=17cm$$

Ans. 3: (c)

Solution: $6\times7+5\times6=42+30=72$

$7\times8+4\times5=56+20=76$

$8\times9+2\times3=72+6=78$

Ans. 4: (a)

Solution: B lives on top floor. Since D lives between B and F. D will be just below B and just above F. Since there are two persons between F and G. Hence the position of G will be second from bottom combining two statements $(a)'$. There is exactly one person between C and E' and (b). E and A live on successive floors we can say that C lives on bottom floor, E lives on third floor and A lives on fourth floor.

Ans. 5: (d)

Solution: 35 -------------------

There are 8 ways to fill the place after '5'. The next place can filled in 7 way and the next place can be filled in 6 way. The last place can be filled in 5 ways.

Hence total numbers of telephone numbers

$$=8\times7\times6\times5=1680$$

Ans. 6: (b)

Solution: Let us denote the cost price and the selling price by CP and SP respectively.

From the question

$$18,000-CP=CP-16,800$$

$$\Rightarrow 2(CP)=18,000+16,800=34,800 \Rightarrow CP=17,400$$

Hence in order to make a profit of 25%, the watch should be sold for

$$17,400\times\frac{125}{100}=21,750$$

Ans. 7: (d)

Solution: A leap year has 52 weeks and 2 days. In order to ensure that there are exactly 52 Sundays, none of the last two days should be a Sunday. Hence the problem reduces to 'What is the probability that none of the last two days is a Sunday'.

Sample space

$$=\{(\text{Sun,Mon}),(\text{Mon,Tue}),$$

$$(\text{Tue,Wed}),(\text{Wed,Thus}),(\text{Thus,Fri}),(\text{Fri,Sat}),(\text{Sat,Sun})\}$$

Out of these 7 outcomes 5 outcomes are favorable to the required event.

Hence required probability $=\frac{5}{7}$.

Ans. 8: (b)

Solution: Let initially there be $2x, 3x$ and $5x$ students in the three classes. Also suppose that after increase the number of students in the there classes becomes $4y, 5y$ and $7y$.

From the question

$2x+20=4y$ (I)

$3x+20=5y$ (II)

$5x+20=7y$ (III)

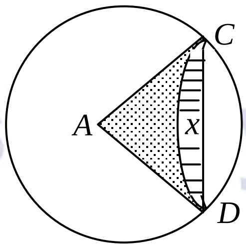
Solving these equations we obtain

$$x=y=10$$

Therefore, total number of students before increases $=2x+3x+5x=10x=10\times 10=100$

Ans. 9: (c)

Solution:



Area of each circle $=\pi(6)^2=36\pi cm^2$

Hence, area of sector $ACD=$ area of sector

$$BCD=\frac{36\pi cm^2}{6}=6\pi cm^2$$

Now consider the left circle. Triangle ACD is an equilateral triangle.

Area of triangle ACD

$$=\frac{\sqrt{3}}{4}(6)^2=9\sqrt{3}$$

Area of region $x=6\pi-9\sqrt{3}$

Hence total area of the shaded region

$$= 2\left(9\sqrt{3}\right) - 2\left(6\pi - 9\sqrt{3}\right)$$

$$= 18\sqrt{3} - 12\pi + 18\sqrt{3} = 36\sqrt{3} - 12\pi$$

Ans. 10: (b)

Solution: Since they walk in opposite directions their relative speed is $2+3=5$ rounds per hour.

Therefore, they cross each other 5 times in 1 hour and 2 times in $\frac{1}{2}$ hour.

Time duration from $8\ AM$ to $9:30\ AM = 1.5$ hour.

Hence the cross 7 times before $9:30\ AM$.

Ans. 11: (d)

Solution: Total number of days in a leap year $=366$. In 52 complete weeks each day will occur once. This means during these 52 weeks there will be 52 Saturdays and 52 Sundays. Since every second Saturday and every fourth Saturday is a holiday, hence there will be $\frac{52}{2}=26$ Saturdays which are holiday.

Hence total number of holidays during 52 complete weeks $=26+52=78$

The 365^{th} day will be Friday and 366^{th} day is a Saturday. Since 366^{th} day is not second Saturday. Hence this will not be a holiday. Total holidays during the leap year $=78$

Total working days $=366-78=288$

Ans. 12: (b)

Solution: The hour hand moves at $0\cdot 5^0$ per minute.

The minute hand moves at 6^0 per minute.

The angle between minute hand and our hand should be 180^0 in our case.

Suppose they coincide for the first time after t minutes (Assuming that both the hour hand and minute hand point towards the '12' mark initially).

Hence, $6t - 0\cdot 5t = 180^0$

$$\Rightarrow 5\cdot 5t = 180^0 \Rightarrow t = \frac{360}{11}$$

Hence they will first coincide after $\frac{360}{11}$ minutes

Hence in one day ($=720$ minutes) they will coin coincide

$$\frac{720}{360/11} \text{ times } = 22 \text{ times.}$$

Ans. 13: (c)

Solution: After meting the ratio of copper to nickel $=5:11$

Hence amount of copper in the final alloy

$$=20\times\frac{5}{16}=\frac{25}{4}kg$$

Amount of nickel in the final mixture

$$=20\times\frac{11}{16}=\frac{55}{4}kg$$

Let the amount of copper and nickel in the first bar be $2x$ and $5x$ and in the second bar $3y$ and $5y$ respectively, then

$$2x+3y=\frac{25}{4}$$

$$\Rightarrow 8x+12y=25 \qquad \text{(I)}$$

Also, $5x+5y=\dfrac{55}{4}$

$$\Rightarrow 4x+4y=11 \qquad \text{(II)}$$

Multiplying equation (II) by 2 and subtracting it from equation (I) gives

$$4y=25-22\Rightarrow y=\frac{3}{4}$$

Therefore equation (II) gives

$$x=\frac{11-4y}{4}=\frac{11-3}{4}=2$$

Hence total weight of first alloy

$$=2x+5x=7x=7\times 2=14kg$$

Ans. 14: (b)

Solution: Between 10^1 and 10^2 there are two integers 11 and 20 such that the sum of their digits is 2.

Between 10^2 and 10^3 there are three integers $110,101$ and 200 such that their digit sum is 2.

We can generalize this and say that between 10^n and 10^{n+1}, there are $(n+1)$ integers whose digit sum is 2.

Hence between 10^6 and 10^7 there are 7 positive integers such that the sun of their digits is 2.

Ans. 15: (a)

Solution: Let us say that students have weights a,b,c,d and e. We assumes that the weights of students form nondecreasing sequence.

From the question

$a+b+c+d=160$

and, $b+c+d+e=180$

The total weight of the students can be written in the following ways

(a) $160+e$ or, (b) $180+a$

Hence e is 20 more than a

The highest possible average. This will occur when $a=b=c=d=40$ and $e=60$

Hence the highest possible average of class

$$=\frac{160+60}{5}=44$$

The least possible value of e will give the minimum possible average. In this case

$b=c=d=e=45$ and $a=25$

Hence minimum possible average

$$=\frac{180+25}{5}=41$$

Hence the difference between maximum possible average and minimum possible average

$=44-41=3\,kg$

Ans. 16: (b)

Solution: The volume of larger cube $=5^3=125\,cm^3$

Volume of each smaller cubes $=1^3=1cm^3$

Hence there will be 125 smaller cubes.

