# Test Your fiziks concepts!

## Topic: Quantum Mechanics

### (For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

**Q.** The Hamiltonian for two particles with angular momentum quantum numbers $l_1 = l_2 = 1$,

is $$\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right]$$

If the operator for the total angular momentum is given by $\hat{L} = \hat{L}_1 + \hat{L}_2$, then the possible energy eigenvalues for states with $l = 2$, (where the eigenvalues of $\hat{L}^2$ are $l(l+1)\hbar^2$) are

(a) $3\in, 2\in, -\in$ (b) $6\in, 5\in, 2\in$

(c) $3\in, 2\in, \in$ (d) $-3\in, -2\in, \in$

**Ans.: (a)**

**Solution.:** $\hat{H} = \frac{\in}{\hbar^2}\left[\left(\hat{L}_1 + \hat{L}_2\right)\cdot\hat{L}_2 - \left(\hat{L}_{1z} + \hat{L}_{2z}\right)^2\right] = \frac{\in}{\hbar^2}\left[\hat{L}_1\hat{L}_2 + \hat{L}_2^2 - \left(L_{1z} + L_{2z}\right)^2\right]$

where $\hat{L} = \hat{L}_1 + \hat{L}_2 \Rightarrow \hat{L}^2 = \hat{L}_1^2 + \hat{L}_2^2 + \hat{L}_1\hat{L}_2 + \hat{L}_2\hat{L}_1 = \hat{L}_1^2 + \hat{L}_2^2 + 2\hat{L}_1\hat{L}_2 \quad \because \left[\hat{L}_1, \hat{L}_2\right] = 0$

$\Rightarrow \hat{L}_1\hat{L}_2 = \frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right)$ and $\hat{L}_z = \hat{L}_{1z} + \hat{L}_{2z}$

$$\hat{H} = \frac{\in}{\hbar^2}\left[\frac{1}{2}\left(\hat{L}^2 - \hat{L}_1^2 - \hat{L}_2^2\right) + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left[\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} - \frac{\hat{L}_2^2}{2} + \hat{L}_2^2 - \hat{L}_z^2\right] = \frac{\in}{\hbar^2}\left(\frac{\hat{L}^2}{2} - \frac{\hat{L}_1^2}{2} + \frac{\hat{L}_2^2}{2} - \hat{L}_z^2\right)$$

Now $\langle\hat{H}\rangle = \frac{\in}{\hbar^2}\left[\frac{\langle\hat{L}^2\rangle - \langle\hat{L}_1^2\rangle + \langle\hat{L}_2^2\rangle}{2} - \langle\hat{L}_z^2\rangle\right] = \frac{\in}{\hbar^2}\left[\ell(\ell+1)\hbar^2 - \ell_1(\ell_1+1)\hbar^2 + \ell_2(\ell_2+1)\hbar^2 - m_\ell^2\hbar^2\right]$

for $\ell_1 = \ell_2 = 1$ and $\ell = 2$; $\langle\hat{H}\rangle = \frac{\in}{\hbar^2}\left[\frac{6\hbar^2 - 2\hbar^2 + 2\hbar^2}{2} - m_\ell^2\hbar^2\right] = \in\left[3 - m_\ell^2\right]$

For $\ell = 2$, $m_\ell = -2, -1, 0, +1, +2$ and $m_\ell^2 = 0, 1, 4$

$\therefore$ The possible values of $\langle H\rangle$ are

$\langle H\rangle = \in[3-4] = -\in$ for $m_\ell^2 = 4$; $\langle H\rangle = \in[3-1] = 2\in$ for $m_\ell^2 = 1$

$\langle H\rangle = \in[3-0] = 3\in$ for $m_\ell^2 = 0$. Thus, correct option is (a).