Surface area of larger cube $=6\cdot 5^2=150\,cm^2$

Total surface area of smaller cubes $125\cdot 6=750cm^2$

Hence, $\frac{\text{Surface area of larger cube}}{\text{Total surface area of smaller cube}}=\frac{150}{750}=\frac{1}{5}$

Hence the required ratio $=1:5$

Ans. 17: (d)

Solution: The percentage increase in 1996.

$$=\frac{40-25}{25}\times 100=60\%$$

Percentage in 1997 $=\frac{60-40}{40}\times 100=50\%$

Percentage increase in $2001=\frac{75-50}{50}\times 100=50\%$

Percentage increase in $2002=\frac{80-75}{75}\times 100=\frac{20}{3}\%$

Hence percentage increase in 1996 as compared to previous year is maximum.

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Ans. 18: (c)

Solution: At $t=4s$, the velocity of first stone is $v_1 = 98-(9\cdot 8)\times 4 = 98-39\cdot 2 = 58\cdot 8\, m/s$.

At $t=4s$, the velocity of second stone $= 0-(9\cdot 8)\times 4 = -39\cdot 2\, m/s$

Hence relative speed $= |v_1 - v_2| = |v_2 - v_1|$

$$= |58\cdot 8 - (-39\cdot 2)| = 98 m/s$$

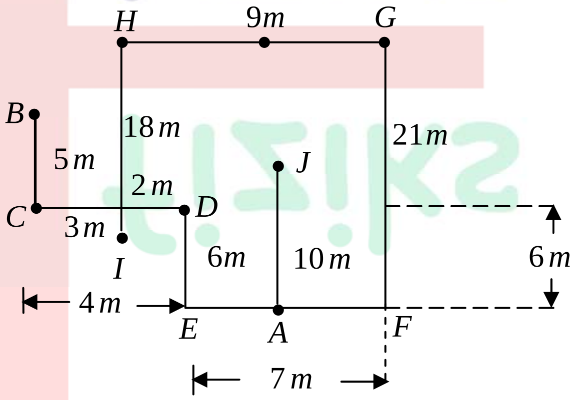
Ans. 19: (c)

Solution: All Dictionaries are books but no Book is a Printer. Similarly no Dictionaries is a printer.

Ans. 20: (d)

Solution: The situation of the problem is shown below in the figure. Hence the distance DI is

$$\sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13} m$$



Ans. 21: (d)

Solution: $\because E_1^{\parallel} = E_2^{\parallel}$ and $D_1^{\perp} = D_2^{\perp} \Rightarrow \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \Rightarrow \dfrac{E_1^{\perp}}{E_2^{\perp}} = \dfrac{\varepsilon_2}{\varepsilon_1}$

$$\Rightarrow E_2^{\perp} > E_1^{\perp} \Rightarrow |\vec{E}_2| > |\vec{E}_1| \qquad \because \varepsilon_1 > \varepsilon_2$$

$$\because D_1^{\perp} = D_2^{\perp} \text{ and } D_1^{\parallel} = \varepsilon_1 E_1^{\parallel}, D_2^{\parallel} = \varepsilon_2 E_2^{\parallel} \Rightarrow \frac{D_1^{\parallel}}{D_2^{\parallel}} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$\Rightarrow D_2^{\parallel} < D_1^{\parallel} \Rightarrow |\vec{D}_2| < |\vec{D}_1| \qquad \because \varepsilon_1 > \varepsilon_2$$

Ans. 22: (c)

Solution: $z = \Sigma \exp(-E_n/kT) \;\Rightarrow\; \langle E\rangle = \dfrac{1}{z}\sum_n E_n e^{-E_n/kT}$

$$C_v = \left.\frac{\partial \langle E\rangle}{\partial T}\right|_{N,V} = -\frac{\partial \ln z}{\partial T}\langle E\rangle + \frac{1}{kT^2}\langle E^2\rangle$$

Thus, $C_v = \dfrac{1}{kT^2}(\langle E^2\rangle - \langle E\rangle^2) \Rightarrow C_V = \dfrac{1}{kT^2}(\Delta E)^2$

Ans. 23: (a)

Solution: $\frac{dy}{dx}=\frac{y^2}{x}\Rightarrow\frac{dy}{y^2}=\frac{dx}{x}\Rightarrow-\frac{1}{y}=\ln x+C'$

$y(1)=1\Rightarrow-\frac{1}{1}=\ln 1+C'\Rightarrow C'=-1\Rightarrow-\frac{1}{y}=\ln x-1\Rightarrow y=\frac{1}{1-\ln x}$ as $x\to 0$, y blows up

Ans. 24: (b)

Solution: If $A=(p_x-bx)$ is conserved then Poisson bracket $[p_x-bx,H]=0$

and $[p_x-bx,H]=(p_x+ax)(-b-a)=0\Rightarrow b=-a$

Ans. 25: (a)

Solution: $B_1\times 2\pi\frac{a}{2}=\mu_0\frac{i}{\pi a^2}\left(\frac{\pi a^2}{4}\right)\Rightarrow B_1=\frac{\mu_0 i}{4\pi a}$ ….. (i)

$B_2\times 2\pi(2a)=\mu_0 i \quad \Rightarrow B_2=\frac{\mu_0 i}{4\pi a}$ ….. (ii)

Thus $\frac{B_1}{B_2}=1$

Ans. 26: (b)

Solution: $\psi(r,\theta,\varphi)=\frac{1}{\sqrt{8\pi a^3}}e^{\frac{-r}{2a}}$

$$\langle r^2\rangle=\iiint\psi^*\psi r^4 dr\sin\theta d\theta d\phi=\frac{1}{8\pi a^3}\int_0^\infty\int_0^\pi\int_0^{2\pi}e^{-r/a}r^4\sin\theta dr d\theta d\phi=\frac{1}{8\pi a^3}\cdot 4\pi\int_0^\infty e^{-r/a}r^4 dr$$

$=3(2a)^2=12a^2$

one can compare the wave function of hydrogen atom with Bohr radius $a_0=2a$

most probable distance,

$$\frac{d}{dr}r^2e^{-r/a}=0,\ r_p=2a,\ \frac{r_p^2}{\langle r^2\rangle}=\frac{(2a)^2}{12a^2}=\frac{1}{3}$$

Ans. 27: (c)

Solution: $I=\oint_C\frac{\sin z}{2z-\pi}$ pole $\Rightarrow 2z-\pi=0\Rightarrow z=\frac{\pi}{2}$

Residue at $z=\frac{\pi}{2}$ $\because|z|=2$ so it will be lies within the contour

$$I_{(emg)}=\oint_C\frac{e^{iz}}{2\left(z-\frac{\pi}{2}\right)}=\sum R\times 2\pi i$$

$$\text{Res}\Big|_{z=\frac{\pi}{2}}=\frac{\left(z-\frac{\pi}{2}\right)e^{iz}}{2\left(z-\frac{\pi}{2}\right)}=\frac{e^{i\pi/2}}{2}=\frac{i}{2}$$ (taking imaginary part); Residue = $\frac{1}{2}$

Now $I=\frac{1}{2}\times 2\pi i=\pi i$

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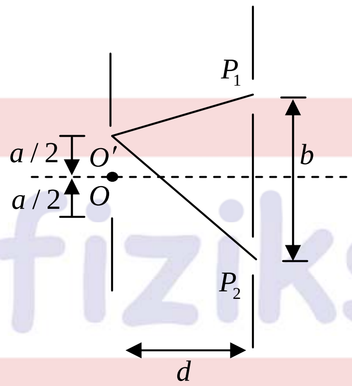
Ans. 28: (b)

Solution: $\frac{dS}{dt}=\sum_{\alpha}\dot{\vec{p}}_{\alpha}.\vec{r}_{\alpha}+\vec{p}_{\alpha}.\dot{\vec{r}}_{\alpha}$ $\quad \left\langle\frac{dS}{dt}\right\rangle=\frac{1}{\tau}\int_0^{\tau}\frac{dS}{dt}dt=\frac{S(\tau)-S(0)}{\tau}=0$

$$\left\langle\sum_{\alpha}\vec{p}_{\alpha}.\dot{\vec{r}}_{\alpha}\right\rangle=-\left\langle\sum_{\alpha}\dot{\vec{p}}_{\alpha}.\vec{r}_{\alpha}\right\rangle\Rightarrow 2\langle T\rangle=-\left\langle\sum_{\alpha}\vec{F}_{\alpha}.\vec{r}_{\alpha}\right\rangle$$

Ans. 29: (d)

Solution:



If the path difference $O'P_2-O'P_1=\frac{\lambda}{2}$

The minima of the interference pattern produced by O will fall on the maxima produced by O'

Now $O'P_2=\left[d^2+\left(\frac{b}{2}+\frac{a}{2}\right)^2\right]^{1/2}\approx d+\frac{1}{2d}\left(\frac{b}{2}+\frac{a}{2}\right)^2$

$$O'P_1=\left[d^2+\left(\frac{b}{2}-\frac{a}{2}\right)^2\right]^{1/2}\approx d+\frac{1}{2d}\left(\frac{b}{2}-\frac{a}{2}\right)^2$$

$$\Rightarrow O'P_2-O'P_1\approx\frac{ab}{2d}\quad(\because d>>b,a)$$

Thus $\frac{\lambda}{2}=\frac{ab}{2d}\Rightarrow d=\frac{ab}{\lambda}$

Ans. 30: (a)

Solution:

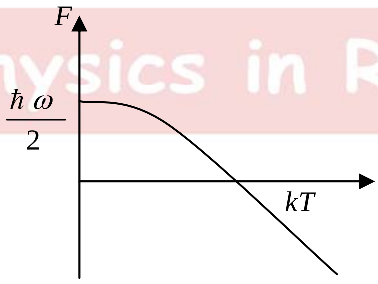


Figure: Plot of F verses

$$F=\frac{N\hbar\omega}{2}+NkT\ln\left(1-e^{-\hbar\omega/kT}\right)$$

Case 1: Now, if $kT\to 0\Rightarrow e^{-\hbar\omega/kT}\to 0\Rightarrow F\to\frac{N\hbar\omega}{2}$

Ans. 31: (a)

Solution: $\frac{dL_z}{dt}=\frac{1}{i\hbar}\left[L_z,H\right]=\frac{1}{i\hbar}\left[L_z,c\vec{\alpha}.\vec{p}+\beta mc^2\right]$

$$i\hbar\frac{dL_z}{dt}=c\left[xp_y,\alpha_x p_x\right]-c\left[yp_x,\alpha_y p_y\right]=\frac{i\hbar c}{i\hbar}\left[p_y\alpha_x-p_x\alpha_y\right]=c\left[p_y\alpha_x-p_x\alpha_y\right]$$

Ans. 32: (b)

Solution: $y=E_0\sin\left(2\pi f_0 t\right)$.

The Fourier transform is:

$$F\left(y\right)=\frac{E_0}{2}\left[\delta\left(f+f_0\right)\right]-\delta\left[f-f_0\right]$$

In Fourier space $\overline{f}=f_0$, $\overline{A}=\frac{E_0}{2}$.

Ans. 33: (c)

Solution: $\omega^2=\beta k+\alpha k^3$

$$2\omega\frac{d\omega}{dk}=\beta+3k^2\Rightarrow\frac{d\omega}{dk}=\frac{\beta+3\alpha k^2}{2\omega}\qquad(1)$$

also $\omega.\frac{\omega}{k}=\beta+\alpha k^2$ (2)

divide (1) and (2)

$$\frac{d\omega/dk}{\omega\left(\omega/k\right)}=\frac{\beta+3\alpha k^2}{2\omega}\times\frac{1}{\beta+\alpha k^2}$$

$$\because\frac{d\omega}{dk}=\frac{\omega}{k}$$

$$\Rightarrow 2\left(\beta+\alpha k^2\right)=\beta+3\alpha k^2\Rightarrow\beta=\alpha k^2\Rightarrow k=\sqrt{\frac{\beta}{\alpha}}$$

Ans. 34: (b)

Solution: $E'_x=0,E'_y=0,E'_z=\frac{\sigma}{\varepsilon_0}$ and $B'_x=0,B'_y=0,B'_z=0$

$$E_x=E'_x=0,\ E_y=\gamma\left(E'_z+vB'_z\right)=\gamma\frac{\sigma}{\varepsilon_0},\ \ E_z=\gamma\left(E'_z-vB'_y\right)=\frac{\gamma\sigma}{\varepsilon_0}$$

$$B_x=B'_x,\ B_y=\gamma\left(B'_y-\frac{vE'_z}{c^2}\right)=-\frac{\gamma v\sigma}{\varepsilon_0 c^2},\ B_z=\gamma\left(B'_z-\frac{vE'_y}{c^2}\right)=0$$

Ans. 35: (c)

Solution: $E_2^1 = \int_0^{a/4} H_p \phi_2^* \phi_2 dx$

$$\Delta V = \int_0^{a/4} V_0 \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right) dx = \frac{2}{a} V_0 \int_0^{a/4} \frac{1}{2}\left[1 - \cos\frac{4\pi x}{a}\right] dx$$

$$= \frac{2}{a} V_0 \left[\frac{a}{8} - \frac{\sin\frac{4\pi}{4}}{\frac{4\pi}{a}}\right] = V_0\left[\frac{1}{4}\right] \simeq 0.25\, V_0$$

Ans. 36: (a)

Solution: $p = \frac{\partial F}{\partial q} = \omega q \cot 2\pi Q$ ....(1)

$$P = -\frac{\partial F}{\partial Q} = \omega q^2 \pi coec^2 2\pi Q$$

Put $q = \sqrt{\frac{P}{\pi\omega}} . \frac{1}{coesc 2\pi Q} = \sqrt{\frac{P}{\pi\omega}} \sin 2\pi Q$ in equation (1) one will get

$$p = \omega\sqrt{\frac{P}{\pi\omega}} . \frac{\cot 2\pi Q}{coesc 2\pi Q} = \sqrt{\frac{\omega P}{\pi}} . \cos 2\pi Q \qquad \Rightarrow \frac{q}{p} = \frac{1}{\omega} \tan 2\pi Q$$

Ans. 37: (a)

Solution: It is voltage doublers circuit in which C_1 will be charged to maximum value input that is $1V$.

So $v(t) = (\cos\omega t - 1)$ according to KVL.