**Note:**

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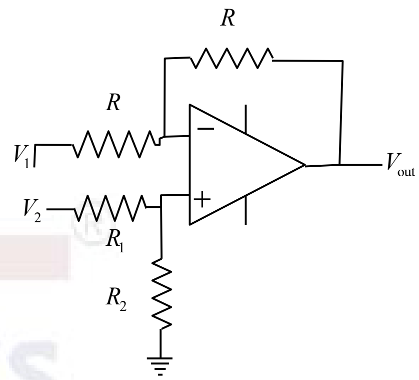
# Test Your fiziks concepts!

## Topic: Electronics

### (For IIT-JAM, JEST, TIFR and CUET Aspirants)

**Q.** In the given circuit, with an ideal op-amp for what value of $\frac{R_1}{R_2}$ the output of the amplifier $V_{\text{out}} = V_2 - V_1$?




(a) 1

(b) 1/2

(c) 2

(d) 3/2

**Ans.: (a)**

**Solution.:** From Superposition theorem

$$V_{\text{out}} = V_{01} + V_{02} = -\frac{R}{R}V_1 + \left(1 + \frac{R}{R}\right)\frac{R_2}{R_1 + R_2}V_2 = -V_1 + \frac{2R_2}{R_1 + R_2}V_2$$

$$\because V_{\text{out}} = V_2 - V_1 \text{ so } \frac{2R_2}{R_1 + R_2} = 1 \Rightarrow R_2 = R_1 \Rightarrow \frac{R_1}{R_2} = 1$$

**Note:**

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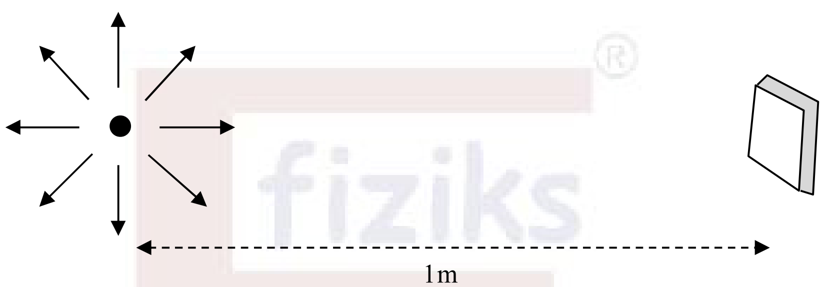
# Test Your fiziks concepts!

## Topic: Modern Physics

### (For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

**Q.** A point source emitting photon of 2 eV energy and 1 W of power is kept at a distance of $1m$ from a small piece of a photoelectric material of area $10^{-4}$ m$^2$. If the efficiency of generation of photoelectrons is 10%, then the number of photoelectrons generated are $f\times10^{12}$ per second. The value of $f$ is: (Given: $1\text{eV}=1.6\times10^{-19}\text{ J}$)




(a) 1.50 (b) 2.00 (c) 2.50 (d) 3.00

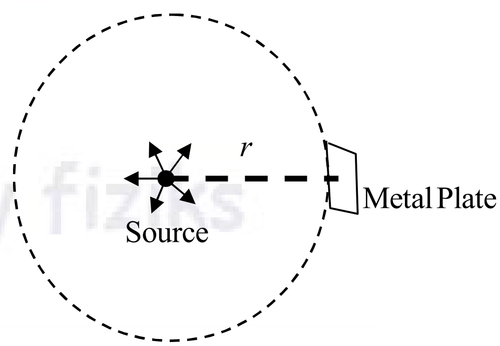
**Ans.: (c)**

**Solution.:**

Energy emitted in one second is $E=1J$.

The number of Photon emitted in one second is

$$n=\frac{E}{h\nu}=\frac{1J}{2\times1.6\times10^{-19}J}=3.125\times10^{18}\text{ photons}$$




Now, area of sphere of radius $r=4\pi r^2$

Number of photons per unit area $=\frac{n}{4\pi r^2}$

Number of photons incident on metal plate of area $10^{-4}\text{m}^2$ is $n'=\frac{n}{4\pi r^2}\times10^{-4}\text{m}^2$

$$\Rightarrow n'=\frac{3.12\times10^{18}}{4\times3.14\times1^2}\times10^{-4}=2.48\times10^{13}\text{photons}$$

Given the efficiency of generation of Photoelectron = 10%

Thus, number of photoelectrons = 10% of $n'=\frac{10}{100}\times2.48\times10^{13}=2.48\times10^{12}\Rightarrow f=2.48$

**Note:**

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# Test Your fiziks concepts!