Ans. 38: (b)

Solution: $\tan(ka + \delta_0) = \left[\frac{1}{\tan(ka)} + \frac{2mV_0}{k\hbar^2}\right]^{-1}$

If the incident particles have small velocities, $ka \ll 1$, we have $\tan(ka) \simeq ka$ and $\tan(ka + \delta_0) \simeq \tan(\delta_0)$.

$$\tan\delta_0 \simeq \frac{ka}{1 + 2mV_0 a/\hbar^2} \quad \Rightarrow \quad \sin^2\delta_0 = \frac{1}{1 + \frac{1}{\tan^2\delta_0}} \simeq \frac{k^2a^2}{k^2a^2 + \left(1 + 2mV_0 a/\hbar^2\right)^2}$$

$$\sigma_0 = \frac{4\pi}{k^2}\sin^2\delta_0 = \frac{4\pi a^2}{k^2a^2 + \left(1 + 2mV_0 a/\hbar^2\right)^2}$$

Ans. 39: (c)

Solution: $\tilde{f}(k)=\int_{-\infty}^{\infty} dx\, e^{ikx}\left(\alpha\delta(x)+\beta\delta'(x)+\gamma\delta''(x)\right)$

$$\int_{-\infty}^{\infty}\alpha\delta(x)e^{ikx}dx=\alpha \Rightarrow \int_{-\infty}^{\infty}\beta\delta'(x)e^{ikx}dx=\beta\left[e^{ikx}\delta(x)\Big|_{-\infty}^{\infty}-\int_{-\infty}^{\infty}ike^{ikx}\delta(x)dx\right]=-i\beta k$$

$$\int_{-\infty}^{\infty}\gamma\delta''(x)e^{ikx}dx=-\gamma k^2$$

Ans. 40: (d)

Solution: $Z=\left(\frac{2\pi mkT}{h^2}\right)^{3N/2}\frac{V^N}{\lfloor N}$ and $U=\frac{3NkT}{2}\Rightarrow kT=\frac{2}{3}\frac{U}{N}$

$$Z=\left(\frac{4\pi mU}{3h^2N}\right)^{3N/2}\frac{V^N}{\lfloor N}$$

Ans. 41: (a)

Solution: $e^A Be^{-A}=B+[A,B]+\frac{1}{\lfloor 2}[A,[A,B]]+\frac{1}{\lfloor 3}[A,[A,[A,B]]]......$

$$A(t)=\exp\left(\frac{itH}{\hbar}\right)A\exp\left(-\frac{itH}{\hbar}\right)=X+\frac{it}{\hbar}[H,X]+\frac{1}{\lfloor 2}[H,[H,X]]+...$$

$$=X+\frac{t}{m}P-\frac{(\omega t)^2}{\lfloor 2}X-\frac{(\omega t)^3}{\lfloor 3}\frac{1}{m\omega}P+\frac{(\omega t)^4}{\lfloor 4}X+\frac{(\omega t)^5}{\lfloor 5}\frac{1}{m\omega}P$$

$$X(t)=X\left[1-\frac{(\omega t)^2}{\lfloor 2}+\frac{(\omega t)^4}{\lfloor 4}+..\right]+\frac{1}{m\omega}P\left[(\omega t)-\frac{(\omega t)^3}{\lfloor 3}+\frac{(\omega t)^5}{\lfloor 5}...\right]$$

$$X(t)=X\cos\omega t+\frac{1}{m\omega}P\sin\omega t$$

Ans. 42: (b)

Solution: $I_E=I_C+I_B=(\beta+1)I_B$

KVL in input loop gives,

$$\therefore\ I_B=\frac{V_{CC}-V_{BE}}{R_B+(\beta+1)R_E}=\frac{20-0.7}{430K+51\times 1K}\Rightarrow I_B=40\mu A$$

$$I_C=\beta_{IB}=50\times 40\mu A=2000\mu A=2mA$$

$$V_C=V_{CC}-I_CR_C=20-2mA\times 2K\quad\Rightarrow V_C=16V$$

Ans. 43: (c)

Solution: $V_{eff}=\frac{J^2}{2mr^2}-\frac{k}{r^n},\ \frac{\partial V_{eff}}{\partial r}=-\frac{J^2}{mr^3}+\frac{nk}{r^{n+1}}=0$

$$\because J=mr^2\omega\quad\Rightarrow\frac{m^2\omega^2r^4}{mr^3}=\frac{nk}{r^{n+1}}\Rightarrow\omega^2\propto\frac{1}{r^{n+2}}\Rightarrow\omega\propto r^{-(n+2)/2}\Rightarrow T\propto r^{\frac{n}{2}+1}$$

$$\frac{T_2}{T_1}=\left(\frac{2R}{R}\right)^{\frac{n+2}{2}}=2^{\frac{n}{2}+1}$$

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Ans. 44: (c)

Solution: $\frac{dx}{dt}=2\sqrt{1-x^2}\Rightarrow\frac{dx}{\sqrt{1-x^2}}=2dt$ $\sin^{-1}x=2t+c$

$x=0,\ t=0$ so, $c=0$

$x=\sin 2t$ (x should not be greater than 1 at $x=1$)

$1=\sin 2t$ $\sin\frac{\pi}{2}=\sin 2t,\ t=\frac{\pi}{4}$

so, $x=\sin 2t$ $0\le t<\frac{\pi}{4}$

$=1$ $t\ge\frac{\pi}{4}$

Ans. 45: (d)

Solution: $T=\frac{1}{f}=\frac{1}{500Hz}=2ms$

$$I=\frac{CdV}{dt}=2\times10^{-6}\times\frac{3}{2\times10^{-3}}=30\times10^{-3}\qquad\Rightarrow I=3mA$$

Ans. 46: (a)

Solution: $d=\frac{1}{\kappa}=\frac{\lambda_0}{4\pi},\ \frac{\in_I}{\in_R}=\sqrt{3}=\frac{\sigma}{\omega\in}$

$$\kappa=\omega\sqrt{\frac{\in\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}-1\right]^{1/2}\Rightarrow\kappa=\omega\sqrt{\frac{\in\mu}{2}}=\frac{2\pi}{\lambda_0}\Rightarrow\sqrt{\in\mu}=\frac{\sqrt{2}}{\omega}\frac{2\pi}{\lambda_0}$$

$$K=\sqrt{k^2+\kappa^2}=\omega\left[\in\mu\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}\right]^{1/2}$$

$$\frac{E_0}{B_0}=\frac{\omega}{K}=\frac{\omega}{\omega\left[\in\mu\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}\right]^{1/2}}=\frac{1}{\sqrt{2\in\mu}}=\frac{1}{\sqrt{2}\times\frac{\sqrt{2}}{\omega}\times\frac{2\pi}{\lambda_0}}=\frac{\lambda_0\omega}{4\pi}=\frac{\lambda_0\times2\pi c/\lambda_0}{4\pi}=\frac{c}{2}$$

Ans. 47: (c)

Solution: For one particle $\langle\mu\rangle=(\mu_0)p+(-\mu_0)(1-p)=\mu_0(2p-1)=\mu_0\left(2.\frac{2}{3}-1\right)=\frac{\mu_0}{3}$ Where

$p=\frac{2}{3}$

$$\langle\mu_1^2\rangle=(\mu_0)^2p+(-\mu_0)^2(1-p)=\mu_0^2$$

$$(\Delta\mu)^2 = \langle\mu^2\rangle - \langle\mu\rangle^2 = \mu_0^2 - \mu_0^2(2p-1)^2 = 4\mu_0^2 p(1-p),$$

$$\langle\Delta\mu\rangle = 2\mu_0\sqrt{p(1-p)} = 2\mu_0\sqrt{\frac{2}{3}\left(1-\frac{2}{3}\right)} = \frac{2\sqrt{2}}{3}\mu_0$$

$$\frac{\langle\Delta\mu\rangle}{\langle\mu\rangle} = \frac{\frac{2\sqrt{2}\mu_0}{3}}{\frac{\mu_0}{3}} = 2\sqrt{2}$$

Ans. 48: (d)

Solution: Let Fourier series is $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$

$$\because c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx} dx$$

$$\Rightarrow c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^x e^{-inx} dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(1-in)x} dx = \frac{1}{2\pi}\frac{1}{(1-in)}\left[e^{(1-in)x}\right]_{-\pi}^{\pi} = \frac{1}{2\pi}\frac{1}{(1-in)}\left[e^{(1-in)\pi} - e^{-(1-in)\pi}\right]$$

$$\Rightarrow c_n = \frac{1}{2\pi}\frac{1}{(1-in)}\left[e^{\pi}e^{-in\pi} - e^{-\pi}e^{in\pi}\right] = \frac{1}{2\pi}\left(\frac{1+in}{1+n^2}\right)\left[e^{\pi} - e^{-\pi}\right](-1)^n \qquad \because e^{\pm in\pi} = (-1)^n$$

$$\Rightarrow c_n = \left(\frac{1+in}{1+n^2}\right)\frac{\sinh\pi}{\pi}(-1)^n \qquad \because \sinh\pi = \frac{e^{\pi} - e^{-\pi}}{2}$$

Thus Fourier series is $$f(x) = \frac{\sinh\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\left(\frac{1+in}{1+n^2}\right)e^{inx} \text{ ....}(1)$$

Let us derive the real Fourier series

$$\because (1+in)e^{inx} = (1+in)(\cos nx + i\sin nx) = (\cos nx - n\sin nx) + i(\cos nx + \sin nx)$$

$\because n$ varies from $-\infty$ to $+\infty$, equation (1) has corresponding term with $-n$ instead of n.

Thus

$$\because (1-in)e^{-inx} = (1-in)(\cos nx - i\sin nx) = (\cos nx - n\sin nx) - i(\cos nx + \sin nx)$$

Let's add these two expressions;

$$(1+in)e^{inx} + (1-in)e^{-inx} = 2(\cos nx - n\sin nx), \qquad n = 1, 2, 3......$$

For $n = 0$, $\quad (-1)^n\left(\frac{1+in}{1+n^2}\right)e^{inx} = 1$

Thus, $$f(x) = \frac{\sinh\pi}{\pi} + 2\frac{\sinh\pi}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{(\cos nx - n\sin nx)}{1+n^2}$$

$$f(x) = \frac{2\sinh\pi}{\pi}\left[\frac{1}{2} - \frac{1}{1+1^2}(\cos x - \sin x) + \frac{1}{1+2^2}(\cos 2x - 2\sin 2x) - + ...\right]$$

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Ans. 49: (c)

Solution: $L=\frac{1}{2}M\dot{x}^2+\frac{1}{2}m\left(\dot{x}^2+\ell^2\dot{\theta}^2+2\ell\dot{x}\dot{\theta}\cos\theta\right)+mg\ell\cos\theta$.

The equations of motion obtained from varying x and θ are

$$(M+m)\ddot{x}+m\ell\ddot{\theta}\cos\theta-m\ell\dot{\theta}^2\sin\theta=0,$$

$$\ell\ddot{\theta}+\ddot{x}\cos\theta+g\sin\theta=0.$$

If θ is small, we can use the small – angle approximations, $\cos\theta\approx1-\theta^2/2$ and $\sin\theta\approx\theta$. Keeping only the terms that are first – order in θ, we obtain

$$(M+m)\ddot{x}+m\ell\ddot{\theta}=0,$$

$$\ddot{x}+\ell\ddot{\theta}+g\theta=0.$$

The first equation expresses momentum conservation. Integrating it twice gives

$$x=-\left(\frac{m\ell}{M+m}\right)\theta+At+B$$

The second equation is $F=ma$ in the tangential direction. Eliminating $\ddot{x}$ gives

$$\ddot{\theta}+\left(\frac{M+m}{M}\right)\frac{g}{\ell}\theta=0.$$

Ans. 50: (a)

Solution: For constant potential transition probability

$$p_{if}(i)=4\frac{\left|\langle\psi_f|V|\psi_1\rangle\right|^2}{h^2\omega_{fi}^2}\left(\sin^2\frac{\omega_{fi}t_i}{2}\right)$$

at $t_f=\frac{t_i}{2}$,

$$p_{if}=\frac{4\left|\langle\psi_f|V|\psi_i\rangle\right|^2}{h^2\omega_{fi}^2}\sin^2\frac{\omega_{fi}(t_f)}{2}$$

$$p_{if}(f)=\frac{4\left|\langle\psi_f|V|\psi_i\rangle\right|^2}{h^2\omega_{fi}^2}\sin^2\left(\frac{\omega_{fi}t_i}{4}\right)$$

$$\frac{p_{if}(f)}{p_{if}(i)}=\frac{\sin^2\left(\frac{\omega_{fi}t_i}{4}\right)}{\sin^2\left(\frac{\omega_{fi}t_i}{2}\right)}=\frac{\dfrac{\sin^2\left(\frac{\omega_{fi}t_i}{4}\right)}{\frac{\omega_{fi}^2t_i^2}{4^2}}\dfrac{\omega_{fi}^2t_i^2}{4^2}}{\dfrac{\sin^2\left(\frac{\omega_{fi}t_i}{2}\right)\left(\frac{\omega_{fi}t_i}{2}\right)^2}{\frac{\omega_{fi}^2t_i^2}{4}}}\qquad t_I\to0$$

$$=\frac{4\omega_{fi}^2t_i^2}{16\omega_{fi}^2t_i^2}=\frac{1}{4}\Rightarrow\frac{p_{if(f)}}{p_{if(i)}}=\frac{1}{4}$$

Ans. 51: (c)

Solution: $\phi_B = \int_S \vec{B}.d\vec{a} = \int_r^w \frac{\mu_0 I}{2\pi r} L dr = \frac{\mu_0 IL}{2\pi R} \ln\left(\frac{r+w}{r}\right)$

$$\Rightarrow I = -\frac{1}{R}\frac{d\phi_B}{dt} = \frac{\mu_0 IL}{2\pi R}\left[\frac{1}{r+w} - \frac{1}{r}\right]\frac{dr}{dt} = \frac{\mu_0 ILwv}{2\pi Rr(r+w)}$$

Ans. 52: (a)

Solution: $E = E_0 + A(Ka) + B(Ka)^2 + C(Ka)^3 + D(Ka)^4$

$$\therefore \frac{\partial E}{dK} = Aa + 2Ba^2K + 3Ca^3K^2 + 4Da^4K^3$$

and $\frac{\partial^2 E}{\partial K^2} = 2Ba^2 + 6Ca^3K + 12Da^4K^2$

At the bottom of the conduction band, $K = 0$

$$\therefore \frac{\partial^2 E}{\partial K^2} = 2Ba^2$$

$$m^* = \frac{\hbar^2}{\partial^2 E/\partial K^2} = \frac{\hbar^2}{2Ba^2} = \frac{(1.05\times10^{-34})^2}{2\times6.4\times10^{-19}\times(3.2\times10^{-10})^2}$$

$$= 8.4\times10^{-32}\ kg$$

$$\therefore \frac{m^*}{m} = \frac{8.4\times10^{-32}}{9.1\times10^{-31}} = 0.092 \simeq 0.1$$

Ans. 53: (d)

Solution: $\frac{dN_2}{dt} = W - \frac{N_2}{\tau_{21}}$ and $\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$

In equilibrium condition $\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$

$$\Rightarrow N_2 = \tau_{21}W = 10^{22}\times9\times10^{-9} = 9\times10^{13}\,cm^{-3}$$

$$N_1 = \frac{\tau_1 N_2}{\tau_{21}} = \frac{3\times10^{-3}\times9\times10^{13}}{9\times10^{-9}} = 3\times10^{19}\,cm^{-3}$$

Ans. 54: (b)

Solution: $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$ so $p_3 = 1 - \left(\frac{1}{2} + \frac{1}{3}\right) = \frac{1}{6}$