## Topic: Quantum Mechanics

### (For CSIR NET-JRF, GATE, JEST and TIFR Aspirants)

**Q.** A quantum particle of mass $m$ is moving in a one-dimensional potential

$$V(x) = V_0\theta(x) - \lambda\delta(x),$$

where $V_0$ and $\lambda$ are positive constants, $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the Dirac delta function. The leading contribution to the reflection coefficient for the particle incident from the left with energy $E >> V_0 > \lambda$ and $\sqrt{2mE} >> \frac{V_0\hbar}{\lambda}$ is

(a) $\frac{V_0^2}{4E^2}$ (b) $\frac{V_0^2}{8E^2}$ (c) $\frac{m\lambda^2}{2E\hbar^2}$ (d) $\frac{m\lambda^2}{4E\hbar^2}$

**Ans.: (c)**

**Solution.:**

$$V(x) = V_0\theta(x) - \lambda\delta(x) \Rightarrow V(x) = \begin{cases} 0, & x < 0 \\ V_0 - \lambda\delta(x), & x \geq 0 \end{cases}$$

$$\psi(x) = \begin{cases} A\exp(ik_1x) + B\exp(-ik_1x), & x \leq 0 \\ C\exp ik_2x, & x \geq 0 \end{cases} \quad \text{where } k_1 = \sqrt{\frac{2mE}{\hbar^2}}, k_2 = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$$

The wave function is continuous $A + B = C$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (V_0 - \lambda\delta(x))\psi(x) = E\psi(x)$$

Integrating both sides: $\frac{-\hbar^2}{2m}\int_{-\delta}^{\delta}\frac{\partial^2\psi}{\partial x^2}dx + \int_{-\delta}^{\delta}(V_0 - \lambda\delta(x))\psi(x)dx = \int_{-\delta}^{\delta}E\psi(x)dx$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\delta}^{\delta} + \int_{-\delta}^{\delta}V_0\psi(x)dx - \int_{-\delta}^{\delta}\lambda\psi(x)\delta(x)dx = \int_{-\delta}^{\delta}E\psi(x)dx$$

$$-\frac{\hbar^2}{2m}\frac{d\psi}{dx}\bigg|_{-\delta}^{\delta} + 0 - \lambda\psi(0) = 0 \Rightarrow -\frac{\hbar^2}{2m}\left[Cik_2 - (Aik_1 - Bik_1)\right] - \lambda(A + B) = 0$$

$$-\frac{\hbar^2}{2m}\left[(A + B)ik_2 - (Aik_1 - Bik_1)\right] - \lambda(A + B) = 0 \qquad \because C = A + B$$

$$-\frac{\hbar^2}{2m}A\left(i(k_2 - k_1) + \frac{2m\lambda}{\hbar^2}\right) = -\frac{\hbar^2}{2m}B\left(-\frac{2m\lambda}{\hbar^2} - i(k_1 + k_2)\right) \Rightarrow \frac{B}{A} = \frac{\left(i(k_2 - k_1) + \frac{2m\lambda}{\hbar^2}\right)}{\left(-\frac{2m\lambda}{\hbar^2} - i(k_1 + k_2)\right)}$$


H.No. 40-D, G.F., Jia Sarai, Near IIT, Hauz Khas, New Delhi-16
Phone: 011-26865455/+91-9871145498, Website: www.physicsbyfiziks.com, Email: fiziks.physics@gmail.com

$$R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+(k_1-k_2)^2}{\frac{4m^2\lambda^2}{\hbar^4}+(k_1+k_2)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}-\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\left(\sqrt{\frac{2mE}{\hbar^2}}+\sqrt{\frac{2m(E-V_0)}{\hbar^2}}\right)^2}$$

$$\Rightarrow R=\left|\frac{B}{A}\right|^2=\frac{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1-\sqrt{1-\frac{V_0}{E}}\right)^2}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{2mE}{\hbar^2}\left(1+\sqrt{1-\frac{V_0}{E}}\right)^2}=\frac{\frac{4m^2\lambda^2}{\hbar^4}}{\frac{4m^2\lambda^2}{\hbar^4}+\frac{8mE}{\hbar^2}}=\frac{1}{1+\frac{2E\hbar^2}{m\lambda^2}}=\frac{m\lambda^2}{2E\hbar^2}$$

Since $E>>V_0>\lambda$ and $\sqrt{2mE}>>\frac{V_0\hbar}{\lambda}$.

**Note:**

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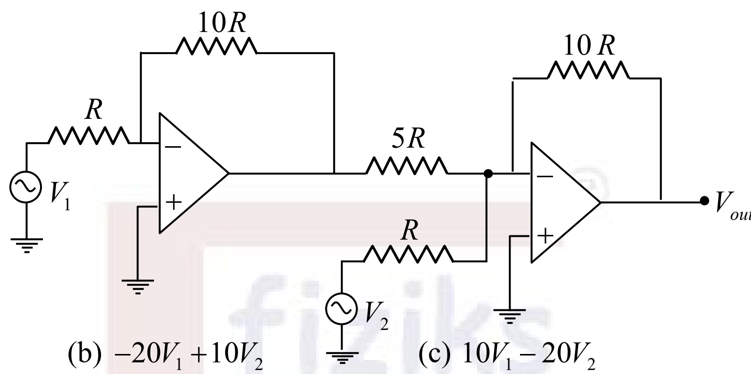
# Test Your fiziks concepts!

## Topic: Electronics

### (For IIT-JAM, JEST, TIFR and CUET Aspirants)

**Q.** In the circuit shown in the figure, both OPAMPs are ideal. The output for the circuit $V_{out}$ is




(a) $20V_1 + 10V_2$ (b) $-20V_1 + 10V_2$ (c) $10V_1 - 20V_2$ (d) $20V_1 - 10V_2$

**Ans.: (d)**

**Solution.:** Output of first op-amp is $V_{01} = -\frac{10R}{R}V_1 = -10V_1$.

Thus $V_{out} = -\frac{10R}{5R}(-10V_1) + \left(-\frac{10R}{R}\right)V_2 \Rightarrow V_{out} = 20V_1 - 10V_2$

**Note:**

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# Test Your fiziks concepts!

## Topic: Modern Physics

### (For PGT: KVS, NVS, DSSSB, State Education Boards, etc.)

**Q.** Consider the $\alpha$-decay ${}^{90}Th^{232} \rightarrow {}^{88}Ra^{228}$. In an experiment with one gram of ${}^{90}Th^{232}$, the average count rate (integrated over the entire volume) measured by the $\alpha$-detector is 3000 counts $s^{-1}$. If the half life of ${}^{90}Th^{232}$ is given as $4.4\times10^{17}$ s, then the efficiency of the $\alpha$-detector is: Given: Avogadro's number = $6.023 \times 10^{23}$ mol$^{-1}$

(a) 0.53 (b) 0.63 (c) 0.73 (d) 0.83

**Ans.: (c)**

**Solution.:**

**Solution.:** Actual decay rate $\left|\frac{dN}{dt}\right| = N\lambda = \left(\frac{1\,gm}{232}\times 6.02\times10^{23}\right)\times\frac{0.693}{4.4\times10^{17}} = 4086\,s^{-1}$

Measured decay rate $= 3000\,s^{-1}$. So, efficiency of $\alpha$-detector $= \frac{3000}{4086} = 0.73$

**Note:**

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