$P_1 = \frac{1}{2}$, $P_2 = 1/3$ and $P_3 = 1/6\cdot$

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$$S=-k_B\left(\frac{1}{2}\ln 1/2+1/3\ln 1/3+1/6\ln 1/6\right)\cdot$$

$$S=-k_b\left(\frac{1}{2}(\ln 1-\ln 2)+\frac{1}{3}(\ln 1-\ln 3)+\frac{1}{6}(\ln 1-\ln 6)\right)$$

$$S=k_B\left[\frac{1}{2}\ln 2+\frac{1}{3}\ln 3+\frac{1}{6}\ln 2+\frac{1}{6}\ln 3\right]$$

$$S=k_B\left[\frac{1}{2}\ln 2+\frac{1}{6}\ln 2+\frac{1}{3}\ln 3+\frac{1}{6}\ln 3\right]$$

$$=k_B\left[\frac{3\ln 2+\ln 2}{6}+\frac{2\ln 3+\ln 3}{6}\right]=k_B\left(\frac{4\ln 2}{6}+\frac{3\ln 3}{6}\right)$$

$$S=k_B\left[\frac{2}{3}\ln 2+\frac{1}{2}\ln 3\right]$$

Ans. 55: (a)

Solution: Our task is to find $u(x,t)$. The general solution of one dimensional wave equation is given by $u(x,t)=\sum_{n=1}^{\infty}\left(B_n\cos\lambda_n t+B_n^*\sin\lambda_n t\right)\sin\frac{n\pi}{L}x$ where B_n's and B_n^*'s are unknowns to be determined and $\lambda_n=\frac{cn\pi}{L}$.

In this problem the initial velocity $u_t(x,0)$ is zero, hence all the B_n^*'s are zero.

B_n's are the Fourier sine series coefficients of $u(x,0)$.

$$B_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx\ ,\ =\frac{2}{1}\int_0^1 k\left[x(1-x)\right]\sin n\pi x dx$$

$$=2k\int_0^1\left(x-x^2\right)\sin n\pi x dx$$

Let us evaluate the two integrals separately

$$\int_0^1 x\sin n\pi x dx=\left\{\frac{-x\cos n\pi x}{n\pi}\right\}_0^1+\frac{1}{n\pi}\int_0^1\cos n\pi x dx=\frac{-(-1)^n}{n\pi}$$

$$\int_0^1 x^2\sin n\pi x dx=\left\{\frac{-x^2\cos n\pi x}{n\pi}\right\}_0^1-\int_0^1 2x\left[\frac{-\cos n\pi x}{n\pi}\right]dx$$

$$=\frac{-(-1)^n}{n\pi}+\frac{2}{n\pi}\int_0^1 x\cos n\pi x dx$$

$$= -\frac{-(-1)^n}{n\pi} + \frac{2}{n\pi}\left[\frac{x\sin n\pi x}{n\pi} + \frac{1}{n^2\pi^2}\cos n\pi x\right]_0^1$$

$$= \frac{-(-1)^n}{n\pi} + \frac{2}{n\pi}\cdot\frac{1}{n^2\pi^2}(\cos n\pi - 1) = \frac{-(-1)^n}{n\pi} + \frac{2}{n^3\pi^3}(\cos n\pi - 1)$$

Hence, $$\int_0^1 (x - x^2)\sin n\pi x = \frac{-1(-1)^n}{n\pi} + \frac{(-1)^n}{n\pi} - \frac{2}{n^3\pi^3}(\cos n\pi - 1)$$

Thus, $$B_n = 2k\left(\frac{-2}{n^3\pi^3}\right)(\cos n\pi - 1)$$

For even n, $B_n = 0$

For odd n, $B_n = \frac{8k}{n^3\pi^3}$

Hence the required solution is

$$u(x,t) = \sum_{n=1}^{\infty}\frac{8k}{n^3\pi^3}\cos n\pi t\cdot\sin n\pi x = \frac{8k}{\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^3}\cos n\pi t\sin n\pi x \text{ , } n \text{ odd.}$$

In the expanded form the solution can be written as

$$u(x,t) = \frac{8k}{\pi^3}\left(\cos\pi t\sin\pi x + \frac{1}{27}\cos 3\pi t\sin 3\pi x + \frac{1}{125}\cos 5\pi t\sin 5\pi x + \cdots\right)$$

Ans. 56: (c)

Solution: Guided velocity $v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 3\times10^8\sqrt{1-\left(\frac{6.5}{7.2}\right)^2} = 1.29\times10^8\ ms$

$$\Rightarrow t = \frac{2l}{v_g} = \frac{2\times150}{1.29\times10^8} = 232\ ns.$$

Ans. 57: (c)

Solution: Take the angle θ between the thin tube and fixed horizontal line through the pivot and the distance r of the centre of mass of the thin rod from the pivot of the tube, as shown in figure, as the generalized coordinates. We have

$$T = \frac{1}{2}M\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{24}MI^2\dot{\theta}^2, \qquad V = 0$$

and the Lagrangian $L = \frac{1}{2}M\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{1}{24}MI^2\dot{\theta}^2$

$$p_\theta = \frac{\partial L}{\partial\dot{\theta}} = Mr^2\dot{\theta} + \frac{MI^2\dot{\theta}}{12}, \qquad \dot{\theta} = \omega$$

$$p_\theta = Mr^2\omega + \frac{MI^2\omega}{12}$$

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Ans. 58: (c)

Solution: Case I: Four steps in x - axis

Two steps in $+x$ and two steps in $-x$

$\therefore$ probability $= {}^4c_2 \times \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^2 = \frac{6}{4^4}$

Case II: Four steps in y - axis

Two steps in $+y$ and two steps in $-y$

$\therefore$ probability $= {}^4c_1 \times {}^2c_1 \left(\frac{1}{4}\right)^2 \left(\frac{1}{4}\right)^2 = \frac{6}{4^4}$

Case 3: Two steps in $+x$ and two steps in $-y$ axis

$\Rightarrow$ One step in each $+x, -x, +y \,\&\, -y$

$\therefore$ probability $= {}^2c_1 \times {}^2c_2 \left(\frac{1}{4}\right)^4 = \frac{4}{4^4}$

$\therefore$ Answer $= \frac{16}{4^4} = \frac{1}{16}$

Ans. 59: (c)

Solution: $V(r) = kr^2$,Trial wave function, $\psi = e^{-\alpha r^2}$

Normalising, ψ , $|A|^2\, 4\pi \int_0^\infty e^{-2\alpha r^2} r^2 dr = 1 \Rightarrow 4\pi |A|^2 \frac{1}{2}\left(\frac{1}{2\alpha}\right)^{3/2} \frac{\sqrt{\pi}}{2} = 1 \Rightarrow |A| = \left(\frac{2\alpha}{\pi}\right)^{3/4}$

$\therefore \quad \psi = \left(\frac{2\alpha}{\pi}\right)^{3/4} e^{-\alpha r^2}$

$$\langle T \rangle = -\frac{\hbar^2}{2m} 4\pi \int_0^\infty \left(\frac{2\alpha}{\pi}\right)^{3/2} e^{-\alpha r^2} \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)\left(e^{-\alpha r^2}\right) r^2 dr$$

$$= -\frac{\hbar^2}{2m}\left(\frac{2\alpha}{\pi}\right)^{3/2} 4\pi \int_0^\infty e^{-\alpha r^2}\left(4\alpha^2 r^2 - 6\alpha\right) e^{-\alpha r^2} r^2 dr$$

$$= -\frac{2\pi\hbar^2}{m}\left(\frac{2\alpha}{\pi}\right)^{3/2}\left[\int_0^\infty 4\alpha^2 r^4 e^{-2\alpha r^2} dr - 6\alpha \int_0^\infty r^2 e^{-2\alpha r^2} dr\right]$$

$$= -\frac{2\pi\hbar^2}{m}\left(\frac{2\alpha}{\pi}\right)^{3/2}\left[4\alpha^2 \frac{1}{2}\left(\frac{1}{2\alpha}\right)^{5/2} \frac{3}{4}\sqrt{\pi} - \frac{6\alpha}{2}\left(\frac{1}{2\alpha}\right)^{3/2} \frac{\sqrt{\pi}}{2}\right]$$

$$= -\frac{2\pi\hbar^2}{m}\left(\frac{2\alpha}{\pi}\right)^{3/2}\left[\frac{3}{8}\sqrt{\frac{\pi}{2\alpha}} - \frac{3}{4}\sqrt{\frac{\pi}{2\alpha}}\right] = -\frac{2\pi\hbar^2}{m}\left(\frac{2\alpha}{\pi}\right)^{3/2} \times -\frac{3}{8}\sqrt{\frac{\pi}{2\alpha}} = \frac{2\pi\hbar^2}{m} \times \frac{3}{8} \times \frac{2\alpha}{\pi}$$

$$\langle T \rangle = \frac{3}{2}\frac{\hbar^2 \alpha}{m}$$

Now, $\langle V \rangle = 4\pi\left(\frac{2\alpha}{\pi}\right)^{3/2} \int_0^\infty r^2 k r e^{-2\alpha r^2} dr \Rightarrow 4\pi\left(\frac{2\alpha}{\pi}\right)^{3/2} \frac{1}{2} k \frac{1}{(2\alpha)^2} \Gamma 2$

$$\langle V\rangle=\frac{2k}{(2\pi)^{1/2}\alpha^{1/2}}$$

$$\langle E\rangle=\langle T\rangle+\langle V\rangle=\frac{3}{2}\frac{\hbar^2\alpha}{m}+\frac{2k}{(2\pi)^{1/2}\alpha^{1/2}}$$

$$\frac{d\langle E\rangle}{d\alpha}=0\Rightarrow\frac{3}{2}\frac{\hbar^2}{m}-\frac{1}{2}\frac{2k}{(2\pi)^{1/2}}\alpha^{-3/2}=0\ \ldots\ldots\ldots\ldots(1)$$

$$\alpha^{-3/2}=\frac{3\hbar^2(2\pi)^{1/2}}{2km}\Rightarrow\alpha^{-1/2}=\left(\frac{3\hbar^2(2\pi)^{1/2}}{2km}\right)^{1/3}$$

From equation (1) $\frac{3}{2}\frac{\hbar^2\alpha}{m}=\frac{1}{2}\frac{2k}{(2\pi)^{1/2}}\alpha^{-1/2}$

$$\langle E\rangle=\frac{3}{2}\frac{2k}{(2\pi)^{1/2}\alpha^{1/2}}=\frac{3k}{(2\pi)^{1/2}\alpha^{1/2}}\Rightarrow\frac{3k}{(2\pi)^{1/2}}\cdot\left[\left(\frac{3\hbar^2(2\pi)^{1/2}}{2km}\right)^{1/3}\right]=3\left(\frac{9k^4\hbar^4}{16\pi^2m^2}\right)^{1/6}$$

Ans. 60: (c)

Solution: Heat capacity is defined as $C_V=AT+BT^3$

where $A=\frac{3}{2}Nk_B^2\frac{1}{E_F}$ and $B=\frac{12\pi^4}{5}Nk_B\frac{1}{\theta_B^3}$

Thus $\frac{A_x}{A_y}=\frac{E_{F_y}}{E_{F_x}}=\frac{6eV}{4eV}=\frac{6}{4}=\frac{3}{2}$

and $\frac{B_x}{B_y}=\frac{\theta_{Dy}^3}{\theta_{Dx}^3}=\left(\frac{320}{180}\right)^3=(1.7)^3=4.8$

Ans. 61: (d)

Solution: The electric field is $\vec{E}(r,t)=-\frac{\partial\vec{A}}{\partial t}=-\frac{\mu_0I_0c}{2\pi\sqrt{(ct)^2-r^2}}\hat{z}$

Ans. 62: (c)

Solution: (i) $\Delta^{++}\to p+\pi^+$

$uuu\to uud+u\bar{d}\quad\Rightarrow\quad uu\to ud+\bar{d}$

(ii) $\Sigma^+\to n+\pi^+$

$uus\to uud+\bar{u}d\quad\Rightarrow\quad ds\to ud+\bar{d}$

(iii) $\Sigma^-\to n+\pi^-$

$dds\to udd+\bar{u}d\quad\Rightarrow\quad ds\to ud+\bar{u}$

(iv) $\Delta^-\to n+\pi^-$

$ddd\to udd+\bar{u}d\quad\Rightarrow\quad dd\to ud+\bar{u}$

Ans. 63: (d)

Solution: $\vec{a}_1 = a\hat{x}$ and $\vec{a}_2 = \frac{a}{2}\left(x+\sqrt{3}\hat{y}\right)$; assuming $a_3 = \hat{z}$

$$\vec{b}_1 = 2\pi\frac{\vec{a}_2\times\vec{a}_3}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)} = \frac{2\pi}{a}\left(\hat{x}-\frac{\hat{y}}{\sqrt{3}}\right)$$

$$\vec{b}_2 = 2\pi\frac{\vec{a}_3\times\vec{a}_4}{\vec{a}_1\cdot\left(\vec{a}_2\times\vec{a}_3\right)} = \frac{4\pi}{a}\hat{y}$$

Area of the first Brillouin zone $=\left[b_1\times b_2\right] = \frac{2\pi}{a}\left(\frac{4\pi}{\sqrt{3}a}\right) = \frac{8\pi^2}{\sqrt{3}a^2}$

Ans. 64: (b)

Solution:

x	y	xy	x^2
-2	-1	2	4
1	1	1	1
3	2	6	9
$\sum x = 2$	$\sum y = 2$	$\sum xy = 9$	$\sum x^2 = 14$

Now, $a = \frac{n\Sigma xy - n\Sigma x\Sigma y}{n\Sigma x^2 - \left(\Sigma x\right)^2} = \frac{3\times 9 - 2\times 2}{3\times 14 - 2^2} = \frac{23}{28} = 0.82$

$$b = \frac{\Sigma y - a\Sigma x}{n} = \frac{2-\frac{23}{28}\times 2}{3} = \frac{5}{19} = 0.26$$

$y = ax + b$

$y = 0.82x + 0.26$

Ans. 65: (c)

Solution: $V_o = AV_m\sin\omega t \Rightarrow \frac{dV_o}{dt} = AV_m\omega\cos\omega t$

$$\Rightarrow S.R = \left.\frac{dV}{dt}\right|_{\max} = AV_m\omega = A\cdot 2\pi fV_m$$

$\because 20\log_{10} A = 40 \Rightarrow A = 100$

$$\Rightarrow V_m = \frac{S.R}{A\cdot 2\pi f_m} = \frac{1/10^{-6}}{100\times 2\pi\times 20\times 10^3} = \frac{1}{4\pi} = 0.0796\ V \Rightarrow V_m = 79.6\ mV$$

Ans. 66: (a)

Solution: $\dfrac{E_6}{E_4}=\dfrac{J'(J'+1)}{J''(J''+1)}=\dfrac{6(6+1)}{4(4+1)}=\dfrac{42}{20}=2.1$

$$\Rightarrow E_6 = 2.1\times E_4 = 2.1\times 148 = 310\,keV$$

Ans. 67: (a)

Solution: $\dfrac{dP}{dT}=\dfrac{L}{T\left(V_g-V_l\right)}\approx\dfrac{L}{TV_g}=\dfrac{dP}{P}=\dfrac{(a-bT)\,dT}{R}=\left(\ln\dfrac{P}{P_c}\right)=\dfrac{\left(aT-b\dfrac{T^2}{2}\right)_{T_c}^{\frac{T_c}{2}}}{R}$

$$\Rightarrow \ln\frac{P}{P_c}=\frac{1}{R}\left(a\left(\frac{T_c}{2}-T_c\right)-b\left(\frac{T_c^2}{8}-\frac{T_c^2}{2}\right)\right)$$

$$\ln\frac{P}{P_c}=\frac{1}{R}\left(a\left(\frac{T_c}{2}-T_c\right)-b\left(\frac{T_c^2}{8}-\frac{T_c^2}{2}\right)\right)\Rightarrow\frac{1}{R}\left(-a\frac{T_c}{2}-\left(-b\frac{3T_c^2}{8}\right)\right)$$

$$\frac{1}{R}\left(\left(b\frac{3T_c^2}{8}\right)-a\frac{T_c}{2}\right)$$

$$P=P_c\exp\left(\frac{1}{R}\left(\left(b\frac{3T_c^2}{8}\right)-a\frac{T_c}{2}\right)\right)$$

Ans. 68: (c)

Solution: Branching ratio $=\dfrac{\left(\dfrac{dN}{dt}\right)_A}{\left(\dfrac{dN}{dt}\right)_\beta}=\dfrac{\left(T_{1/2}\right)_A}{\left(T_{1/2}\right)_\beta}\Rightarrow\left(T_{1/2}\right)=\dfrac{\left(T_{1/2}\right)}{B.R.}=\dfrac{100}{0.70}$

$$\Rightarrow \left(T_{1/2}\right)_\beta = 142.9 \text{ hours}$$

Ans. 69: (c)

Solution: Number of laser modes in the cavity of volume V is

$$N=8\pi V\frac{\Delta\lambda}{\lambda^4}=8\pi\left(2\times10^{-2}\right)^3\times\frac{0.5\times10^{-9}}{\left(6.893\times10^{-7}\right)^4}=4.45\times10^{11}$$

Ans. 70: (b)

Solution: $V_0=\dfrac{6.6}{0.5}=13.2\simeq13$

Binary equivalent $=1101$

Ans. 71: (a)

Solution: Taking origin at $0\cdot 65, h=0\cdot 01$ and $x=0\cdot 6538$

$$\therefore p=\frac{0\cdot 6538-0\cdot 65}{0\cdot 01}=38$$

The difference table is

x	p	$10^7 y$	$10^7\Delta y$	$10^7\Delta^2 y$	$10^7\Delta^3 y$	$10^7\Delta^4 y$	$10^7\Delta^5 y$	$10^7\Delta^6 y$
			76349					
$0\cdot 63$	-2	6270463		–955				
			75394		-4			
$0\cdot 64$	-1	6345857		–959		1		
			74435		–3		1	
$0\cdot 65$	0	6420292		–962		2		-4
			73473		-1		–3	
$0\cdot 66$	1	6493765		–963		-1		
			72510		-2			
$0\cdot 67$	2	6566275		–965				
			71545					
$0\cdot 68$	3	6637820						

Using Gauss's forward interpolation formula

$$\frac{p(p-1)}{2!}\Delta^2 y+\frac{(p+1)p(p-1)}{3!}\Delta^3 y-1+\cdots$$

$$10^7 y_0 38=6420292+(0\cdot 38)(73473)+\frac{(0\cdot 38)(0\cdot 38-1)}{2}(-962)+\cdots=6448325$$

$$\therefore y_0 38=0\cdot 6448325$$

Ans. 72: (a)

Solution: $\bar{\nu}=2B(J+1)=2B(3+1)=8B_1=83.03\,cm^{-1}$

$\Rightarrow B_1=10.38$ [For ${}^1HCl^{35}$ molecule]

For ${}^1HCl^{35}$: $\mu_1=\frac{1\times 35}{1+35}amu=\frac{35}{36}amu$

For ${}^1HCl^{37}$: $\mu_2=\frac{1\times 37}{1+37}amu=\frac{37}{38}amu$

Since $\frac{B_2}{B_1}=\frac{\mu_1}{\mu_2}\Rightarrow B_2=\frac{\mu_1}{\mu_2}\times B_1=\frac{35}{36}\times\frac{38}{37}\times 10.38$

$B_2=10.36\,cm^{-1}$

The shift in the spectral line $=8B_1-8B_2=8\left(B_1-B_2\right)$

$$=8\left(10.38-10.36\right)=0.16cm^{-1}$$

Ans. 73: (b)

Solution: Probability $\left|\langle\phi_0|\phi_1\rangle\right|^2, \phi_0=\sqrt{\frac{2}{L}}\cos\frac{\pi x}{L},\ \phi_1=\sqrt{\frac{2}{2L}}\cos\frac{\pi x}{2L}$

Since the wall of box are moved suddenly then

$$\text{Probability }=\left|\int_{-L/2}^{L/2}\sqrt{\frac{2}{L}}\cdot\sqrt{\frac{1}{L}}\frac{\cos\pi x}{L}\cdot\frac{\cos\pi x}{2L}dx\right|^2=\left|\frac{\sqrt{2}}{L}\frac{1}{2}\int_{-L/2}^{L/2}\frac{2\cos\pi x}{L}\cdot\frac{\cos\pi x}{2L}dx\right|^2$$

$$\Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\int_{-L/2}^{L/2}\left[\cos\left(\frac{3\pi x}{2L}\right)+\cos\left(\frac{\pi x}{2L}\right)\right]dx\right|^2\Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\left[\frac{2L}{3\pi}\sin\frac{3\pi x}{2L}+\frac{2L}{\pi}\sin\frac{\pi x}{2L}\right]_{-L/2}^{L/2}\right|^2$$

$$\Rightarrow\left|\frac{\sqrt{2}}{L}\cdot\frac{1}{2}\left[\frac{2L}{3\pi}\left(\sin\frac{3\pi}{4}+\sin\frac{3\pi}{4}\right)+\frac{2L}{\pi}\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{4}\right)\right]\right|^2\Rightarrow\left|\frac{2}{3\pi}+\frac{2}{\pi}\right|^2=\left|\frac{8}{3\pi}\right|^2$$

Ans. 74: (b)

Solution: (a) We know the invariant formula:

$$c^2t_{12}'^2-x_{12}'^2=c^2t_{12}^2-x_{12}^2$$

in frame K' both events occur at same point then $x_{12}'=0$ then

$$c^2t_{12}'=c^2t_{12}^2-\left(x_2-x_1\right)^2=c^2\left(t_2-t_1\right)^2-\left(x_2-x_1\right)^2$$

$$=\left(ct_2-ct_1\right)^2-\left(x_2-x_1\right)^2=\left(6-1\right)^2-\left(5-2\right)^2=16$$

$$t_{12}'^2=\frac{16}{\left(3\times10^8\right)^2}$$

$$t_{12}'=\frac{4}{3\times10^8}\cong1.3\times10^{-8}s\cong13ns$$

Ans. 75: (c)

Solution: $\frac{\partial F}{\partial M}=0\Rightarrow F=-2aM+6bM^5=0\Rightarrow\ F=2M(-a+3bM^4)=0\Rightarrow M=\pm\left(a/3b\right)^{1/4}$
